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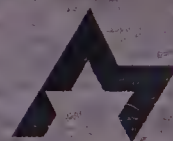
TEACHER'S EDITION
HOLT MATHEMATICS SYSTEM

BOOK 6



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Teacher's Edition

Holt

Mathematics

System

Book Six

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Mathematics System*

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TORONTO

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SCOPE AND SEQUENCE

The following chart gives an overview of mathematical content presented at this grade level and the grade levels below. It shows the extent to which each mathematical topic is presented from grade to grade in

the HOLT MATHEMATICS SYSTEM. It is hoped that this will help you to better prepare your lessons for a successful teaching experience.

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TEACHER'S COMMENTARY

THE HOLT MATHEMATICS SYSTEM

Two fundamental premises which underlie an effective learning system are:

1. Motivation is an important prerequisite to learning.
2. Each student may require a different motivational technique.

The HOLT MATHEMATICS SYSTEM (HMS) was developed to incorporate these premises. The various components of HMS provide a variety of motivational and instructional devices with which to reach students.

Teachers can present opportunities for learning through a medium best suited to the particular abilities and learning style of the individual student. The components of HMS are:

Readiness

Student's Book
Teacher's Edition

Grade 1

Student's Book
Teacher's Edition
Activity Masters
BFA Computational Skills Kit I

Grade 2

Student's Book
Teacher's Edition
Activity Masters
BFA Computational Skills Kit I
BFA Math Problem Solving I

Grade 3

Student's Book
Teacher's Edition
BFA Computational Skills Kit I
BFA Math Problem Solving I
Duplicating Masters

Grade 4

Student's Book
Teacher's Edition
BFA Computational Skills Kit II
BFA Math Problem Solving II
Duplicating Masters
Calculator Activity Masters

Grade 5

Student's Book
Teacher's Edition
BFA Computational Skills Kit II
BFA Math Problem Solving II
Duplicating Masters
Calculator Activity Masters

Grade 6

Student's Book
Teacher's Edition
BFA Computational Skills Kit II
BFA Math Problem Solving II
Duplicating Masters
Calculator Activity Masters

TEACHING STRATEGIES

There is no one best way to teach mathematics to all students. Therefore, the HOLT MATHEMATICS SYSTEM is adaptable to many teaching styles.

Four different ways in which the program can be taught are:

- Teach the whole class together; have all students work on the same material at the same time, with the same written assignments.
- Teach the whole class together; have all students work on the same material at the same time, but differentiate the written assignments.
- Group the children and teach each group separately; each group may be doing different lessons on the same day.
- Use a continuous progress approach, letting each student progress at his or her own rate; every student could be working on a different lesson on any particular day. Suggestions in the *Teacher's Edition* provide assistance in differentiating the assignments.

THE STUDENT'S BOOK

Language

While it is important to teach students to become better readers, even within the framework of mathematics instruction, a deficiency in reading should not stand in the way of learning mathematics. The language used in the *Student's books* is uncomplicated and concepts are presented by means of illustrations or examples rather than by long verbal explanations.

Clear, concise titles, which appear at the top of every lesson, serve as a general description of content. A further note outlining the specific page object and subject matter is included at the bottom of every page.

Content

The major primary grade concepts which appear throughout HMS are as follows:

Number and Numeration
Operations and Properties
Sentences
Problem Solving and Application
Geometry
Measurement
Graphs, Tables

Since students do not fully master any concept on the first encounter, this program of studies is presented in three stages:

- 1st — a thorough introduction
- 2nd — reinforcement and mastery
- 3rd — maintenance and extension.

Understanding is consistently reinforced as students bring their growing knowledge and maturity to bear upon more abstract concepts and more difficult skills.

Developmental Aspects of Lessons

The HOLT MATHEMATICS SYSTEM is “developmental” in that each lesson is sequenced to proceed from an initial activity, through a learning stage, and finally to practice exercises. This lesson style (**display**, **development**, and **drill**) was adopted to give children an understanding of the concepts in the lessons through active participation in the development of the concepts or skills, followed by practice in the use of these concepts.

Each lesson follows a definite pattern: (1) **display** — an *initial activity* where hands-on materials such as blocks are used to teach the concept; the initial activity appears in the side column of the *Teacher's Edition* for every lesson. (2) **development** — a *learning stage* which uses pictures and other visual hints to develop the concept; the learning stage is the first part of the Exercises where the child is guided, through the use of coloured numerals or other hints, to the pattern of response. To provide immediate reinforcement, the answer to some of the developmental items, indicated by circles, are placed in the back of the *Student's book*. (3) **drill** — *exercises* which drill the concept or skill presented in the display; the exercises are the items which the children do individually to demonstrate understanding.

Basic Skills

HMS embodies the philosophy that it is important for the student to develop a concept or skill meaningfully. However, a meaningful development needs to be followed by practice. To become a proficient user of mathematics one has to practise the skills that have been acquired. Accordingly, ample practice is provided to diagnose areas of difficulty and to maintain skills.

Throughout the program, a large number of computational exercises are presented with over 4000 in each book, Grades 2 to 6. Additional exercises may be found in the other HMS components.

Exercises which extend the developmental items presented in the lesson are starred. Additional challenges are provided by the **Braintickers** which appear throughout the book.

HMS repeatedly asks students to discover patterns. These experiences are intended to help build a sense of relationship between numbers and to develop self-reliance when a problem is tackled. Discovering patterns is an enjoyable activity since it is usually accompanied by a sense of anticipation.

Diagnosing

Diagnostic materials appear throughout HMS in a variety of formats. The **Basic Skills Check Ups**, which are part of HMS from Readiness to Grade 6, provide experience in the type of format that is often used on standardized tests.

Cumulative Reviews in Grades 3 to 6 enable the teacher to pinpoint areas of the child's achievement or deficiency prior to studying other concepts in the text. Based on the results, one can determine what combination of learning experiences will best help each child.

All **Check Ups** and **Cumulative Reviews** are keyed in the *Teacher's Edition* to the pages on which the skills were presented.

The **Chapter Test** at the end of each chapter can also be used diagnostically. Each exercise in the test is referenced in the *Teacher's Edition* to a specific objective and page number. This provision enables the teacher to review specific concepts and skills needing improvement.

A **Cumulative Test Item Bank**, located at the front of each *Teacher's Edition* (Grades 1 to 6), covers a representative sampling of the entire year's objectives. There are two matching items per objective in the Grades 3 to 6 tests, thereby offering the opportunity for repeat testing. The **Cumulative Test** for Grades 1 and 2 is presented in a multiple-choice type format.

Maintaining Skills

Keeping acquired skills sharp is important to HMS. Maintenance pages therefore appear regularly throughout the pupil books. **Keeping Fit** (Grades 1 and 2) and **Tune Up** (Grades 3 to 6) are mixed frequently with stimulating **Practice** pages to help reinforce specific facts and concepts.

At the end of each book are collected additional **Extra Practice** exercises, keyed to the appropriate pages in the *Student's book*.

Additional and supplementary practice material in the form of duplicating masters, computational and problem solving skills kits, and calculator activity masters are also available and are keyed to the program.

Testing

A complete testing program for monitoring students' progress is provided within HMS. **Chapter Tests** (called “Think” in Grades 1 and 2) at the end of each chapter are designed to help evaluate the extent of mastery of the essential chapter content. In the *Teacher's Edition*, test items are referenced to pupil page numbers and specific objectives which are listed for each chapter. An invaluable measuring device, the **Chapter Tests** allow teachers to diagnose the particular strengths and weaknesses of each student.

Additional test materials and supplementary tests are available in the form of *duplicating masters* (Grades 3 to 6).

Measurement

The measurement section in HMS is completely SI metric. The strand features a “hands-on” approach to measurement augmented by written exercises which further broaden and reinforce the concepts.

Often a page of nonstandard, informal units of measure will help introduce the activities and exercises developing the established measurement system.

Problem Solving

Solving problems is one of the major strands in HMS. We start problem solving early and use it as a tool for reinforcing basic facts.

The development of problem-solving skills is very gradual. It is based essentially on (1) interpreting action pictures, (2) joining and separating sets of objects, and (3) solving word problems without pictures. Word problems are first presented in the form of mini-problems which contain a picture dictionary to help children read the printed words. Mini-problems contain only those words which are necessary for an understanding of the problems.

From these careful beginnings, problem solving in HMS is expanded to include:

- reading word problems
- choosing a number sentence to fit a word problem
- writing number sentences
- estimating the answer
- recognizing extraneous and insufficient information
- reading information from tables or graphs
- choosing the correct operation
- solving word problems related to careers
- multiple-step problems
- drawing pictures to help solve problems
- establishing and using a problem-solving technique

to mention a few.

The challenge of providing children with an opportunity to grow systematically in the ability to solve problems is met by incorporating word problems into many of the lessons throughout the program.

In addition to this rich and systematic program of developing problem-solving skills, HMS includes a lesson on problem solving related to a selected career in each chapter. This career strand is of considerable importance to this program and is therefore discussed in detail.

Career Strand

Making children aware of existing careers makes mathematics learning relevant, realistic, and motivational.

Specifically, the purpose of the HMS career strand is twofold:

- To stimulate career awareness in children by presenting them with problems which deal with situations related to various careers, and
- To provide teachers with the essential information about various careers.

Appropriate learning experiences or activities are provided in the *Teacher's Edition* for lessons dealing with careers. These lessons are adapted to the developmental level of the child.

The careers are chosen to focus on certain specific objectives:

- To help the child develop an awareness of physical skills
- To develop an awareness of self and others
- To help students develop self-reliance
- To develop an awareness of a multitude of careers
- To develop social awareness

Each chapter highlights one or more specific careers and provides the opportunity to discuss others.

Activity Pages

Interspersed throughout the texts are **activity** pages. **Activity** pages provide motivation and active participation on the part of the child. Concepts are developed or practised through the use of activities.

Chapter Themes

In an effort to motivate children and to interrelate mathematics to other subject areas, some chapters in these books are oriented toward certain themes. For example, the theme of a chapter may pertain exclusively to transportation, fairy tales, the sea, the farm, the community, the circus, foreign lands, and others. The illustrations in these chapters emphasize the theme of the chapter. These themes are identified in the **Chapter Overviews**.

THE TEACHER'S EDITION

The *Teacher's Edition* is the key to using HMS. All references to components of this program, as they apply to each lesson, are provided literally at one's fingertips. With this type of manual, the teacher can easily direct children to other practice materials, guide them to activities, and provide them with projects that will extend their mathematical horizons.

The layout for each individual lesson contains a reduced version of the pupil page with answers superimposed. Surrounding this are the related lesson commentaries closely positioned to allow quick and easy access.

Front of Teacher's Edition

A **Scope and Sequence** chart displays three years of topics and the extent to which they are presented in HMS. Using the chart, it is easy to tell, at a glance, where any particular lesson falls in the flow and scheme of the whole Mathematics System.

An **Activity Reservoir** section, consisting of mathematical games and activities, provides a framework for enjoyable practice work throughout the year. These games and activities are keyed into individual lessons, but each may be adapted and used at the discretion of the teacher.

A **Problem of the Week** section consists of challenging mathematical puzzles and problems. These are for additional motivation. They can be offered to children via the bulletin board or a special problem box.

A **Cumulative Test Item Bank** is supplied for the evaluation of the children's achievement with respect to part or all of the entire year's work. Pupil edition page references which are located along the margins of this section, allow the teacher to select items which test the appropriate desired objective. This format also allows the teacher to test on a regular basis or periodically spot check, as the particular situation may require.

Chapter Overview

Chapter Overviews are appropriately interleaved before each chapter. Each overview consists of the following parts:

An **Introduction** explains what content is to be studied in the chapter.

Objectives for the chapter are stated in behavioural terms.

Background provides a meaningful setting for the mathematical concepts and skills taught in the chapter.

Materials lists the materials that are suggested for use in teaching the lessons.

Career Awareness describes the career to be studied in that chapter and provides background information for discussion. When necessary a caption is given for the photograph illustrating the career (Grades 1 and 2).

Lesson Commentaries

Daily lesson commentaries generally provide varied approaches to teaching the lessons. Each lesson commentary, in the side columns of the *Teacher's Edition*, contains the following categories:

Objectives for each lesson are stated in behavioural terms. These objectives state very specifically what a child ought to be able to do at the end of the lesson.

Pacing suggestions are provided for some lessons to indicate how assignments may be differentiated.

Level A: a minimum course

Level B: an average course

Level C: an extended course

Vocabulary lists new words and terms introduced in the lesson.

Materials lists stationary supplies, props, demonstration aids helpful for teaching the lesson.

Related Aids keys the appropriate supplementary components of the program to the particular lesson.

Background provides a meaningful setting for mathematical concepts on which the lesson is based.

Suggestions usually provide readiness-type learning experiences which encourage children's involvement. This section contains the **Initial Activity** comments that may be completed before using the pupil page.

Using the Book provides specific teaching instructions for the lesson.

Activities provide varied learning experiences such as mathematical games, research projects, experiments, and so on that represent additional practice, enrichment, or extension. The activities are usually listed in order of difficulty.

Extra Practice supplies additional exercises which are appropriate to the content of the particular page. The assignment of these **Extra Practice** exercises, whether they be used orally as review preceding the next page, placed on cards or chalkboards as remedial exercises or perhaps for fast finishers, is of course left up to the discretion of the teacher.

SUPPLEMENTARY MATERIALS

Duplicating Masters provide extra practice for selected lessons, graph paper, dot paper, cutouts for activities, nets for geometry and additional chapter tests.

BFA Computational Skills Kits I (Grades 1 to 3) and *II* (Grades 4 to 6) provide a diagnostic/prescriptive program for both instruction and practice. Simple placement tests help identify each child's level.

BFA Math Problem Solving I and *II* offer instruction and practice in solving math story problems. The kits are organized into five sections: Addition/Subtraction; Multiplication/Division; Application; Review; Enrichment.

Calculator Activity Masters provides an opportunity to use the calculator to explore topics formerly thought to be too difficult and tedious. The exercises are related directly to the concepts developed in the *HMS Student's book*. Emphasis is on generalizing concepts, using larger numbers, and extension of concepts.

ACTIVITY RESERVOIR

BINGO

Use: To practise basic facts, arithmetic operations, mental computations, and naming geometric figures and shapes.

Materials: Blank game sheets, counters, paper and pencils for computations (if necessary, problem cards (optional))



Players: Any number

The Game: Distribute the blank game sheets. Have the players randomly select and write in numerals and/or words from a predetermined, displayed set of answers (e.g., numerals from 0 to 35; fractions from $\frac{1}{2}$ to $\frac{7}{7}$ inclusive; decimals from 0.0 to 2.9 inclusive; even dollar amounts from \$10 to \$40; names of plane and 3D shapes). Circulate to be sure that (a) all squares on the gamesheet are used and (b) no one number, word, shape, etc. has been repeated.

Call out or display a problem (e.g., 4×8 ; $\frac{4}{6} - \frac{3}{6}$; $\$2.25 + \3.75 , etc.) and have the players cover that number if it appears *anywhere* on their sheet. (Using “under the ‘B’” and “under the ‘G’” as in regular Bingo makes for too drawn out a game.)

There are various ways of declaring a winner:

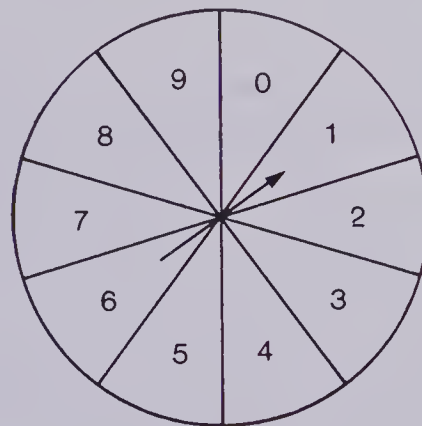
- (a) five markers in a row vertically, horizontally, or diagonally.
- (b) four corners covered.
- (c) the whole sheet covered.

Emphasize that the first person to cover the required squares and raise her/his hand is the winner. Variations (a) and (b) are quickly completed. You may wish to put the call-out problems on cards and reward the winners by having them as the “caller” for the next round.

THE P.V. GAME

Use: To practise place-value skills involving comparison of numbers

Materials: Construct a spinner using the digits 0 to 9. (Blank dice or number cards can also be used.)



Players: Any number (or two teams)

The Game: On a piece of paper, each player draws three blanks: ____ ____ ____ A designated player (or caller) twirls the spinner and calls out the indicated digit. The players may place this digit in any one of their three blank spaces. The caller repeats this procedure two more times. Each player uses the digits to build a “personal number”. The player with the greatest number is the winner (or, the team to which the winner of the round belongs scores one point. The first team to score five points is the overall winner.). Students will soon develop strategies for placing the digits in the most potentially favourable positions.

Variations: 1. Use 4 or 5 blanks to extend the game to thousands and ten thousands.

2. Play the same game for least number.

3. Practise comparing decimal numbers by using 4 to 6 blanks arranged for 2 decimal places.

4. Practise adding, subtracting, and comparing the answers by having the players draw sets of blanks for addition and subtraction.

$$\begin{array}{r} \text{---} \text{---} \text{---} \\ + \text{---} \text{---} \text{---} \\ \hline \end{array} \qquad \begin{array}{r} \text{---} \text{---} \text{---} \\ - \text{---} \text{---} \text{---} \\ \hline \end{array}$$

The numbers obtained by spinning are placed in the blanks. The player with the greatest sum (or difference) wins.

FRACTION CARDS

Use: To practise equivalence and comparison of fractions

Materials: 2 sets of playing cards. Set A should show:

0, 1, $\frac{0}{2}$, $\frac{1}{2}$, $\frac{2}{2}$, $\frac{0}{4}$, $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, $\frac{4}{4}$, $\frac{0}{8}$, $\frac{1}{8}$, ..., $\frac{8}{8}$, $\frac{0}{16}$, $\frac{1}{16}$, ..., $\frac{16}{16}$.

Set B (a different colour) should show: 0, 1, $\frac{0}{3}$, $\frac{1}{3}$, $\frac{2}{3}$,

$\frac{3}{3}$, $\frac{0}{6}$, $\frac{1}{6}$, ..., $\frac{6}{6}$, $\frac{0}{9}$, $\frac{1}{9}$, ..., $\frac{9}{9}$, $\frac{0}{12}$, $\frac{1}{12}$, ..., $\frac{12}{12}$.

Players: Two or more

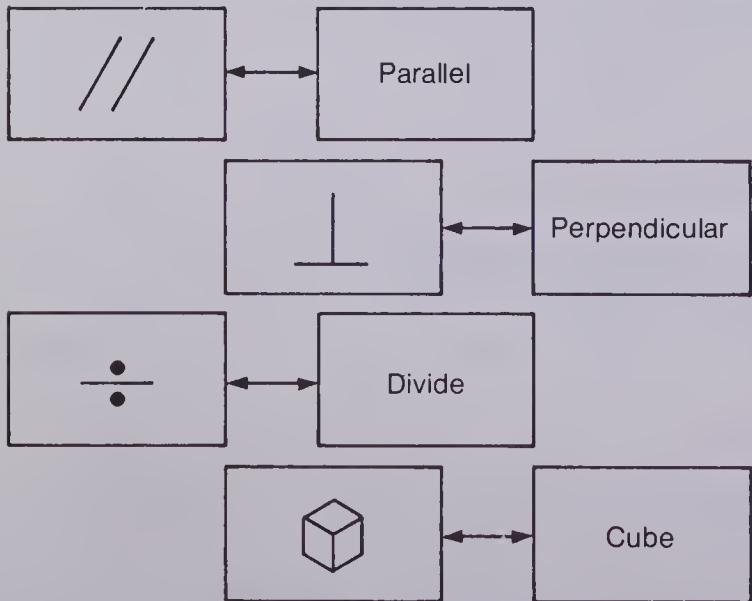
The Games: Card decks as described can be used to play many ordering and matching games, three of which are as follows.

Fraction Fishing. Cards are shuffled and scattered face down on a desk. Players simultaneously draw a card from the collection. The player with the greatest fraction wins that set. If the cards are equal, the first player to say "equal" wins that set. The player with the most sets when the card collection is depleted is the overall winner.

Think Quick. Shuffle and scatter the cards as for "Fraction Fishing". Provide a series of instruction cards such as: "Choose any 6 cards. Put them in order, smallest to largest"; "Choose 2 cards. Write any fraction that would come between"; "Choose any card. Write an equivalent fraction for it"; "Choose any card. Add $\frac{1}{2}$ to the fraction shown. Write the sum"; "Choose any card. Subtract it from 1. Write the difference"; etc. Players read and race to complete the instruction. The first one to correctly complete the task wins the round. First player to win 5 times is the overall winner.

Fraction Snap. Shuffle the cards and divide the deck between two players. The players simultaneously flip a card face up on table. First player to identify equivalence, when it occurs, wins all cards that have been played. Player with the most cards after a predetermined amount of time is the overall winner.

Variation: The cards described above can be used as the basis for an "equivalence deck" with which to play "Snatch and Match". Select a number of equivalent fraction cards and augment the deck to yield about 50 pairs with cards such as these.



Deal all cards to players (about 6 to 12). Each player begins by matching and discarding any matching cards they hold. The dealer then "snatches" 1 card (unseen) from the player to the immediate left and "matches" it to a card in hand, if possible. This "Snatch and Match" process continues around the circle of players, till someone matches the last card in his/her hand. That player is the winner.

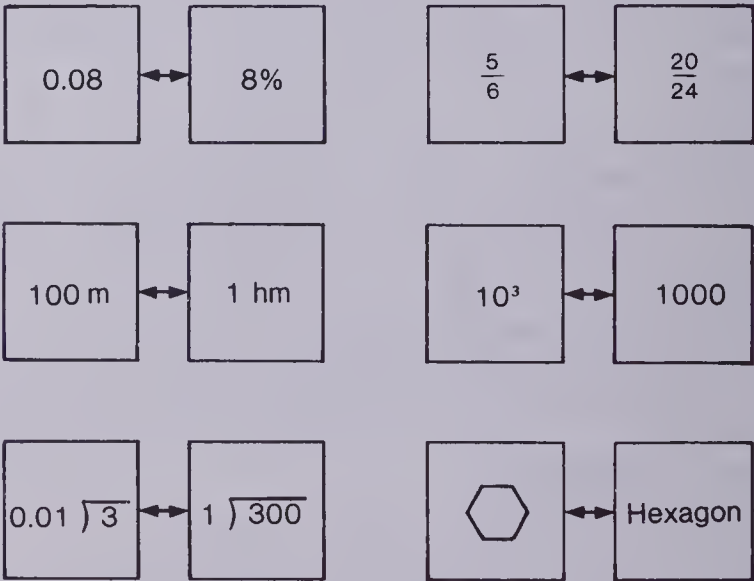
CONCENTRATION

Use: To practise basic facts and recognition, renaming, and matching skills

Materials: Index cards

Players: From two to four

The Game: Prepare 10 to 15 pairs (20 to 30 cards in all) of appropriate facts cards. Shuffle the cards and place them face down on a desk in a regular array. To begin, the first player turns over a card and names the figure (or word) which appears. The same player then turns over and names a second card. If the two cards match, that player scores one point, removes these cards from the array, and takes another turn. If they do not match, both cards are turned face down and the next player proceeds. Students must "concentrate" to recognize and remember the various card locations. The player with the most points after all cards have been paired is the winner. Some sample card pairs are shown.



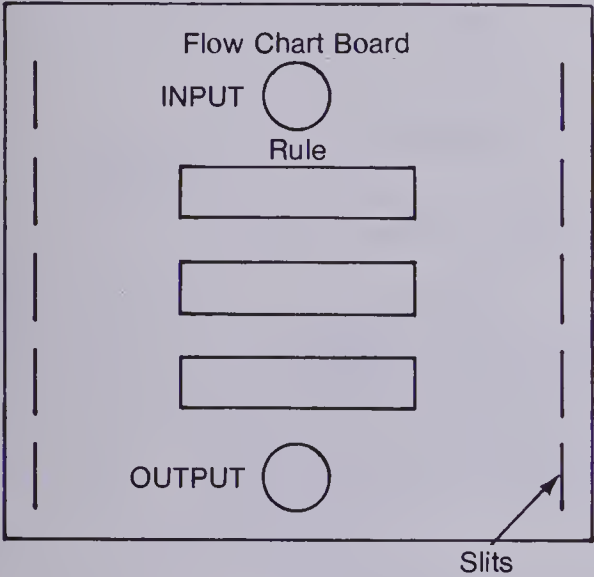
Variation: For an added challenge, include a third card in each set to play "Triple Concentration". Triple sets are easy to make for (a) equivalent fractions; (b) fractions, decimals, pictures of fractional parts; (c) measurement unit relationships (i.e., 2 m, 200 cm, 2000 mm); (d) number expressions (i.e., 15, 5 x 3, 10 + 5); (e) characteristics of shapes (i.e., no flat faces, sphere, no corners).

INPUT-OUTPUT

Use: To practise mental computation

Materials: Tagboard flow chart, number strips appropriate for different skills, blank strips for "output"

Players: Two or more



Sample Input Strips

0.5	3.0	4.5	15
50	\$75	8.1	6.00

Note blank leader space.

Sample Rule Strips

$\times 100$	$\times 0.01$	$\times 1.0$
$\div 10$	$\div 0.1$	$\div 0.5$
$+\$6.50$	-3.05	$\times 0.8$

Use blank strips for "output".

The Game: Student A adjusts the strips so that an "input" and one or more "rules" are presented in the flow chart. Student B is to calculate the "output". Student A checks B's answer. (The use of a calculator to check will add zest to the game.) The two players then reverse roles. There is one point for each correct "output". The player with the most points after 10 turns each is the winner.

- Variations:**
1. Strips can be designed
 - (a) to check equivalence of decimals, fractions, and percents. The output is YES or NO.
 - (b) for which common factors (or multiples) can be calculated.
 - (c) which show fractions. The rule strip could read: "Write the common denominator.", "Write the least common denominator.", "Reduce the fraction." etc.
 2. Instead of scoring each correct output, time each response and keep a record of response times. The player with the lowest time total after 5 turns is the overall winner.

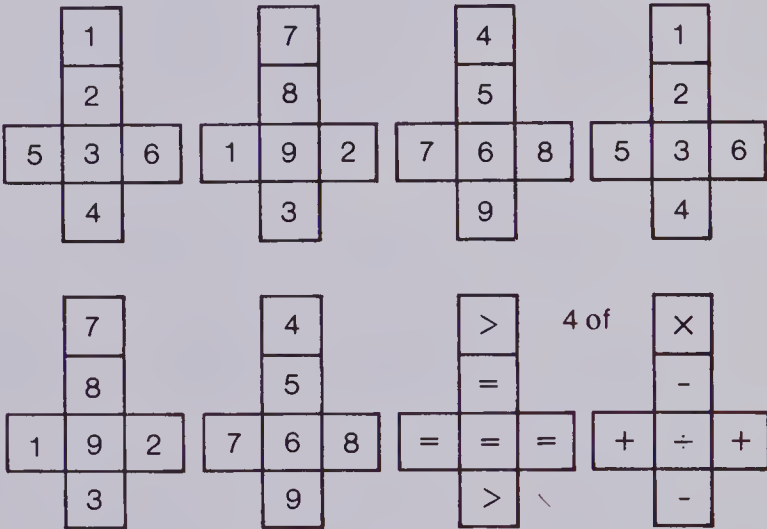
NUMBER SENTENCE

Use: To practise making equations, and to identify equations and inequations

Materials: 11 dice (nets for which are shown)

Players: Two to four

The Game: Players take turns rolling all 11 dice. Each player tries to make an equation using as many dice as possible. Each die used scores one point. The player with the highest total after 7 turns is the winner. (A player who is forced to make an inequation scores an automatic 2 points.) The numbers and symbols on the dice are as shown.



- Variations:** The game can be further complicated by:
- (a) having each player achieve a score equal to the value expressed by the equations (e.g., $55 \div 5 = 11 \times 1$ scores 11, not 9) thereby presenting the need to choose between possible equations.
 - (b) allowing anyone who identifies an error to correct it and score instead.
 - (c) allowing the use of parentheses to increase a potential equation value.

PRO(BABILITY) GOLF

Use: To practise number theory principles
Materials: A course card, score sheets, 2 regular dice
Players: From two to four

The Game: Players take turns rolling both dice till they get what is called for at each hole. Each roll of the dice counts as one “stroke”. The number of “strokes” for each hole should be recorded on the score sheet. At the end of nine holes, the player with the *lowest* score wins.

Course Card		
Hole	Par	On the dice:
1	3	Sum is a prime number.
2	2	Sum is an even number.
3	4	Sum is divisible by 4.
4	3	Both dice show odd numbers.
5	4	Sum is a multiple of 3.
6	4	Sum > 10.
7	5	Sum < 3.
8	3	Both dice show composite numbers.
9	4	A six shows on one or both dice.
Total→32		

Score Sheets

Names: A _____ C _____
B _____ D _____

Hole	Par	Scores:			
		A	B	C	D
1	3				
2	2				
3	4				
4	3				
5	4				
6	4				
7	5				
8	3				
9	4				
Total	32				

Variations: 1. Alter the conditions listed under “On the dice” on the Course Card to suit the topics being studied.
2. Change the numbers on the dice to make the computations more difficult.

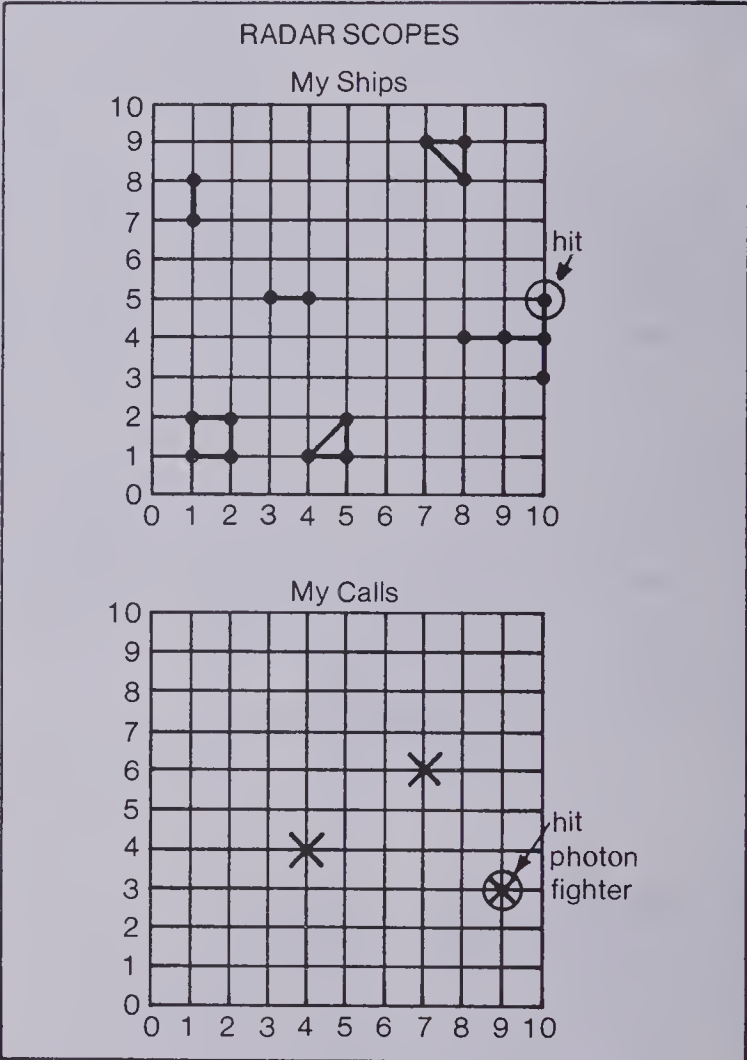
RADAR SCOPES

Use: To practise locating and graphing ordered pairs
Materials: Graph paper (HMS — DM69)
Players: Two

The Game: Each player labels a Radar Scope with axis numbers and the following types of ships:

- 2 photon fighters
- 2 star cruisers
- 1 gyro lab
- 1 star base

Sample



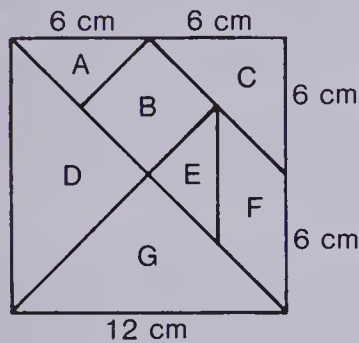
The ships may be plotted in any attitude anywhere on the plane (sample shows play after 3 turns).
Players (a) take turns calling ordered pairs in an attempt to name occupied points on the opponent’s unseen grid, (b) keep a record of calls and hits made and received, (c) call “partial hit on (*photon fighter*)” if one of their points is named by the opponent, (d) call “(*photon fighter*) disintegrated” when all points in a particular ship have been identified. The winner is the first player to name all of the opponent’s 19 points.

Variations: 1. To simplify the game (a) reduce the size of the grid, (b) reduce the number of ships, (c) simplify the shape of the ships.
2. To complicate the game (a) increase the size of the grid and the number and/or shape of ships, (b) increase the number of quadrants in the plane and use integers.

TANGRAMS

Use: To practise making and duplicating patterns; calculating perimeter and area

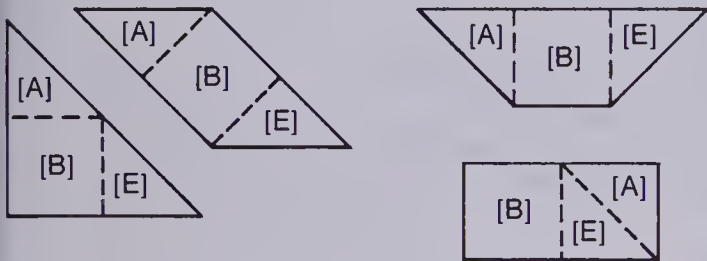
Materials: Tangram shapes made from heavy cardboard (as shown)



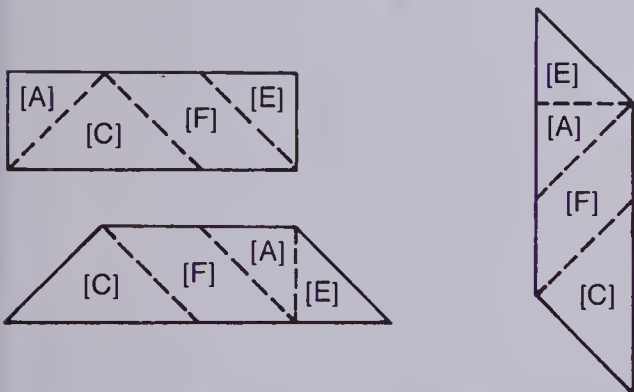
Players: Any number

The Game: Provide the players with some drawn-to-scale outlines, samples of which are shown below. Have them (a) use their tangram pieces to cover each outline exactly and (b) draw in the outline of each piece to show where it fits. (Dotted lines and square brackets indicate answers.)

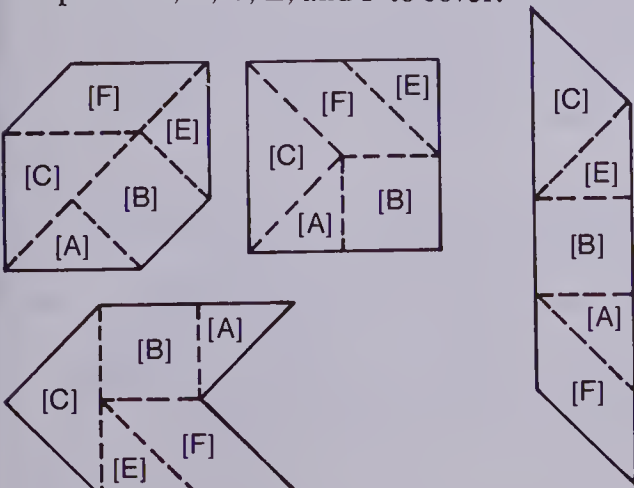
Use pieces A, B, and E to cover:



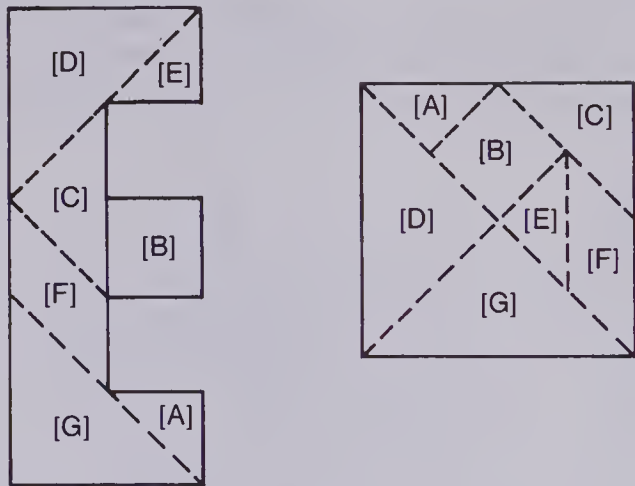
Use pieces A, C, E, and F to cover:



Use pieces A, B, C, E, and F to cover:



Use all 7 pieces to cover:



Variations: 1. There are many more outlines which can be prepared and distributed at varying levels of difficulty.

2. Have the children prepare their own outlines for exchange with classmates.

3. Have the tangram pieces prepared on centimetre grid paper. Have the children calculate the area of some of the outlines and/or present the following challenges:

- (i) Measure and list the perimeter of each Tangram piece.
- (ii) Arrange the seven Tangram pieces in order by area from smallest to largest (rounded to nearest square centimetre). List the order.
[A, E; B, C, F; D, G]
- (iii) What do you notice? [A = E, B = C = F; D = G]
- (iv) Cover D with smaller pieces. Which pieces cover it exactly? [A, B, E or A, E, F]
- (v) Use all 7 pieces to make a square.
What is the perimeter? [48 cm]
What is the area? [144 cm²]
What fraction of the total area is D? [$\frac{1}{4}$]

What is the area of D?
[$\frac{1}{4} \times 144 = 36 \text{ cm}^2$]
Complete this chart.

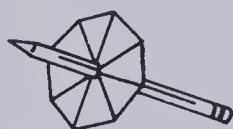
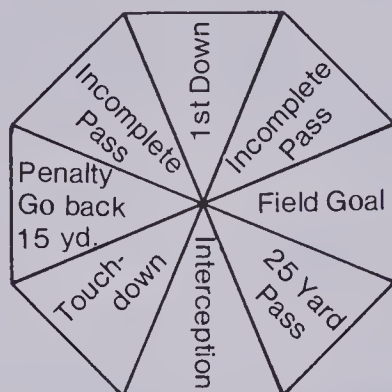
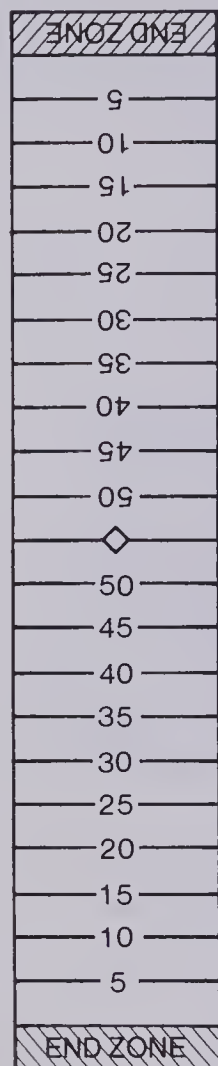
Piece	Fraction of Total Area	Area of Piece
A	$[\frac{1}{16}]$	[9 cm ²]
B	$[\frac{1}{8}]$	[18 cm ²]
C	$[\frac{1}{8}]$	[18 cm ²]
D	$\frac{1}{4}$	36 cm ²
E	$[\frac{1}{16}]$	[9 cm ²]
F	$[\frac{1}{8}]$	[18 cm ²]
G	$[\frac{1}{4}]$	[36 cm ²]

- (vi) Pieces B, C, and F have equal areas. What about their perimeters?
- 4. Have the children calculate the area of some of the outlines shown under "The Game".

FOOTBALL

Use: To practise operation skills, problem solving, etc.

Materials: Playing board, spinner, question cards, coin or marker for use as a ball



Place pencil through spinner.

$\frac{5}{6} = \frac{\blacksquare}{24}$	$0.6 \times 0.3 = \blacksquare$
$1 \text{ m} = \blacksquare \text{ dm}$	$18\% = \frac{\blacksquare}{50}$

Sample Question Cards

Players: Two or more

The Game: The question cards are to reflect the skill or topic to be developed. Two teams are formed. The ball is placed on the centre line. A coin is flipped to determine which team plays first. Player(s) on the starting team take turns flipping the spinner. If it stops on 1st down, 25 yard pass, field goal, or touch-down, the player draws a question card, solves it, and moves according to this schedule:

- 1st down: advance ball 10 yards
- 25 yard pass: advance ball 25 yards
- field goal: score 3 points, place ball on the opposing 35 yard line. Opposing team now plays.
- touch-down: score 6 points and draw an extra card for convert point. When complete, place ball on centre line and allow opposing team to play.

If a touch-down is made on a 1st down or a 25 yard pass, follow instructions as for touch-down. If spinner lands on Interception or Incomplete Pass the opposing team spins. If spinner lands on Penalty, the ball is moved back 15 yards and the same team takes another turn.

For an incorrect answer to a problem, the ball is moved 15 yards back and the same team takes another turn.

ERASER

Use: To practise various skills

Materials: Chalkboard (or overhead projector), problem cards



$16.47 \rightarrow ?$
nearest tenth

$$\frac{3}{4} = 0.\blacksquare\blacksquare$$

Name a
prime number.

$$1\frac{1}{2} + 1\frac{1}{2} = \blacksquare$$

Players: Two (or two teams)

The Game: Draw stickpersons on the chalkboard (or on an overhead transparency) so that there is one stick-person per player or team. Players or members of each team take turns selecting and solving problem cards. For each error, the opponent is entitled to erase one part of the stickperson. The object is to solve the 16 problems, keeping as much of the stickperson visible as possible.

Variations: The game can be made more simple or difficult by adjusting the problem cards as necessary and/or changing the number of erasable parts on the stickperson.

SQUARE IT

Use: To practise computation skills
To practise strategy and logic

Materials: Duplicated playing sheets and pencils (or laminated sheets and water soluble markers)

•	•	•	•	•	•
1	3	12	6	15	
8	4	1	3	1	•
•	•	•	•	•	•
4	20	8	1	5	
•	•	•	•	•	•
16	2	1	25	9	
•	•	•	•	•	•
5	6	2	20	4	
•	•	•	•	•	•

Players: Two to four

The Game: Players take turns drawing 1 line either horizontally or vertically between adjacent dots. The player who draws the fourth side to any square gets the number of points shown in that square. When all squares are complete, the winner is the player with the greatest point total.

Variations: Varying degrees of difficulty can be introduced by (a) changing the numbers on the playing sheet (e.g., decimals, mixed numerals, equivalent fractions, combinations of all three); (b) enlarging the playing surface (from 6×6 dots to 9×9 dots); (c) using the playing surface shown, but having the players multiply to yield point totals (i.e., not $2 + 4 + 5 + 1 \dots$ but $2 \times 4 \times 5 \times 1 \dots$ etc.

CODED RIDDLES

Use: To practise operation skills

Materials: Blank duplicating masters

Players: One or more

The Game: As a prototype, prepare and distribute a coded riddle as shown in the sample. Players solve the various exercises, match each answer to a letter of the alphabet as indicated by the code, and thereby yield the surprise answer to the riddle.

Once the children have had sufficient experience with this type of activity, have them prepare coded riddles of their own for distribution to other groups or classes. Have them follow these steps:

- 1. Find a riddle that you think everyone will like.
- 2. Write out the riddle and make blanks like those in the sample for the answer to the riddle.
- 3. Decide on a code to use. Write that code out.
- 4. Take the letters in your answer in order and match them to the necessary numbers in the code.
- 5. Starting with each answer, make up a problem to fit.

You may wish to have available also (a) a supply of riddles or jokes. There are many books of this kind available in libraries, book and variety stores; (b) calculators for checking answers; (c) examples or suggestions for alternate codes, i.e., A = 0, B = 3, C = 6, etc.

Sample:

What do huge, mean, quick-tempered gorillas watch on television?

□□□□□□□□ □□□□ □□□□ □□!

Code:		
A = 1	I = 9	R = 18
B = 2	J = 10	S = 19
C = 3	K = 11	T = 20
D = 4	L = 12	U = 21
E = 5	M = 13	V = 22
F = 6	N = 14	W = 23
G = 7	O = 15	X = 24
H = 8	P = 16	Y = 25
	Q = 17	Z = 26

1. $50 \div 50 = 1$
2. $2 + (2 \times 6) = \square$
3. $\frac{25}{100} = 0.\square\square$
4. $200 \div 10 = \square$
5. $8 \times 8 \div 8 = \square$
6. $(10 - 1) \times 1 = \square$
7. 2 weeks = \square d
8. $100 - \square = 93$
9. $\frac{4}{5} = \frac{16}{\square}$
10. $8000 \div 1000 = \square$
11. $\frac{1}{2} = 0.\square$
12. 2500 cm = \square m
13. $100 \div 4 - 2 = \square$
14. $\frac{1}{4} + \frac{1}{2} + \frac{2}{8} = \square$
15. $\square \times 1 = 14$
16. $5 \times 4 \times 1 = \square$
17. 20 000 m = \square km
18. $100 - \square = 85$

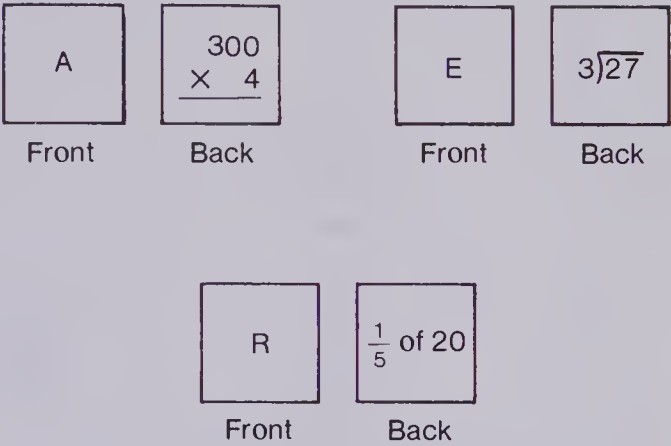
Answer: Anything they want to!

SCRAMBLE

Use: To practise operations skills

To reinforce selected topics

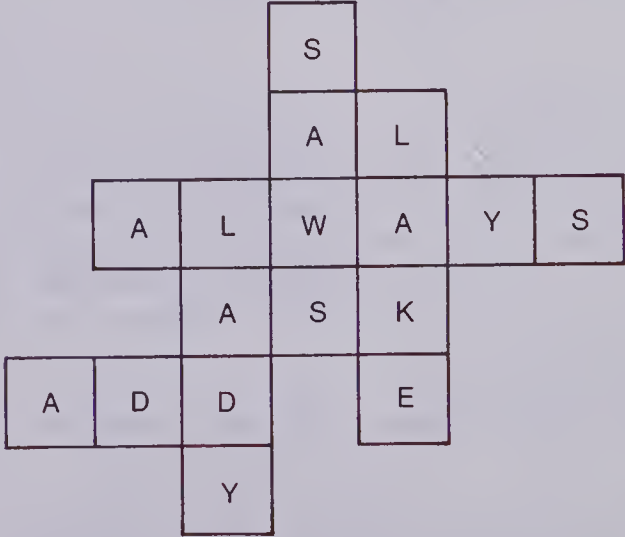
Materials: Scramble cards as shown



Players: Two to four

The Game: Seven cards are dealt to each player. The remaining cards are randomly displayed skill side up on the display surface. Players take turns playing cards in hand, scrabble style, to make connecting and/or overlapping words. As cards in hand are used, players may replace these with cards from the random set so that each player has seven cards to use at all times. The mathematical twist in "Scramble" is that players must identify the correct answer for the problem on the back of each card before that card can be played. Players score 1 point per letter for each word completed. The deck should consist of approximately 3 of each consonant and 5 of each vowel.

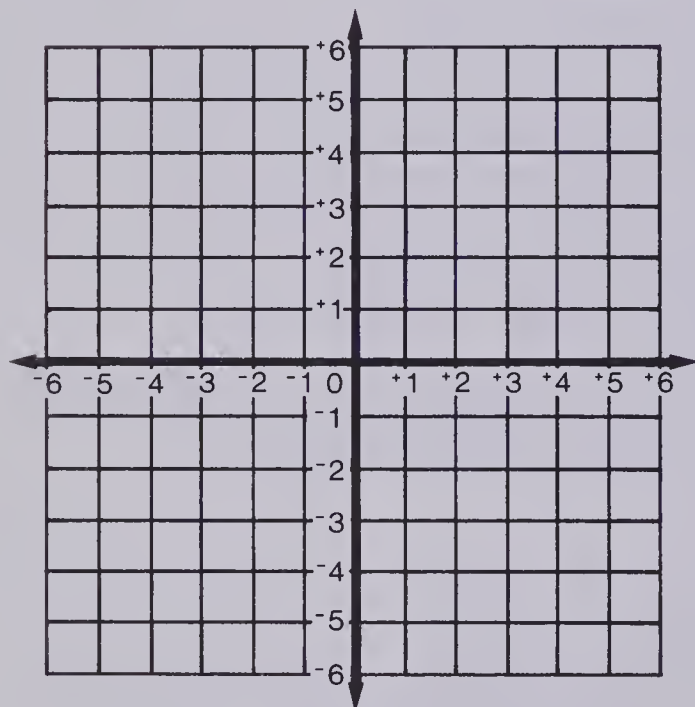
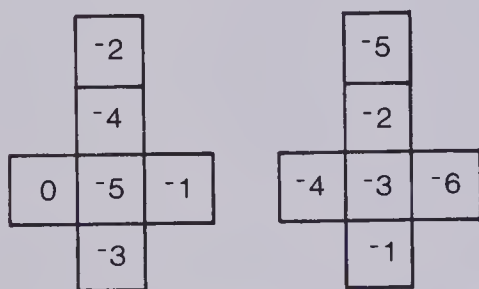
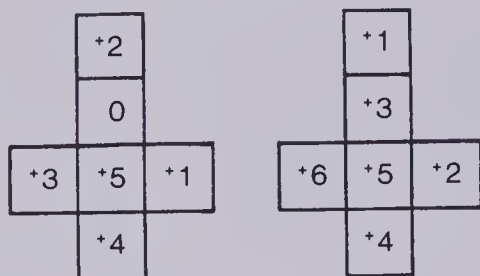
Sample play:



QUADRANT

Use: To identify and plot coordinates

Materials: Demonstration-size 4-quadrant plane (HMS — DM69), 6 markers per player, 4 dice (nets for which are shown), container (box, tin) as dice holder



Players: Two to four

The Game: Players take turns (a) randomly (i.e., with eyes covered) choosing only 2 dice from the container; (b) throwing dice and locating a point identified thereon (*note:* whichever point is convenient, i.e., roll -2 and +5, locate either (-2, +5) or (+5, -2); (c) placing a marker on that point; (d) returning both dice to the container for selection by the next player.

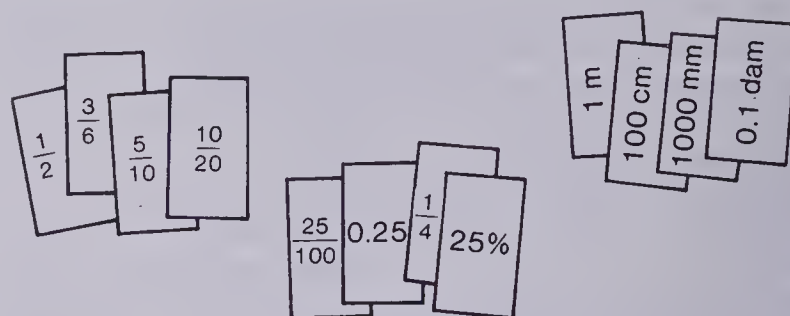
The object is to get all 6 markers into the same quadrant. When all 6 counters are in play, players may choose which of their markers they wish to move.

Variations: The game can be made easier or more difficult by (a) altering the size of the grid (i.e., use first quadrant only without negative integer dice), number of dice, and/or number of markers per player; (b) requiring that markers form straight lines or other regular polygonal shapes.

RUMMY

Use: To review and maintain computation skills, equivalence, selected skills

Materials: Cards (from 48 to 100) as shown



Players: Two to five

The Game: The dealer shuffles the deck and deals 7 cards per player. The remaining cards are piled face down on the desk and are used as the drawing pile. Dealer turns the top card face up as the start of the discard pile. Players take turns (a) drawing a card from either the discard or drawing pile, (b) using cards in hand to make sets of 3 or 4 equivalent expressions, (c) laying their sets face up in front of themselves, (d) discarding one card to the discard pile.

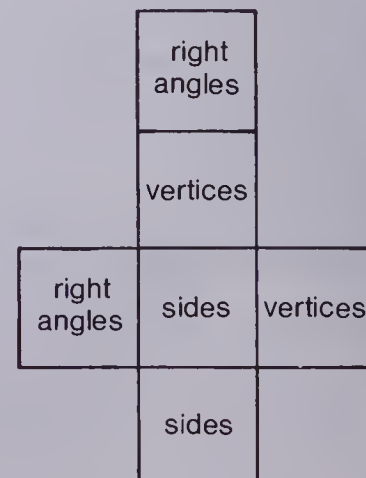
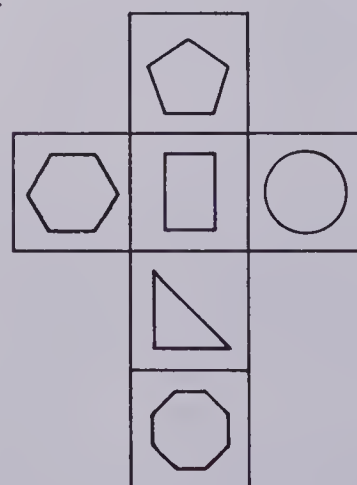
The first player to display all cards as in Rummy wins. If all the cards in the drawing pile are used before the end of the game, the discard pile may be reshuffled and turned face down to start a new drawing pile.

Variations: Use the Rummy format to provide practice in such areas as related facts, time, temperature, units of length, capacity, mass, equivalent fractions and decimal amounts, etc.

ROAD RALLY

Use: To practise identifying properties of shapes
To provide practice with selected figures

Materials: One counter per player, 2 dice marked as shown, Pit Stop cards, game board as shown



Sample Pit Stop cards:
 Note: For each card attempted, players should move
 — 2 spaces forward for a correct response,
 — 2 spaces backward for an incorrect response.

13.58 → ?
nearest tenth

25% = $\frac{\blacksquare}{100}$

Name a
composite
number.

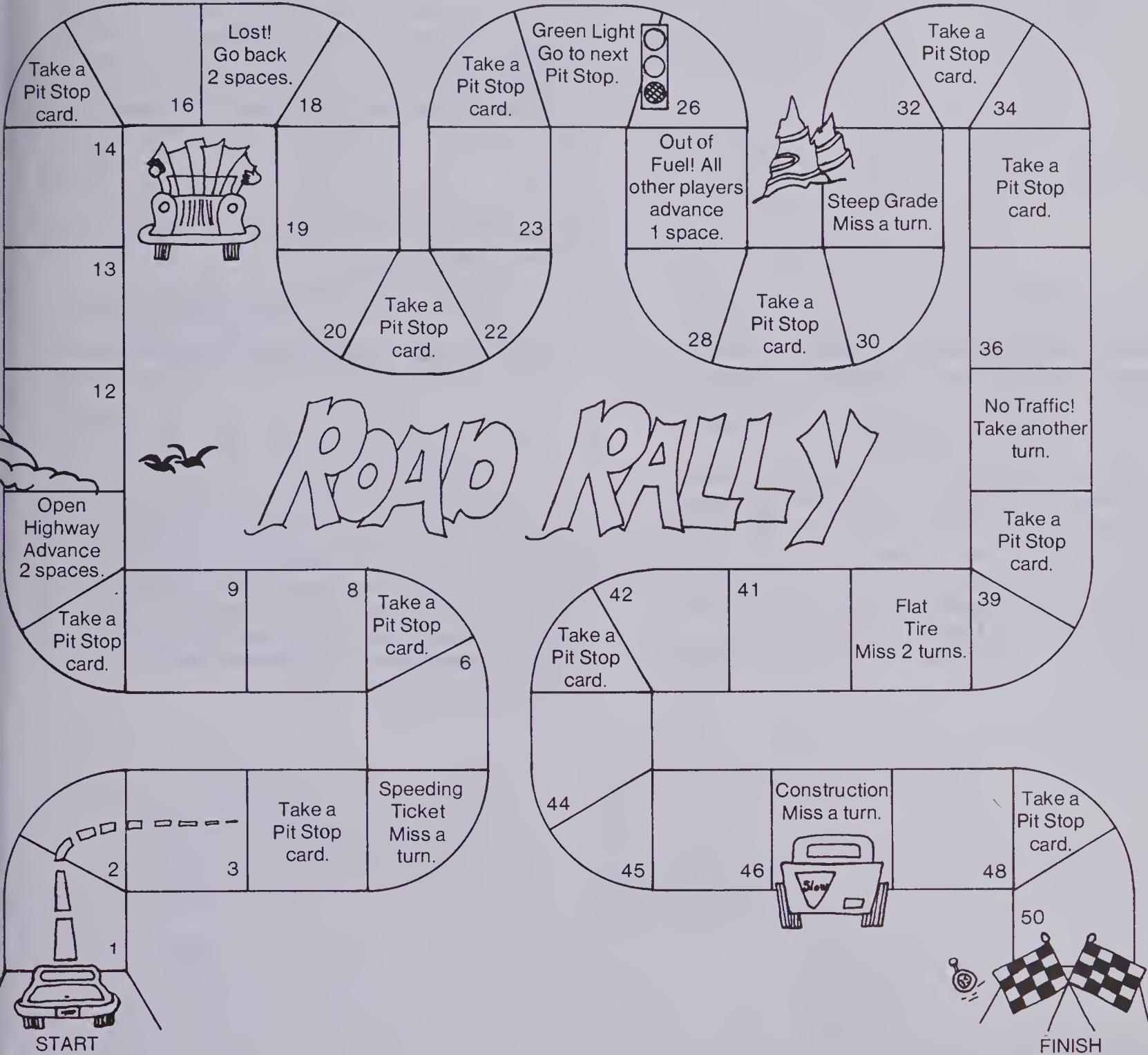
50 cm = \blacksquare m

4.60 > 4.6
true? false?

0.5 × 5 = \blacksquare

3 + 4 × 2 = \blacksquare

$\frac{3}{4} = \frac{\blacksquare}{20}$



Players: Two to four
The Game: Players take turns (a) throwing the dice;
 (b) moving their counter the number of spaces suggested,
 i.e., throw



(c) taking and solving Pit Stop cards as necessary;
 (d) following any special instructions on the game board
 should they land at these particular locations. The
 winner is the first person to finish the course exactly.

500 GRAND

Use: To practise locating points on a grid; computational skills

Materials: Playing board (shown), 2 dice (numbered 1-6), paper and pencils for score keeping

6	345	730	291	75	0	19
5	277	7	181	15	45	688
4	505	96	3	943	10	238
3	0	25	1	550	66	444
2	707	50	100	1	0	112
1	190	34	76	221	699	1
	1	2	3	4	5	6

Players: Two to four

The Game: Players take turns (a) rolling both dice twice to identify 2 sets of coordinates (i.e., roll 5 and 2, 3 and 3 — use coordinates (5, 2) and (3,3); (b) identifying the numbers on the playing board at these locations (i.e., 0 and 1); (c) multiplying these two numbers together.

Players keep a cumulative total of their products. The player closest to 500 000 after 4 rounds is the winner. *Note:* You may wish to (a) point out that the locations on this grid are squares, not intersection points; (b) have a calculator available to check answers.

Variations: The game can be made more simple or more difficult by (a) adjusting the numbers shown on the grid; (b) changing the operation (i.e., addition instead of multiplication); (c) increasing or decreasing the number of turns per round.

ITZA FACT!

Use: To practise basic facts (+, −, ×, ÷)

Materials: Deck of 45 cards numbered as shown below

Players: Two to four

The Game: Five cards are dealt to each player and one card is left face up from which to begin play. Players in turn (a) play an appropriate card held in hand on top of the last “face up” card, (b) take another card from the top of the deck if a card in hand cannot be played, (c) attempt to be the first to play all cards in hand.

Sample play:

Last face-up card —

9
3

This card could be interpreted as value 12 (addition), value 6 (subtraction), value 27 (multiplication), or value 3 (division).

The next player in turn may play any card with a sum, difference, product, or quotient of 3, 6, 12, or 27.

3
4

 or

4
2

 or

5
2

 etc.

(product 12) (sum 6) (difference 3)

The deck should contain:

1
1

,

2
1

,

2
2

,

3
1

,

3
2

,

3
3

4
1

,

4
2

,

4
3

,

4
4

, ...

9
1

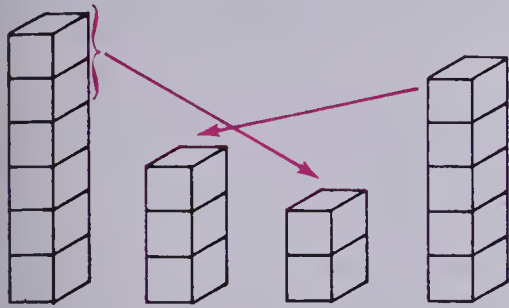
 ...

9
9

Variations: The game can be made more difficult and/or quicker moving by (a) increasing the scope of the deck; (b) allowing more than one appropriate card to be played at a time; (c) playing in rounds, i.e., when each round is over, score a penalty point for each card remaining in hand. The overall winner is the player with the *lowest* penalty point total after 4 rounds, etc.

PROBLEMS OF THE WEEK

1. Pile 16 cubes in 4 piles as shown.



A move consists of moving one or more cubes from one pile to another. In two moves make all the piles the same height.

Can you make an arrangement of the 16 cubes into 4 piles which, in order to have equal-sized piles, requires

(a) 3 moves? $7 + 3 + 3 + 3$

(b) 4 moves? $6 + 6 + 1 + 3$

2. How many digit "9's" are there between 1 and 100? **20**

3. In a 300-page book, how many page numbers will show "7" as one of the digits? **57**

4. How many 2-digit numbers have a digit sum of 8? **8**
List them in order from smallest to largest.

Example

Digit Sum

17 $1 + 7 = 8$ **17, 26, 35, 44, 53, 62, 71, 80**

5. How many palindromes are there between 100 and 1000? (A palindrome is a number which reads the same forwards as backwards, e.g., 101, 232, ...) **90**

6. Continue this pattern.

$$1 + 3 = 4$$

$$1 + 3 + 5 = 9$$

$$1 + 3 + 5 + 7 = 16$$

$$1 + 3 + 5 + 7 + \dots + 41 = ?$$
 441

7. Grandson: Grandpa, how many geese and how many pigs do you have?

Grandfather: I counted 30 heads and 100 feet this morning when I counted the geese and pigs.

Grandson: I can't tell from that.

Can you?

Let A = number of geese

B = number of pigs

$$A + B = 30$$

$$2A + 4B = 100$$

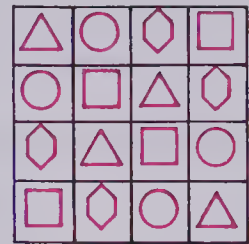
10 geese and 20 pigs.

8. Use the digits for the year and any of the operations of addition, subtraction, multiplication, and division to write as many numbers as you can. You may change the order of the digits. For example, if the year is 1983:

$$3 - 1 - (9 - 8) = 1$$

$$(9 + 1) \div (8 - 3) = 2$$

9. Draw the squares.

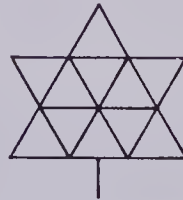


Put these shapes in the squares so that each row contains

a \triangle , \square , \diamond , and \circ and each column contains a

\triangle , \square , \diamond , and \circ .

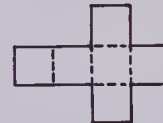
- 10.



How many triangles are in this figure? **17**

11. A wheel has 24 spokes. How many spaces does it have between spokes? **24**

12. This combination of six squares can fold to form a cube. How many other nets can you make using 6 squares that fold to form a cube?



13. This combination of equilateral triangles, when cut out and folded, will form an octahedron. How many other arrangements of equilateral triangles can you make that will fold to form octahedrons?



14. Multiply these using only Roman numerals.

$$\begin{array}{r} \text{VI} \\ \times \text{II} \\ \hline \text{XII} \end{array}$$

$$\begin{array}{r} \text{XI} \\ \times \text{V} \\ \hline \text{LV} \end{array}$$

$$\begin{array}{r} \text{XXXI} \\ \times \text{X} \\ \hline \text{CCCX} \end{array}$$

$$\begin{array}{r} \text{XVII} \\ \times \text{X} \\ \hline \text{CLXX} \end{array} \quad \begin{array}{r} \text{LXXX} \\ \times \text{XI} \\ \hline \text{DCCCLXXX} \end{array}$$

Check by changing each to our numerals and multiplying.

15. Can you make a rectangular array using 50 or fewer coins so that the number of heads in the border equals the number of tails inside?

One arrangement (though not the correct one) is shown.



$$48 (6 \times 8)$$

12 heads \neq 4 tails

16. Which has more value, one of each piece of Canadian currency up to the one hundred dollar bill (i.e., 1¢, 5¢, 10¢, ..., \$100) or a one metre high stack of quarters? **The currency collection equals \$189.91, assuming a silver dollar face value of \$1; the quarter stack, assuming a coin thickness of 1.5 mm, would equal approximately \$166.75.**

17. Write the number of letters contained in each word of this sentence:

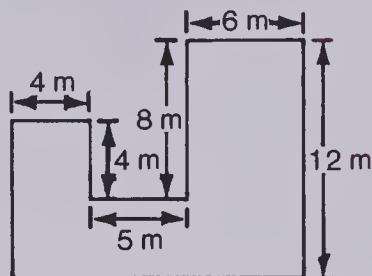
"May I have a large container of toffee?"

3

What number is this?

Write a sentence that does the same thing so that you will remember an approximation of pi.

18. What is the area of this garden? 124 m^2

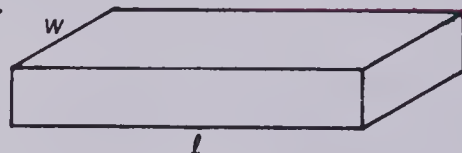


19. What is the volume of this box? 324 cm^3

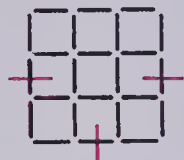
The length is three times the width.

The height is half its width.

The length is 18 cm.



20. Remove 3 line segments to leave 6 equal-sized squares.



Remove those marked with colour.

21. To test whether a number is divisible by 11:

Find the sums of the alternate digits.

If they are equal or differ by 11 or are a multiple of 11, the number is divisible by 11.

Example

(a) 2354 $2 + 5 = 7$ $3 + 4 = 7$ $7 = 7$

2354 is divisible by 11.

(b) 714 395 $7 + 4 + 9 = 20$ $1 + 3 + 5 = 9$

$20 - 9 = 11$

Therefore 714 395 is divisible by 11.

Which of these numbers are divisible by 11?

(c) 7631 (d) 2134 (e) 45 617 (f) 149 523 (g) 1 476 926

d, e, f, g

22. What is the smallest whole number that has exactly 10 different factors? 48

23. What is the smallest whole number that has exactly nine different factors? 36

24. Twin primes are those that differ by 2, e.g., 3 and 5, 11 and 13.

There are only 8 sets of twin primes among the first 100 numbers. What are they? (3, 5); (5, 7); (11, 13); (17, 19); (29, 31); (41, 43); (59, 61); (71, 73)

25. 5 students played in a round-robin chess tournament. Each student played each of the other students once.

How many games were played? 10

$$26. 33 = 3 \times 11 = 11 + 11 + 11 \\ = 10 + 11 + 12$$

33 can be written as the sum of three consecutive numbers.

$$22 = 2 \times 11 = 4 \times 5.5 = 5.5 + 5.5 + 5.5 + 5.5 \\ = 4 + 5 + 6 + 7$$

22 can be written as the sum of four consecutive numbers.

Write:

(a) 50 as sum of 5 consecutive numbers.

$$8 + 9 + 10 + 11 + 12$$

(b) 60 as sum of 5 consecutive numbers.

$$10 + 11 + 12 + 13 + 14$$

(c) 72 as sum of 9 consecutive numbers.

$$4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12$$

Write each as the sum of consecutive numbers.

(d) 9 (e) 30 (f) 28 (g) 14

$$(d) 2 + 3 + 4$$

$$(e) 9 + 10 + 11$$

$$(f) 1 + 2 + 3 + 4 + 5 + 6 + 7$$

$$(g) 2 + 3 + 4 + 5$$

How many of the first 31 numbers can you express as the sum of consecutive numbers?

27. Study this pattern for finding certain squares.

Any two-digit number ending in "5"	\downarrow $\underline{\quad}^2$
Take first digit	\downarrow $\underline{\quad}$
Multiply it by the next greater number	\downarrow $\underline{\quad} \times \underline{\quad}$
Multiply by $100 + 5 \times 5$	\downarrow $\underline{\quad} \times \underline{\quad} \times 100 + 5 \times 5$
Simplify	\downarrow $\underline{\quad} \times 100 + 25$
Correct square	\downarrow $\underline{\quad}$

Let's try it with:

$$25^2 \\ \downarrow \\ 2 \\ \downarrow \\ 2 \times 3 \\ \downarrow \\ 2 \times 3 \times 100 + 5 \times 5 \\ \downarrow \\ 6 \times 100 + 25 \\ \downarrow \\ 625$$

Use the pattern to find these. Use a calculator to check.

$$(a) 15^2$$

$$(b) 35^2$$

$$(c) 45^2$$

$$(d) 65^2$$

$$(e) 75^2$$

$$(f) 95^2$$

$$(a) 225$$

$$(b) 1225$$

$$(c) 2025$$

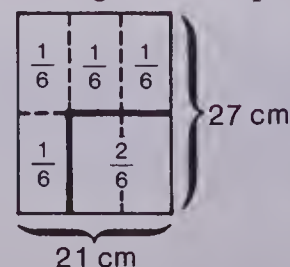
$$(d) 4225$$

$$(e) 5625$$

$$(f) 9025$$

28. Choose any magazine. Roughly, how much (i.e., how many pages) of the magazine is made up of advertisements? Answers will vary.

Use this fraction guide to help measure fractional parts of pages.



Remember:

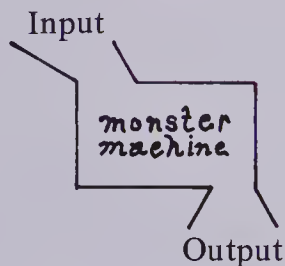
$$\frac{2}{6} = \frac{1}{3}$$

$$\frac{3}{6} = \frac{1}{2}$$

$$\frac{4}{6} = \frac{2}{3}$$

29. Ian is 12 a old. Ian's father is 37 a old. How old was Ian's father when Ian was half as old as he is now? **31**
30. Six grapefruits can be purchased for 98¢. At this price, how much will 21 grapefruits cost? **\$3.43**
31. Jamie is a 5-pin bowler. He bowled 228 and 196 for his first 2 games. What must he bowl on his third game to get an average of 220? **236**
32. Sam decided to save 25¢ on the first day of his summer job and increase his daily savings by 25¢ each day.
- (a) How much had he saved by the end of the first 5 d of work? **\$3.75**
- (b) How much had he saved by the end of the first month (i.e., 22 d)? **\$63.25**
33. How many different ways can 6 doughnuts be arranged on 2 plates? Make a list of the ways. **7**
34. How many different ways can 6 doughnuts be arranged on 3 plates? Make a list of the ways. **27**
35. What is the rule for this machine? **$\times 3 - 1$**

Input	Output
7	20
10	29
5.5	15.5
8.2	23.6
25	74



Solve these inputs or outputs.

Input	Output
5	14
15	44
19.5	57.5
3	8
17	50
200	599
34	101

36. (a) Using the last 4 digits in your phone number, make a true sentence.

Example

Using the digits 8593: $8 = 5 + (9 \div 3)$

- (b) Copy down your telephone number. Group the first two digits or the last two digits to form a number from 10 through 99.

Using mathematical operations, can you construct a true number sentence?

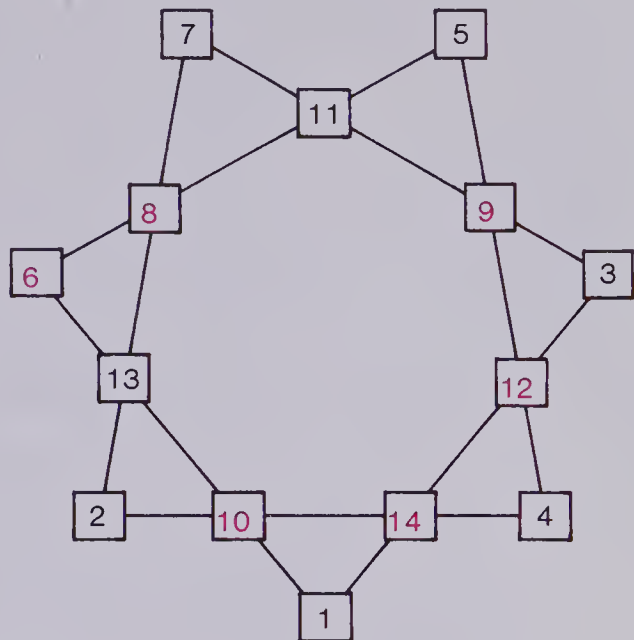
Example

839-8593

$$83 = 9 \times 8 + 5 + 9 - 3$$

37. The sum of the ages of 2 tennis players is 27. One player is 3 a younger than the other. How old are the tennis players? **12, 15**
38. Carol purchased a baseball and a baseball glove for \$32. The glove cost \$20 more than the ball. How much did the ball cost? **\$6**

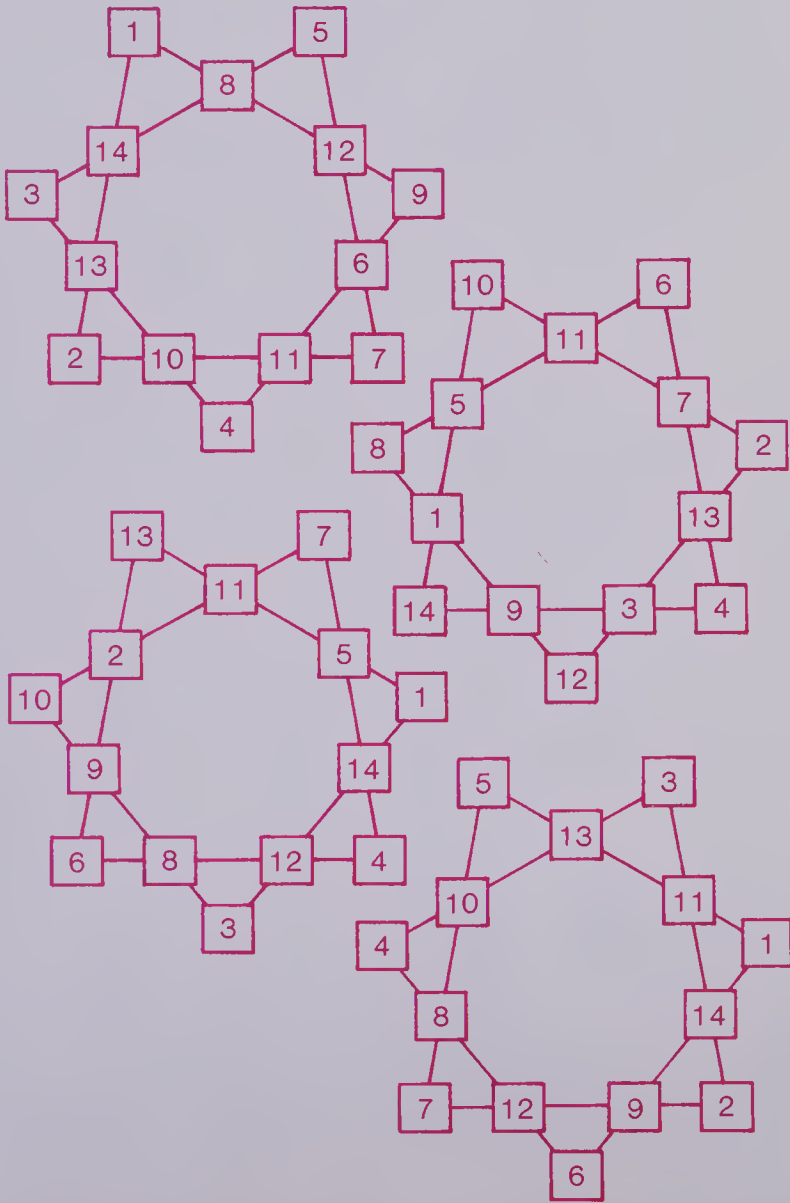
39. Trace the star in this diagram. Include the eight



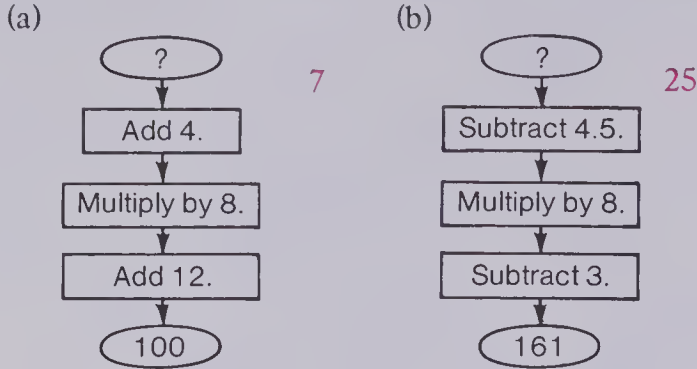
numbers given. Solve equations (a) to (f) and write the solutions on separate squares of paper the size of the squares in the star. Place the squares on the empty boxes so that each row of 4 boxes has the sum 30. Write the correct numbers in the boxes.

- (a) $20 \times 70 = n \times 100$ (b) $30 \times 40 = n \times 100$
 (c) $50 \times 20 = n \times 100$ (d) $20 \times 40 = n \times 100$
 (e) $20 \times 30 = n \times 100$ (f) $30 \times 30 = n \times 100$

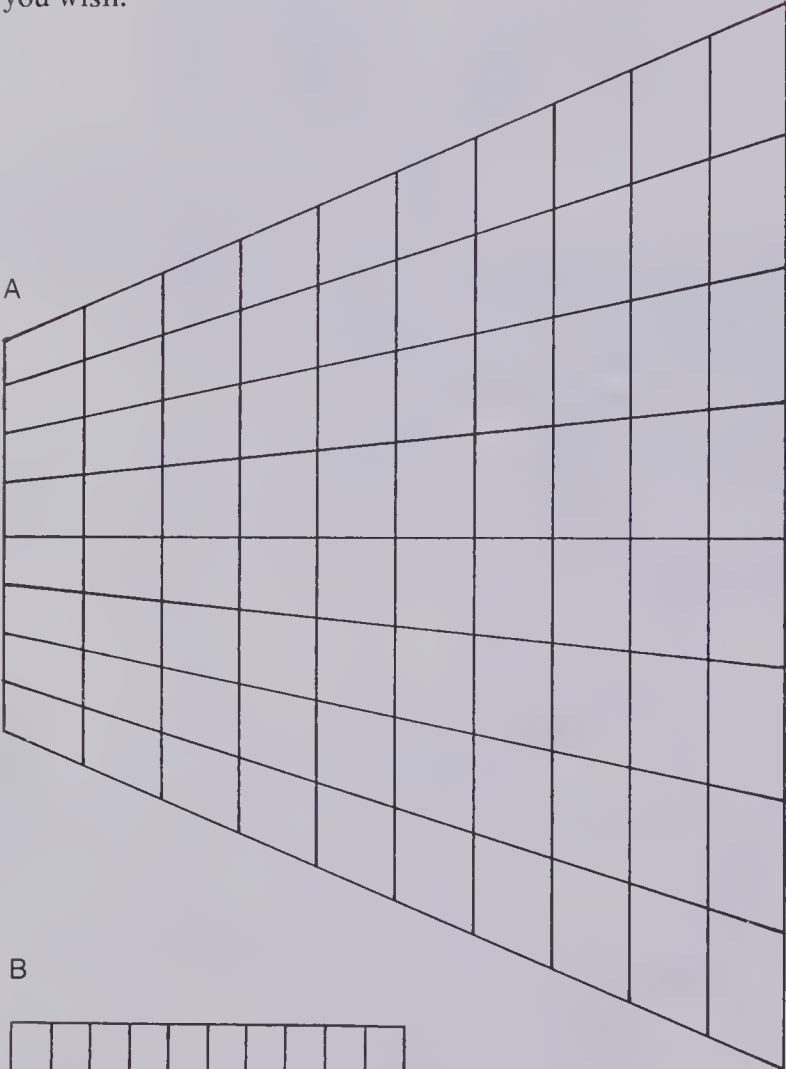
4 variations of the star are shown and may be used for more problems.



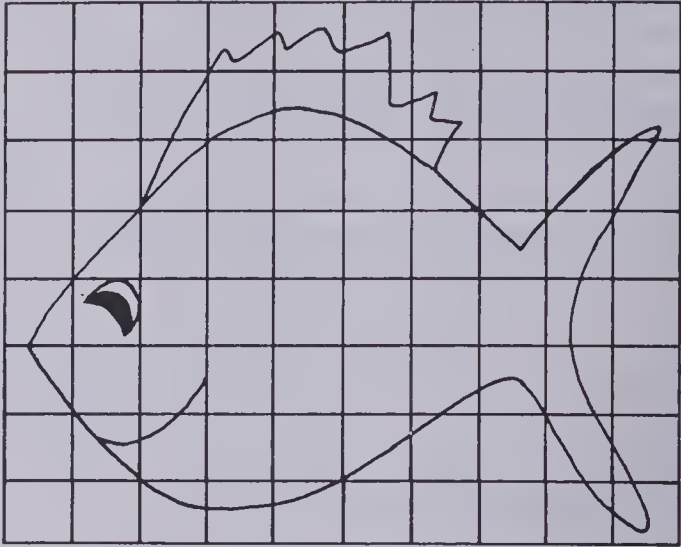
40. Find the missing numbers in these flow charts.



41. Draw each grid A and B as shown, but of any size you wish.



(a) Enlarge the fish using the two different grids.
(b) Use other grids to enlarge the fish.



CUMULATIVE TEST ITEM BANK

These items test a number of the major objectives in this book. In using these test items, select from the set up to the page on which your class is working at present. Set A and Set B are matched items to provide opportunity for pre and post testing or for repeat testing of the objectives. The pages can be readily removed and duplicated if you wish to distribute the test. Be certain that all of the children are familiar with the accepted answer format, whether it involves writing answers directly on a duplicated sheet, or on a separate sheet or workbook page. Demonstrate the sample items on the chalkboard and be sure there are no questions before beginning the test.

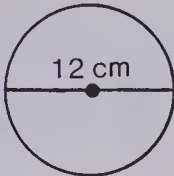
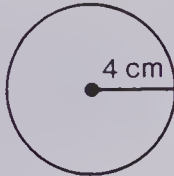

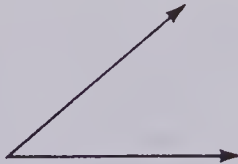
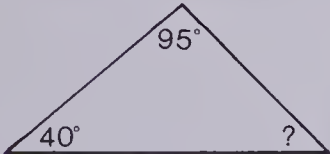
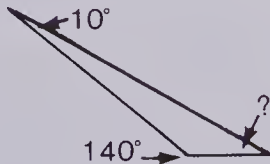
Sample Items

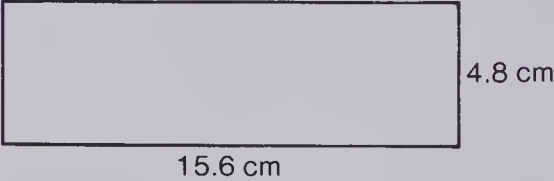
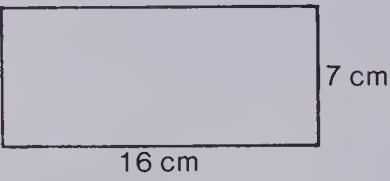
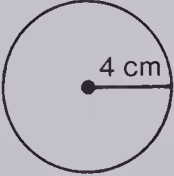
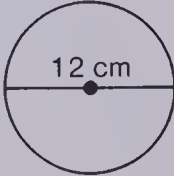
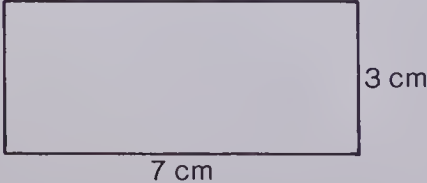
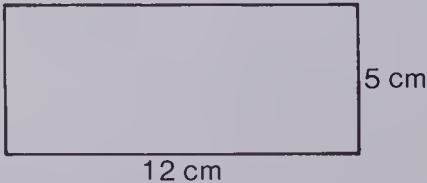
(a) Write as a decimal. $16\frac{1}{10}$	(b) Calculate. $4.6 + 3.27$
(c) Round to the nearest thousand. 6399	(d) Round to the nearest tenth. 14.381

Answers to Test

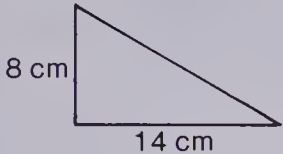
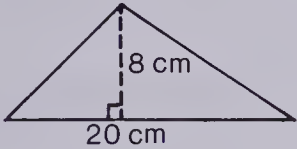
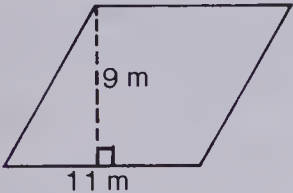

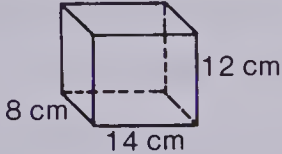
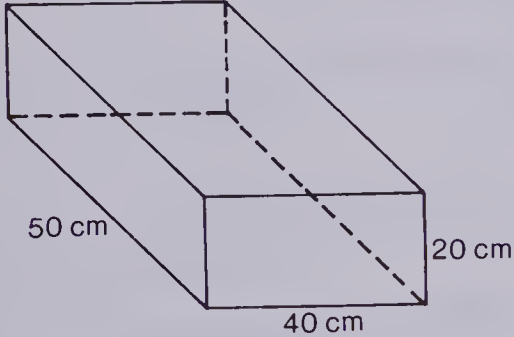
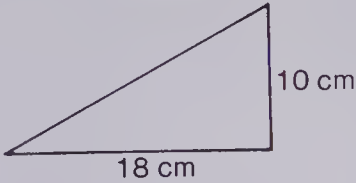
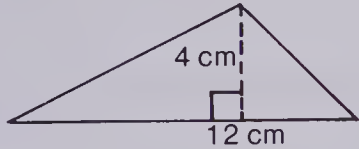
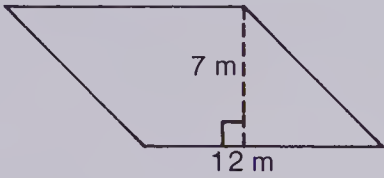

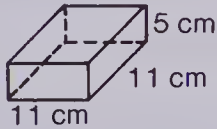
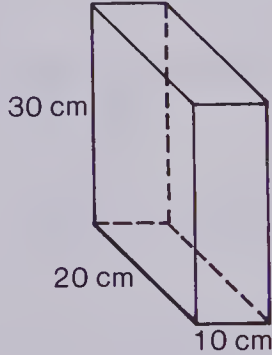
- (a) 53.34 (b) 14.065 2. (a) 48.67 (b) 29.594 3. 11 099
15 682 5. 29 268 6. 18 617 7. 645.4251
8. 438.4138 9. 86.11 10. 185.65 13. (a) 50 000
14. (a) 80 000 (b) 50 000 15. \$21 16. \$24
17. (a) $n = 18$ (b) $n = 53$ 18. (a) $n = 16$ (b) $n = 33$
19. (a) \neq (b) \neq (c) $=$ 20. (a) \neq (b) $=$ (c) $=$ 21. 145.5,
145.9, 154.5, 154.6 22. 243.5, 243.6, 432.4, 432.9
23. 16 cm 24. 12 cm 25. 6 cm 26. 8 cm 27. 130°
28. 40° 29. 45° 30. 30° 31. (a) - (ii), (b) - (iii), (c) - (i)
32. (a) - (iii), (b) - (i), (c) - (ii) 33. 26 950 34. 45 040
35. 9360 36. 8370 37. 38 560 38. 65 590 39. 159 936
40. 536 517 41. \$267.40 42. \$285.12 43. 15 211.8
44. 17 174.5 45. 22.14 46. 37.68 47. 184.0318
48. 359.7776 49. 6.986 93 50. 11.145 72 51. \$560
52. \$320 53. (a) $n = 2632$ (b) $n = 4$ 54. (a) $n = 3680$
56. $n = 11$ 55. \$28; \$12 56. \$90; \$10 57. 81 58. 81
59. \$655 60. \$397 61. 7343 R2 62. 7284 R1 63. 561
64. 362 65. 68 R14 66. 69 R53 67. 4.83 68. 6.54
69. (a) 1400 cm (b) 0.2 hm (c) 6 cm 70. (a) 15 000 m
71. 3000 m (c) 700 mm 71. 40.8 cm 72. 46 cm
73. 25.12 cm 74. 37.68 cm 75. 21 cm^2 76. 60 cm^2
77. 56 cm^2 78. 90 cm^2 79. 80 cm^2 80. 24 cm^2
81. 99 m^2 82. 84 m^2 83. 78.5 cm^2 84. 113.04 cm^2
85. 1344 cm^3 86. 605 cm^3 87. 40 L 88. 6 L
89. (a) 4000 cm^3 (b) 15 mL 90. (a) 5 L (b) 56 cm^3 91. 1 t,
0.5 t, 2 t 92. 8000 g, 8000 kg, 8 000 000 kg 93. (a) 2343 m
94. 9.049 m 94. (a) 805 m (b) 2.507 m 95. (a) 360
96. 1430 96. (a) 480 (b) 6570 97. 312 98. 522 99. 5.9
100. 8.7 101. 26 110 102. 83 260 103. 36 104. 54
105. 600 km; 616 km 106. 2400 km; 2054 km
107. (a) 3 kg (b) 2 kg (c) 2000 cm^3 108. (a) 5 kg (b) 4 kg
109. 6000 cm^3 109. 24:25 or 00:25 110. 17:05
111. 04:30 112. 02:35 113. Ahead 114. Back
115. (a) 1st century (b) 8th century (c) 19th century
116. (a) 1st century (b) 7th century
117. (a) -4°C (b) -21°C
118. 8°C 118. (a) 6°C (b) -32°C (c) 4°C 119. 70 m
120. 2 km south 121. (a), (c) 122. (a), (b) 123. 1, 2, 3,
4, 6, 8, 12, 24 124. 1, 2, 4, 8, 16, 32 125. 1, 2, 5, 10,
25, 50 126. 1, 5, 11, 55 127. (a) 6 (b) 0 (c) 0 128. (a) 257
(b) 0 (c) 0 129. 31, 37 130. 41, 43, 47
131. (a) $18 = 2 \times 3 \times 3$ (b) $150 = 2 \times 3 \times 5 \times 5$
132. (a) $30 = 2 \times 3 \times 5$ (b) $42 = 2 \times 3 \times 7$ 133. 6
134. 2 135. 5 136. 2 137. 21, 24, 27 138. 63, 72, 81
139. 12 140. 30 141. $3072 = (3 \times 1000) + (0 \times 100) +$
 $(7 \times 10) + (2 \times 1) = (3 \times 10^3) + (0 \times 10^2) + (7 \times 10^1) +$
 (2×1) 142. $78\,509 = (7 \times 10\,000) + (8 \times 1000) +$
 $(5 \times 100) + (0 \times 10) + (9 \times 1) = (7 \times 10^4) + (8 \times 10^3) +$
 $(5 \times 10^2) + (0 \times 10^1) + (9 \times 1)$ 143. (a) 36 (b) 490
144. (a) 225 (b) 360 145. $\frac{4}{10}$ or $\frac{2}{5}$ 146. $\frac{6}{8}$ or $\frac{3}{4}$
147. (a) $\frac{4}{5}$ (b) $\frac{4}{7}$ 148. (a) $\frac{2}{5}$ (b) $\frac{5}{7}$ 149. 16 150. 9
151. $\frac{7}{12}$ 152. $\frac{13}{20}$ 153. $\frac{5}{10}$ or $\frac{1}{2}$ 154. $\frac{11}{20}$ 155. $\frac{13}{10} = 1\frac{3}{10}$
156. $\frac{43}{30} = 1\frac{13}{30}$ 157. $4\frac{3}{5}$ 158. $4\frac{5}{8}$ 159. $\frac{6}{4} = 1\frac{1}{2}$
160. $\frac{10}{5} = 2$ 161. (a) 0.71 (b) 0.006 162. (a) 0.15
(b) 0.009 163. $4\frac{1}{10}$ 164. $5\frac{23}{24}$ 165. $3\frac{1}{20}$ 166. $5\frac{3}{10}$
167. 18 168. 15 169. $2\frac{1}{2}$ 170. $2\frac{1}{3}$ 171. $\frac{5}{16}$ 172. $\frac{1}{2}$
173. $n = \frac{1}{4}$ 174. $n = 1\frac{1}{2}$ 175. $\frac{1}{8}$ 176. $\frac{1}{3}$ 177. $\frac{3}{5}$
178. $\frac{21}{32}$ 179. $\frac{7}{48}$ 180. $7\frac{1}{2}$ 181. $\frac{8}{21}$ 182. $2\frac{2}{5}$ 183. $3\frac{3}{8}$
184. $16\frac{1}{9}$ 185. $1\frac{1}{2}$ 186. $1\frac{19}{30}$ 187. (a) 0.2 (b) 0.75
188. (a) 0.4 (b) 0.45 189. 0.7 190. 0.6 191. 3:4
192. 3:5 193. 40 194. 30 195. (a) 82% (b) 85%
196. (a) 74% (b) 65% 197. 0.47 198. 0.09 199. (a) 50
(b) \$1.32 200. (a) 7 (b) \$5.10 201. $\frac{3}{20}$ 202. $\frac{16}{25}$ 203. 13
204. 12 205. \$26.82 206. \$278.61 207. \$26 208. \$70
209. \$21.35 210. \$32.11 211. Sales tax: \$630; total
cost: \$9630 212. Sales tax: \$960; total cost: \$12 960
213. Discount: \$11.40; sale price: \$26.60 214. Discount:
\$11.00; sale price: \$44.00 215. (a) $n = 32$ (b) $n = 5$
(c) $n = 20$ 216. (a) $n = 55$ (b) $n = 9$ (c) $n = 18$ 217. 0.5 h
218. 0.25 h 219. 5 220. 8 221. 7 222. 2 and 12
223. \$600.00 224. \$1050.00 225. (a) $n = 3$ (b) $s = 22$
226. (a) $w = 10$ (b) $n = 6$ 227. (a) $n = 39 - 12$; $n = 27$
(b) $n = 156 - 82$; $n = 74$ 228. (a) $n = 42 - 9$; $n = 33$
(b) $n = 81 - 56$; $n = 25$ 229. (a) $n = 54 + 18$; $n = 72$
(b) $n = 71 + 48$; $n = 119$ 230. (a) $n = 41 + 22$; $n = 63$
(b) $n = 101 + 78$; $n = 179$ 231. (a) $n = 54 \div 6$; $n = 9$
(b) $n = 9.1 \div 7$; $n = 1.3$ 232. (a) $n = 72 \div 9$; $n = 8$
(b) $n = 10.2 \div 6$; $n = 1.7$ 233. (a) $n = 13 \times 8$; $n = 104$
(b) $n = 28 \times 26$; $n = 728$ 234. (a) $n = 21 \times 7$; $n = 147$
(b) $n = 18 \times 1.9$; $n = 34.2$ 235. A(2, 1), B(2, 4), C(3, 2),
D(5, 0) 236. A(0, 2), B(3, 0), C(5, 3), D(2, 5) 239. 1, 3,
5, 7, 9 240. 8, 13, 18, 23, 28 241. 4, 6, 8, 10, 12
242. 4, 10, 16, 22, 28 243. (a) $+10$ (b) -7 (c) $+4$ (d) -5 (e) -2
(f) $+7$ 244. (a) $+7$ (b) -10 (c) -11 (d) -9 (e) -3 (f) $+4$
245. (a) -3 (b) $+7$ (c) $+2$ 246. (a) $+4$ (b) -9 (c) -11
247. (a) $n = +6$ (b) $n = -5$ 248. (a) $n = +3$ (b) $n = -8$
249. (a) $+1$ (b) -16 (c) -3 (d) $+10$ 250. (a) $+1$ (b) -12 (c) -4
(d) $+7$ 253. 75 254. 55 km 255. (a) 75 km (b) 2nd hour
256. (a) 0 km (b) 20 km 257. (a) 3 d (b) 15 d 258. (a) \$20
(b) \$48 259. north 260. south 261. (a) turn (b) flip
(c) slide 262. (a) turn (b) slide (c) flip
265. (a) $AR \rightarrow OT$; $RN \rightarrow TP$; $AN \rightarrow OP$ (b) $R \rightarrow T$;
 $N \rightarrow P$; $A \rightarrow O$ 266. (a) $S \rightarrow K$; $M \rightarrow P$; $O \rightarrow A$
(b) $SM \rightarrow KP$; $OM \rightarrow AP$; $SO \rightarrow KA$ 267. (a) 268. (b)

Page	A	B																																																																
	<p>1. Write as a decimal.</p> <p>(a) $53\frac{34}{100}$</p> <p>(b) $14\frac{65}{1000}$</p> <p>3. $\begin{array}{r} 4381 \\ 6270 \\ + 448 \\ \hline \end{array}$</p> <p>5. $\begin{array}{r} 47\,321 \\ -18\,053 \\ \hline \end{array}$</p> <p>7. Write in columns, then add. $594.46 + 47.3 + 3.6651$</p> <p>9. $\begin{array}{r} 258.6 \\ -172.49 \\ \hline \end{array}$</p> <p>10 11. Write each in a place-value chart.</p> <p>(a) 4 million, 3 hundred twenty-one thousand, four hundred fifty</p> <p>(b) 2 billion, 483 million</p> <p>(c) three million, forty-two thousand, eighty</p> <div><table><tr><td>billions</td><td>hundred millions</td><td>millions</td><td>ten thousands</td><td>thousands</td><td>hundreds</td><td>tens</td><td>ones</td></tr><tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr></table></div> <p>13. Round to the nearest ten thousand.</p> <p>(a) 45 379</p> <p>(b) 71 058</p> <p>15. For UNICEF, Mary collected \$8.73, Jim \$7.29, and Bernie \$4.93. About how much did they collect? (Round to the nearest dollar.)</p> <p>17. Solve.</p> <p>(a) $n + 14 = 32$</p> <p>(b) $n - 45 = 8$</p> <p>20 19. Copy and complete using = or \neq.</p> <p>(a) $76 \bullet 43$</p> <p>(b) $27 \bullet 2.7$</p> <p>(c) $\\$0.75 \bullet 75\text{¢}$</p> <p>21. Arrange in order from smallest to largest. 154.5, 145.5, 154.6, 145.9</p> <p>30 23. What is the diameter of this circle?</p> <div></div>	billions	hundred millions	millions	ten thousands	thousands	hundreds	tens	ones																									<p>2. Write as a decimal.</p> <p>(a) $48\frac{67}{100}$</p> <p>(b) $29\frac{594}{1000}$</p> <p>4. $\begin{array}{r} 9735 \\ 4960 \\ + 987 \\ \hline \end{array}$</p> <p>6. $\begin{array}{r} 83\,426 \\ -64\,809 \\ \hline \end{array}$</p> <p>8. Write in columns, then add. $402.97 + 28.5 + 6.9438$</p> <p>10. $\begin{array}{r} 369.4 \\ -183.75 \\ \hline \end{array}$</p> <p>12. Write each in a place-value chart.</p> <p>(a) 6 million, five hundred seventy-three thousand, eight hundred sixty</p> <p>(b) 7 billion, 965 million</p> <p>(c) nine million, seventy-one thousand, fifty</p> <div><table><tr><td>billions</td><td>hundred millions</td><td>millions</td><td>ten thousands</td><td>thousands</td><td>hundreds</td><td>tens</td><td>ones</td></tr><tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr></table></div> <p>14. Round to the nearest ten thousand.</p> <p>(a) 75 463</p> <p>(b) 53 140</p> <p>16. For UNICEF, Joe collected \$7.85, Steve \$9.31, and Janet \$6.84. About how much did they collect? (Round to the nearest dollar.)</p> <p>18. Solve.</p> <p>(a) $n + 12 = 28$</p> <p>(b) $n - 27 = 6$</p> <p>20. Copy and complete using = or \neq.</p> <p>(a) $95 \bullet 61$</p> <p>(b) $84 \bullet 84$</p> <p>(c) $\\$0.50 \bullet 50\text{¢}$</p> <p>22. Arrange in order from smallest to largest. 243.5, 432.4, 243.6, 432.9</p> <p>24. What is the radius of this circle?</p> <div></div>	billions	hundred millions	millions	ten thousands	thousands	hundreds	tens	ones																								
billions	hundred millions	millions	ten thousands	thousands	hundreds	tens	ones																																																											
billions	hundred millions	millions	ten thousands	thousands	hundreds	tens	ones																																																											

age	A	B												
	<p>25. What is the radius of this circle?</p> 	<p>26. What is the diameter of this circle?</p> 												
40	<p>27. Use a protractor to measure this angle.</p> 	<p>28. Use a protractor to measure this angle.</p> 												
	<p>29. Calculate the measure of the third angle.</p> 	<p>30. Calculate the measure of the third angle.</p> 												
50	<p>31. Match.</p> <table><tr><td>(a) 4 faces</td><td>(i) octahedron</td></tr><tr><td>(b) 6 faces</td><td>(ii) tetrahedron</td></tr><tr><td>(c) 8 faces</td><td>(iii) hexahedron</td></tr></table>	(a) 4 faces	(i) octahedron	(b) 6 faces	(ii) tetrahedron	(c) 8 faces	(iii) hexahedron	<p>32. Match.</p> <table><tr><td>(a) 5 sides</td><td>(i) hexagon</td></tr><tr><td>(b) 6 sides</td><td>(ii) octagon</td></tr><tr><td>(c) 8 sides</td><td>(iii) pentagon</td></tr></table>	(a) 5 sides	(i) hexagon	(b) 6 sides	(ii) octagon	(c) 8 sides	(iii) pentagon
(a) 4 faces	(i) octahedron													
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(a) 5 sides	(i) hexagon													
(b) 6 sides	(ii) octagon													
(c) 8 sides	(iii) pentagon													
	<p>33. $\begin{array}{r} 3850 \\ \times 7 \\ \hline \end{array}$</p>	<p>34. $\begin{array}{r} 5630 \\ \times 8 \\ \hline \end{array}$</p>												
60	<p>35. $\begin{array}{r} 936 \\ \times 10 \\ \hline \end{array}$</p>	<p>36. $\begin{array}{r} 837 \\ \times 10 \\ \hline \end{array}$</p>												
	<p>37. $\begin{array}{r} 482 \\ \times 80 \\ \hline \end{array}$</p>	<p>38. $\begin{array}{r} 937 \\ \times 70 \\ \hline \end{array}$</p>												
	<p>39. $\begin{array}{r} 448 \\ \times 357 \\ \hline \end{array}$</p>	<p>40. $\begin{array}{r} 641 \\ \times 837 \\ \hline \end{array}$</p>												
	<p>41. $\begin{array}{r} \\$53.48 \\ \times 5 \\ \hline \end{array}$</p>	<p>42. $\begin{array}{r} \\$47.52 \\ \times 6 \\ \hline \end{array}$</p>												
70	<p>43. $\begin{array}{r} 5634 \\ \times 2.7 \\ \hline \end{array}$</p>	<p>44. $\begin{array}{r} 4907 \\ \times 3.5 \\ \hline \end{array}$</p>												
	<p>45. $\begin{array}{r} 36.9 \\ \times 0.6 \\ \hline \end{array}$</p>	<p>46. $\begin{array}{r} 94.2 \\ \times 0.4 \\ \hline \end{array}$</p>												
	<p>47. $\begin{array}{r} 38.42 \\ \times 4.79 \\ \hline \end{array}$</p>	<p>48. $\begin{array}{r} 56.48 \\ \times 6.37 \\ \hline \end{array}$</p>												

Page	A	B
	<p>49. $\begin{array}{r} 14.23 \\ \times 0.491 \\ \hline \end{array}$</p>	<p>50. $\begin{array}{r} 17.58 \\ \times 0.634 \\ \hline \end{array}$</p>
80	<p>51. Round money to nearest dollar. Round number to nearest 10. Multiply to find the estimated product. $\begin{array}{r} \\$8.42 \\ \times 66 \\ \hline \end{array}$</p>	<p>52. Round money to nearest dollar. Round number to nearest 10. Multiply to find the estimated product. $\begin{array}{r} \\$4.39 \\ \times 78 \\ \hline \end{array}$</p>
	<p>53. Copy and complete. (a) $7 \times 376 = n$ (b) $4 + 7 + 5 = 4 \times n$</p>	<p>54. Copy and complete. (a) $8 \times 460 = n$ (b) $9 \times 3 = n + 16$</p>
90	<p>55. Mrs. Johnson bought 16 boxes of petunias at \$1.75 a box. How much did she pay? How much change did she receive from \$40?</p>	<p>56. Dick bought 12 record albums at \$7.50 each. How much did he spend on record albums? How much change did he receive from \$100?</p>
	<p>57. $6 \overline{)486}$</p>	<p>58. $4 \overline{)324}$</p>
	<p>59. $\\$65\,500 \div 100$</p>	<p>60. $\\$39\,700 \div 100$</p>
	<p>61. $4 \overline{)29\,374}$</p>	<p>62. $8 \overline{)58\,273}$</p>
	<p>63. $35 \overline{)19\,635}$</p>	<p>64. $43 \overline{)15\,566}$</p>
100	<p>65. $51 \overline{)3482}$</p>	<p>66. $64 \overline{)4469}$</p>
	<p>67. $37 \overline{)178.71}$</p>	<p>68. $92 \overline{)601.68}$</p>
	<p>69. (a) 14 m = ____ cm (b) 20 m = ____ hm (c) 60 mm = ____ cm</p>	<p>70. (a) 15 km = ____ m (b) 30 hm = ____ m (c) 70 cm = ____ mm</p>
	<p>71. Calculate the perimeter.</p> 	<p>72. Calculate the perimeter.</p> 
	<p>73. Calculate the circumference. (Use $\pi = 3.14$.)</p> 	<p>74. Calculate the circumference. (Use $\pi = 3.14$.)</p> 
	<p>75. Calculate the area.</p> 	<p>76. Calculate the area.</p> 

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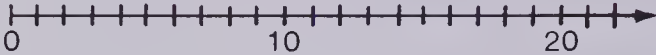
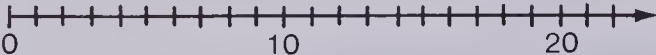
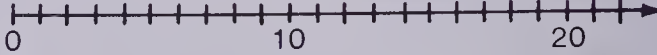
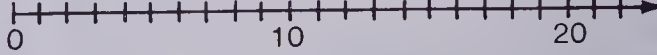
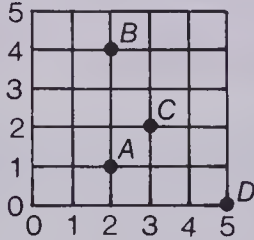
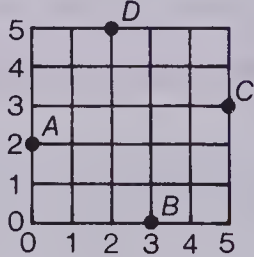
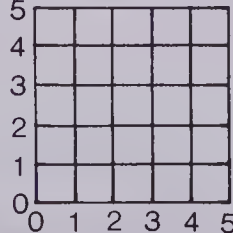
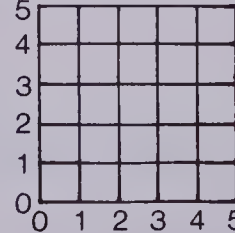
Page	A	B
	<p>77. Calculate the area.</p>  <p>79. Calculate the area.</p>  <p>81. Calculate the area of the parallelogram.</p>  <p>83. Calculate the area of the circle. (Use $\pi = 3.14$.)</p>  <p>85. Calculate the volume.</p>  <p>87. Calculate the capacity in litres.</p>  <p>89. (a) $4 \text{ L} = \underline{\hspace{1cm}} \text{ cm}^3$ (b) $15 \text{ cm}^3 = \underline{\hspace{1cm}} \text{ mL}$</p> <p>91. Complete.</p> <p>$1000 \text{ kg} = \underline{\hspace{1cm}} \text{ t}$</p> <p>$500 \text{ kg} = \underline{\hspace{1cm}} \text{ t}$</p> <p>$2000 \text{ kg} = \underline{\hspace{1cm}} \text{ t}$</p>	<p>78. Calculate the area.</p>  <p>80. Calculate the area.</p>  <p>82. Calculate the area of the parallelogram.</p>  <p>84. Calculate the area of the circle. (Use $\pi = 3.14$.)</p>  <p>86. Calculate the volume.</p>  <p>88. Calculate the capacity in litres.</p>  <p>90. (a) $5000 \text{ cm}^3 = \underline{\hspace{1cm}} \text{ L}$ (b) $56 \text{ mL} = \underline{\hspace{1cm}} \text{ cm}^3$</p> <p>92. Complete.</p> <p>$8 \text{ kg} = \underline{\hspace{1cm}} \text{ g}$</p> <p>$8 \text{ t} = \underline{\hspace{1cm}} \text{ kg}$</p> <p>$8000 \text{ t} = \underline{\hspace{1cm}} \text{ kg}$</p>

Page	A	B
	<p>93. Write as metres. (a) 2 km + 3 hm + 4 dam + 3 m (b) 9 mm + 4 cm + 0 dm + 9 m</p>	<p>94. Write as metres. (a) 5 m + 0 dam + 8 hm + 0 km (b) 2 m + 5 dm + 0 cm + 7 mm</p>
	<p>95. (a) $0.1 \overline{)36}$ (b) $0.5 \overline{)715}$</p>	<p>96. (a) $0.1 \overline{)48}$ (b) $0.8 \overline{)5256}$</p>
	<p>97. $9.5 \overline{)2964}$</p>	<p>98. $8.5 \overline{)4437}$</p>
130	<p>99. $3.7 \overline{)21.83}$</p>	<p>100. $1.7 \overline{)14.79}$</p>
	<p>101. $0.01 \overline{)261.1}$</p>	<p>102. $0.01 \overline{)832.6}$</p>
	<p>103. $0.59 \overline{)21.24}$</p>	<p>104. $0.82 \overline{)44.28}$</p>
140	<p>105. Estimate by rounding, then solve. Mary drives 56 km to work each day. How far does she drive in 11 d?</p>	<p>106. Estimate by rounding, then solve. Tom drives 79 km to work each day. How far does he drive in 26 d?</p>
	<p>107. Complete. (a) 3 L of water has a mass of ____ kg. (b) 2000 cm³ of water has a mass of ____ kg. (c) 2 kg of water has a volume of ____ cm³.</p>	<p>108. Complete. (a) 5 L of water has a mass of ____ kg. (b) 4000 cm³ of water has a mass of ____ kg. (c) 6 kg of water has a volume of ____ cm³.</p>
	<p>109. What time is it 8 h after 16:25? ____</p>	<p>110. What time is it 5 h after 12:05? ____</p>
	<p>111. What time is it 6 h after 22:30? ____</p>	<p>112. What time is it 3 h after 23:35? ____</p>
	<p>113. In the spring the province of British Columbia goes on daylight saving time. Do people set their clocks ahead or back?</p>	<p>114. In the fall when a province returns from daylight saving time to standard time, do people set their clocks ahead or back?</p>
	<p>115. Name the century each year is in. (a) 56 (b) 786 (c) 1876 (d) 2015</p>	<p>116. Name the century each year is in. (a) 99 (b) 666 (c) 1970 (d) 2005</p>
	<p>117. Record the new temperature. (a) The temperature was 8°C; it dropped 12°C. ____ (b) The temperature was -16°C; it dropped 5°C. ____ (c) The temperature was -12°C; it rose 20° C. ____</p>	<p>118. Record the new temperature. (a) It was 12°C; the temperature dropped 6°C. ____ (b) It was -20°C; the temperature dropped 12°C. ____ (c) It was -18°C; the temperature rose 22° C.</p>
150	<p>119. Draw a picture for this problem. Solve. A rose bed is 25 m long and 10 m wide. How much fencing is needed to completely fence the rose bed?</p>	<p>120. Draw a picture for this problem. Solve. Robert jogged 1 km west, 2 km south, and 1 km east. How far from where he started is he?</p>
160	<p>121. Which are divisible by 3? (a) 39 (b) 152 (c) 285</p>	<p>122. Which are divisible by 9? (a) 72 (b) 261 (c) 573</p>

Page	A	B
	123. List the factors of 24.	124. List the factors of 32.
	125. List the factors of 50.	126. List the factors of 55.
	127. (a) $6 + 0 = \underline{\hspace{2cm}}$ (b) $17 \times 0 = \underline{\hspace{2cm}}$ (c) $0 \div 9 = \underline{\hspace{2cm}}$	128. (a) $257 - 0 = \underline{\hspace{2cm}}$ (b) $0 \div 10 = \underline{\hspace{2cm}}$ (c) $4 \times 0 = \underline{\hspace{2cm}}$
	129. Write all the prime numbers between 30 and 40.	130. Write all the prime numbers between 40 and 50.
	131. Write as a product of prime factors.	132. Write as a product of prime factors.
170	(a) $18 = \underline{\hspace{2cm}}$ (b) $50 = \underline{\hspace{2cm}}$	(a) $30 = \underline{\hspace{2cm}}$ (b) $42 = \underline{\hspace{2cm}}$
	133. Write the greatest common factor for: 12 and 30.	134. Write the greatest common factor for: 10 and 16.
	135. Find the missing prime. $50 = 2 \times 5 \times \square$	136. Find the missing prime. $54 = \square \times 3 \times 3 \times 3$
	137. Write the next 3 numbers. 9, 12, 15, 18, $\underline{\hspace{1cm}}$, $\underline{\hspace{1cm}}$, $\underline{\hspace{1cm}}$	138. Write the next 3 numbers. 27, 36, 45, 54, $\underline{\hspace{1cm}}$, $\underline{\hspace{1cm}}$, $\underline{\hspace{1cm}}$
	139. What is the least common multiple for: 4 and 6?	140. What is the least common multiple for: 5 and 6?
	141. Write in expanded notation. 3072	142. Write in expanded notation. 78 509
180	143. (a) $2^2 \times 3^2 = \square$ (b) $10 \times 7^2 = \square$	144. (a) $3^2 \times 5^2 = \square$ (b) $10 \times 6^2 = \square$
	145. $\frac{7}{10} - \frac{3}{10}$	146. $\frac{5}{8} + \frac{1}{8}$
	147. Express in lowest terms. (a) $\frac{12}{15}$ (b) $\frac{8}{14}$	148. Express in lowest terms. (a) $\frac{10}{25}$ (b) $\frac{15}{21}$
	149. Find the missing amount. $\frac{4}{5} = \frac{\square}{20}$ $\square = \underline{\hspace{2cm}}$	150. Find the missing amount. $\frac{3}{8} = \frac{\square}{24}$ $\square = \underline{\hspace{2cm}}$
190	151. $\frac{1}{3}$ $+\frac{1}{4}$ $\underline{\hspace{2cm}}$	152. $\frac{2}{5}$ $+\frac{1}{4}$ $\underline{\hspace{2cm}}$

Page	A	B
200	153. $\begin{array}{r} \frac{4}{5} \\ - \frac{3}{10} \\ \hline \end{array}$	154. $\begin{array}{r} \frac{3}{4} \\ - \frac{1}{5} \\ \hline \end{array}$
	155. $\begin{array}{r} \frac{3}{5} \\ + \frac{7}{10} \\ \hline \end{array}$	156. $\begin{array}{r} \frac{5}{6} \\ + \frac{3}{5} \\ \hline \end{array}$
	157. Write as an equivalent mixed numeral. $\frac{23}{5}$	158. Write as an equivalent mixed numeral. $\frac{37}{8}$
	159. $\frac{1}{4} + \frac{2}{4} + \frac{3}{4}$	160. $\frac{1}{5} + \frac{2}{5} + \frac{3}{5} + \frac{4}{5}$
	161. Express as a decimal. (a) $\frac{71}{100}$ (b) $\frac{6}{1000}$	162. Express as a decimal. (a) $\frac{15}{100}$ (b) $\frac{9}{1000}$
	163. $\begin{array}{r} 1\frac{3}{5} \\ + 2\frac{1}{2} \\ \hline \end{array}$	164. $\begin{array}{r} 2\frac{5}{8} \\ + 3\frac{1}{3} \\ \hline \end{array}$
	165. $\begin{array}{r} 4\frac{3}{10} \\ - 1\frac{1}{4} \\ \hline \end{array}$	166. $\begin{array}{r} 12\frac{4}{5} \\ - 7\frac{1}{2} \\ \hline \end{array}$
	167. $\frac{2}{3}$ of 27	168. $\frac{3}{8} \times 40$
	169. $5 \times \frac{1}{2}$	170. $7 \times \frac{1}{3}$
	171. $\frac{1}{2} \times \frac{5}{8}$	172. $\frac{4}{5} \times \frac{5}{8}$
	173. Solve for n . $4 \times n = 1$	174. Solve for n . $\frac{2}{3} \times n = 1$
	175. $\frac{1}{4}$ of $\frac{1}{2}$	176. $\frac{1}{2}$ of $\frac{2}{3}$
	177. $\frac{4}{5} \times \frac{3}{4}$	178. $\frac{3}{8} \times \frac{7}{4}$
210	179. $\frac{7}{8} \div 6$	180. $5 \div \frac{2}{3}$
	181. $\frac{1}{3} \div \frac{7}{8}$	182. $\frac{4}{5} \div \frac{1}{3}$
	183. $1\frac{1}{2} \times 2\frac{1}{4}$	184. $3\frac{1}{3} \times 4\frac{5}{6}$
	185. $5\frac{1}{2} \div 3\frac{2}{3}$	186. $6\frac{1}{8} \div 3\frac{3}{4}$
	187. Express as a decimal. (a) $\frac{1}{5}$ (b) $\frac{3}{4}$	188. Express as a decimal. (a) $\frac{2}{5}$ (b) $\frac{9}{20}$

Page	A	B
220	<p>189. Round to the nearest tenth. 0.727 272 7...</p> <p>191. Write this ratio in lowest terms. 15:20</p> <p>193. Find the missing number. $\frac{5}{8} = \frac{25}{\square}$ $\square = \underline{\hspace{2cm}}$</p> <p>195. Write as a percent. (a) $\frac{41}{50}$ (b) $\frac{17}{20}$</p>	<p>190. Round to the nearest tenth. 0.575 757...</p> <p>192. Write this ratio in lowest terms. 18:30</p> <p>194. Find the missing number. $\frac{1}{5} = \frac{6}{\square}$ $\square = \underline{\hspace{2cm}}$</p> <p>196. Write as a percent. (a) $\frac{37}{50}$ (b) $\frac{13}{20}$</p>
230	<p>197. Express as a decimal. 47%</p> <p>199. Calculate. (a) 20% of 250 (b) 8% of \$16.50</p>	<p>198. Express as a decimal. 9%</p> <p>200. Calculate. (a) 5% of 140 (b) 6% of \$85.00</p>
240	<p>201. Express as reduced ratios. 15%</p> <p>203. Calculate the average of these numbers. 8, 10, 12, 15, 20</p> <p>205. \$18.87 + \$7.95</p> <p>207. Calculate the yearly interest earned on \$200 if the rate is 13% per year.</p> <p>209. Gina bought a pair of jeans for \$19.95. How much did she pay if the provincial sales tax rate is 7%?</p> <p>211. Calculate the sales tax and the total cost of: A new car for \$9000.00 when the sales tax rate is 7%.</p>	<p>202. Express as reduced ratios. 64%</p> <p>204. Calculate the average of these numbers. 8, 9, 19, 12</p> <p>206. \$193.02 + \$85.59</p> <p>208. Calculate the yearly interest earned on \$500 if the rate is 14% per year.</p> <p>210. Joanne bought a tennis racket for \$25.44 and a tin of tennis balls for \$4.29. How much did she pay altogether if the provincial sales tax rate is 8%?</p> <p>212. Calculate the sales tax and the total cost of: A new truck for \$12 000.00 when the sales tax rate is 8%.</p>
250	<p>213. Track suit, regular \$38.00, save 30% Calculate the discount and the sale price.</p>	<p>214. Tennis racket, regular \$55.00, save 20% Calculate the discount and the sale price.</p>
260	<p>215. Solve. (a) $n + 18 = 50$ (b) $n \times 7 = 35$ (c) $n \div 2 = 10$</p> <p>217. Elephants run at a rate of 40 km/h. How long would an elephant take to run 20 km?</p> <p>219. If you flip a nickel 10 times, how many heads would be likely?</p>	<p>216. Solve. (a) $n + 25 = 80$ (b) $n \times 8 = 72$ (c) $n \div 3 = 6$</p> <p>218. Elephants run at a rate of 40 km/h. How long would an elephant take to run 10 km?</p> <p>220. If you flip a penny 16 times, how many tails would be likely?</p>

Page	A	B
270	<p>221. Using a pair of normal dice, which sum is easiest to roll?</p> <p>223. This year, the Jones family saved 25% on their heating bill by insulating the house. If last year's bill was \$800.00, what did they pay for heating the house this year?</p>	<p>222. Using a pair of normal dice, which sum is hardest to roll?</p> <p>224. This year, the Ross family saved 30% on their heating bill by insulating the house. If last year's bill was \$1500.00, what did they pay for heating the house this year?</p>
	<p>225. Solve, then graph the solution on the number line.</p> <p>(a) $n + 6 = 9$</p>  <p>(b) $s - 12 = 10$</p> 	<p>226. Solve, then graph the solution on the number line.</p> <p>(a) $w - 6 = 4$</p>  <p>(b) $8 + n = 14$</p> 
	<p>227. Write a related subtraction sentence for each. Then solve.</p> <p>(a) $n + 12 = 39$</p> <p>(b) $n + 82 = 156$</p>	<p>228. Write a related subtraction sentence for each. Then solve.</p> <p>(a) $n + 9 = 42$</p> <p>(b) $n + 56 = 81$</p>
	<p>229. Write a related addition sentence for each. Then solve.</p> <p>(a) $n - 18 = 54$</p> <p>(b) $n - 48 = 71$</p>	<p>230. Write a related addition sentence for each. Then solve.</p> <p>(a) $n - 22 = 41$</p> <p>(b) $n - 78 = 101$</p>
	<p>231. Write a related division sentence for each. Then solve.</p> <p>(a) $n \times 6 = 54$</p> <p>(b) $n \times 7 = 9.1$</p>	<p>232. Write a related division sentence for each. Then solve.</p> <p>(a) $n \times 9 = 72$</p> <p>(b) $n \times 6 = 10.2$</p>
280	<p>233. Write a related multiplication sentence for each. Then solve.</p> <p>(a) $n \div 8 = 13$</p> <p>(b) $n \div 26 = 28$</p>	<p>234. Write a related multiplication sentence for each. Then solve.</p> <p>(a) $n \div 7 = 21$</p> <p>(b) $n \div 1.9 = 18$</p>
	<p>235. Write the ordered pair for each lettered point.</p> 	<p>236. Write the ordered pair for each lettered point.</p> 
	<p>237. On the grid locate each point. Label.</p>  <p>(a) (3, 2) (b) (4, 5)</p> <p>(c) (0, 4) (d) (1, 4)</p>	<p>238. On the grid locate each point. Label.</p>  <p>(a) (4, 0) (b) (3, 5)</p> <p>(c) (2, 1) (d) (0, 3)</p>

239. Complete the table.

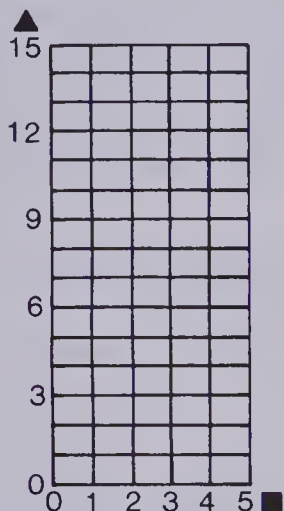
Rule: $2 \times \blacksquare - 3 = \blacktriangle$

\blacksquare	2	3	4	5	6
\blacktriangle					

241. Complete the table. Graph the ordered pairs.

Rule: $2 \times \blacksquare + 4 = \blacktriangle$

\blacksquare	0	1	2	3	4
\blacktriangle					



243. Add. Show each on an integer line.

- (a) $+3 + +7$ (b) $-4 + -3$
 (c) $+8 + -4$ (d) $+4 + -9$
 (e) $-5 + +3$ (f) $-2 + +9$

245. Write the opposite of each.

- (a) $+3$ (b) -7 (c) -2

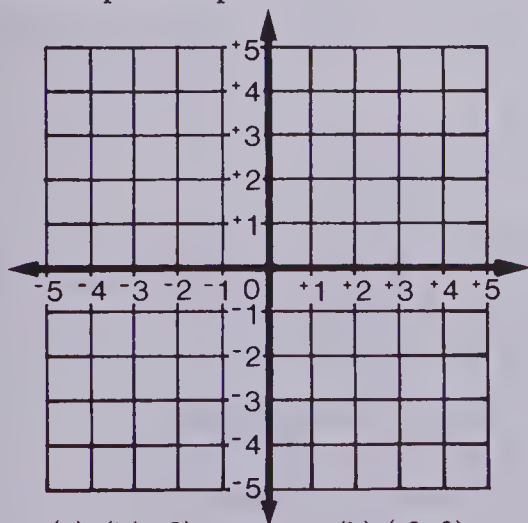
247. Solve.

- (a) $n + -6 = 0$
 (b) $+5 + n = 0$

249. Calculate.

- (a) $+3 - +2$ (b) $-10 - +6$
 (c) $-3 - 0$ (d) $+7 - -3$

251. Graph each point.



- (a) $(+4, -2)$ (b) $(-3, 3)$
 (c) $(0, -4)$ (d) $(-3, -2)$

240. Complete the table.

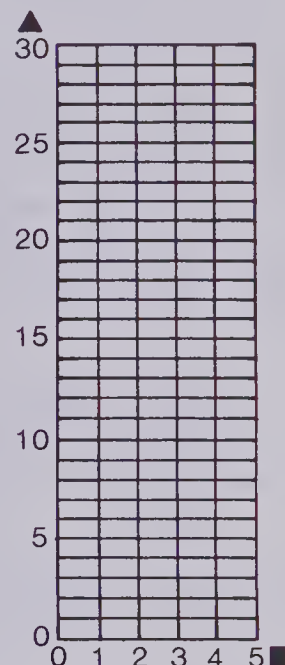
Rule: $5 \times \blacksquare + 8 = \blacktriangle$

\blacksquare	0	1	2	3	4
\blacktriangle					

242. Complete the table. Graph the ordered pairs.

Rule: $6 \times \blacksquare - 2 = \blacktriangle$

\blacksquare	1	2	3	4	5
\blacktriangle					



244. Add. Show each on an integer line.

- (a) $+2 + +5$ (b) $-8 + -2$
 (c) $-6 + -5$ (d) $+3 + -12$
 (e) $-7 + +4$ (f) $-4 + +8$

246. Write the opposite of each.

- (a) -4 (b) $+9$ (c) $+11$

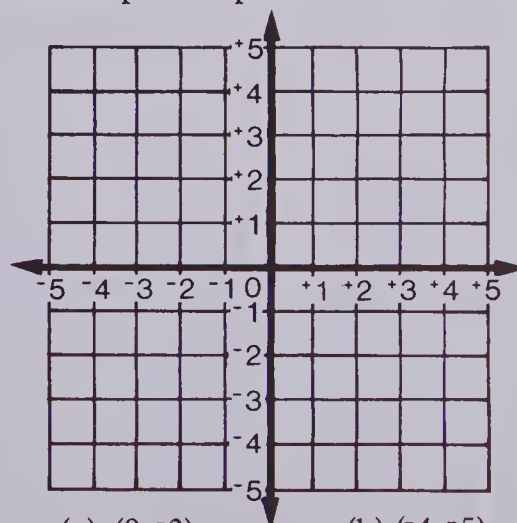
248. Solve.

- (a) $n + -3 = 0$
 (b) $+8 + n = 0$

250. Calculate.

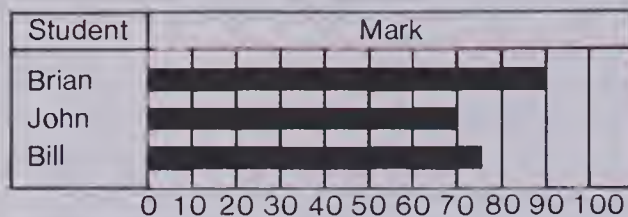
- (a) $+5 - +4$ (b) $-8 - +4$
 (c) $-4 - 0$ (d) $+5 - -2$

252. Graph each point.



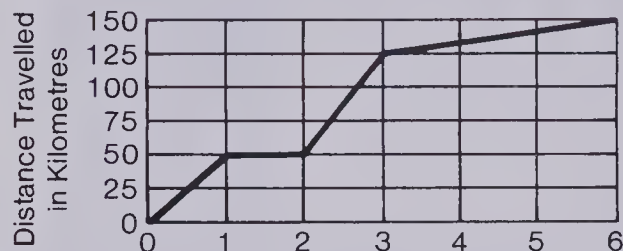
- (a) $(0, -3)$ (b) $(-4, -5)$
 (c) $(+2, -4)$ (d) $(+3, 0)$

253. Marks on Math Test



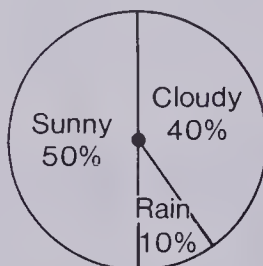
What was Bill's score?

255. Car Rally



- (a) How far did the car travel in the 3rd hour?
 (b) In which hour did the car not travel?

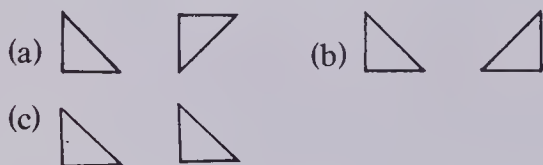
257. Weather in September



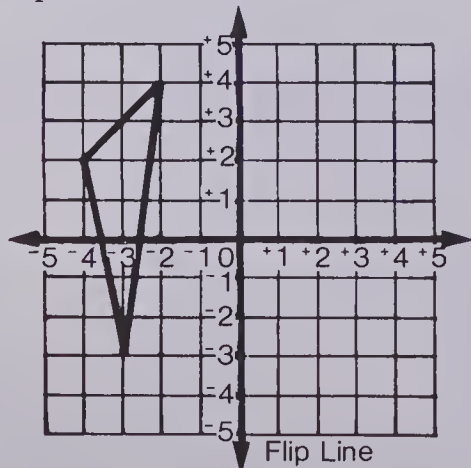
- (a) How many days of rain were there in September?
 (b) How many days of cloud and rain were there altogether?

259. The location of a city is given by 45N, 75W. Is this north or south of the equator?

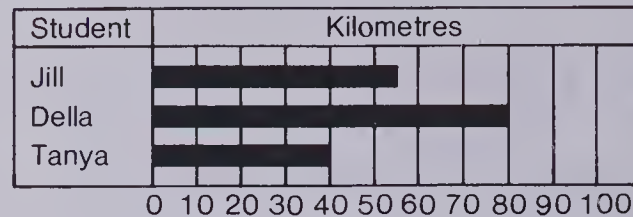
261. Identify each as a slide, flip, or turn.



263. Complete the reflection.

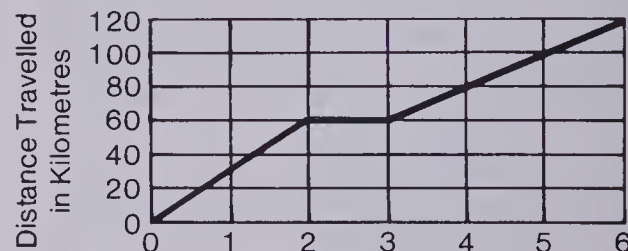


254. Kilometres Jogged in a Month



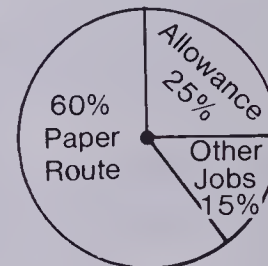
How far did Jill run?

256. Bicycle Trip



- (a) How far did the bicycle travel in the 3rd hour?
 (b) How far did the bicycle travel in the 4th hour?

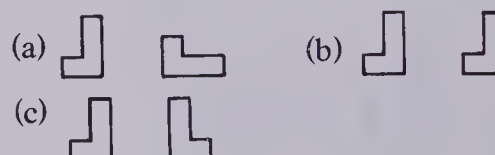
258. Betty's Income



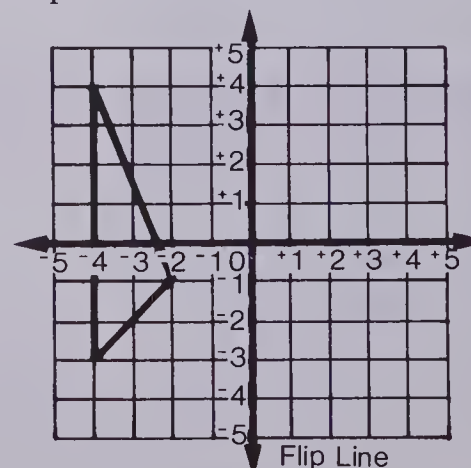
- (a) Betty's total income was \$80. How much was her allowance?
 (b) How much did Betty earn on her paper route?

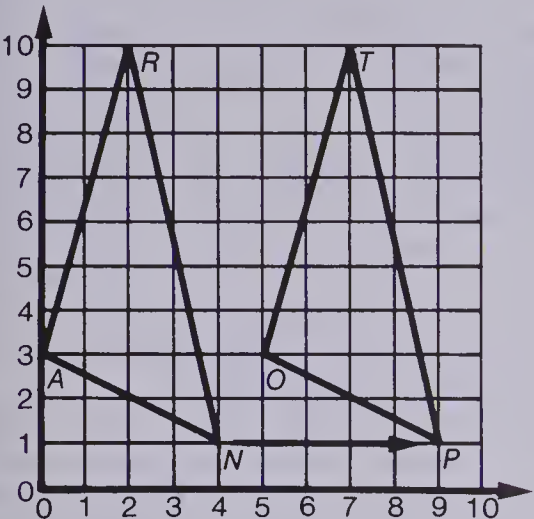
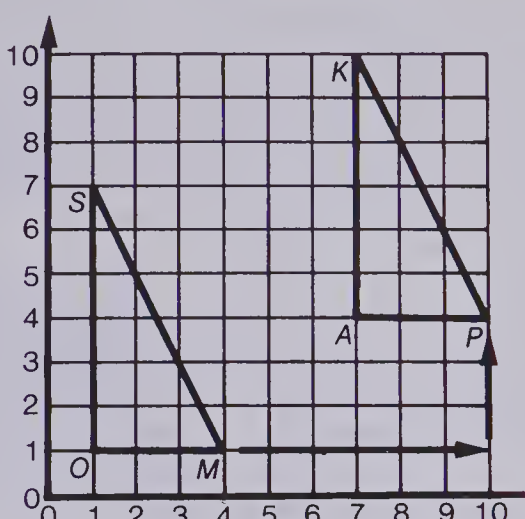



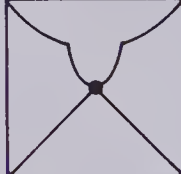
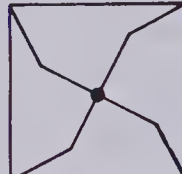
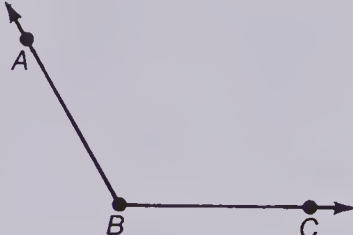
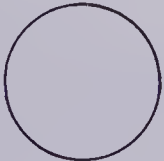
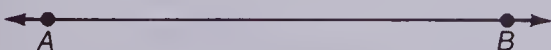
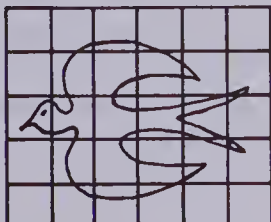
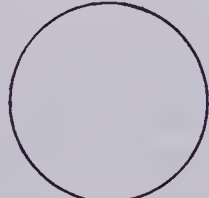
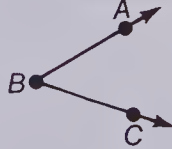
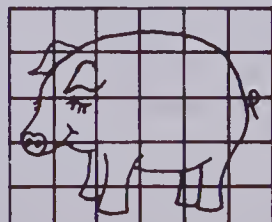
260. The location of a city is given by 30S, 30E. Is it north or south of the equator?

262. Identify each as a slide, flip, or turn.



264. Complete the reflection.



Page	A	B
320	<p>265. Name the corresponding (a) sides (b) vertices.</p> 	<p>266. Name the corresponding (a) vertices (b) sides.</p> 
	<p>267. Which has rotational symmetry?</p> <p>(a)  (b) </p> <p>269. Draw a segment. Construct the perpendicular bisector to the segment.</p> 	<p>268. Which has rotational symmetry?</p> <p>(a)  (b) </p> <p>270. Draw an angle. Construct the bisector.</p> 
330	<p>271. Locate the centre of the circle.</p>  <p>273. Draw a line. Construct another line parallel to the line.</p>  <p>275. Use grid paper. Copy this cartoon so that it is larger.</p> 	<p>272. Locate the centre of the circle.</p>  <p>274. Copy this angle.</p>  <p>276. Use grid paper. Copy this cartoon so that it is larger.</p> 
340	<p>Permission to duplicate this page is granted by Holt, Rinehart and Winston of Canada.</p>	

CHAPTER 1 OVERVIEW

This chapter reviews and develops the concept of place value from billions to ten thousandths. This is intertwined with the addition and subtraction of whole numbers and decimals (to ten thousandths). Also presented are: rounding and estimation of whole numbers and decimal amounts; equations, inequations, and the symbols used therein; and a basic technique for solving word problems.

OBJECTIVES

- A To review and develop the concept of place value from billions to ten thousandths
- B To review addition and subtraction of whole numbers and decimals to ten thousandths
- C To review rounding and estimation procedures
- D To review and/or introduce the concept of equations and inequations
- E To review the commutative and associative properties of addition
- F To solve word problems

BACKGROUND

An introductory chapter such as this provides an excellent opportunity not only to review basic topics previously covered, but to set the precedent for what will be the answer format accepted in your class. Set the standards by displaying an “ideal” written assignment, and be sure to discuss the merits of each point in the format:

- identifiability (name? date? page and question numbers?)
- legibility
- spaces for comment and/or corrections
- time limit

Clarify how responses are to be written (copy and complete? answer only? answers but show rough work?) and establish a system for dealing with corrections (recopy and complete? repairs only? resubmission for remarking?—by when: 3:15?, next day?, etc.).

Once a system has been clarified, be consistent across both time and situation. Be sure to frequently and publicly compliment students on the quality of their exercise workbooks and written assignments.

Rounding and estimation are presented here too, not only because of their close relationship to place value but also to provide a quick method of yielding accurate estimates. These are valuable and often-used skills (about how much time do I have? about how much money do I need? etc.) and ones that require much practice. They will be used later in the text to help establish answer (and number) reasonableness before a calculation is attempted.

When teaching the lessons involving number sentences and equations, be sure to include (a) plenty of practice “balancing” the value on each side of the symbol, (b) a review of the order of operations rules (in this regard you may wish to present a memory aid such as **Bless My Dear Aunt Sally**, i.e., **B**rackets first; **M**ultiplication, **D**ivision, **A**ddition, **S**ubtraction in the order they appear), and (c) the use of related sentences to help solve for unknowns, i.e., $n - 8 = 10$. $10 + 8 = 18$. $\therefore 18 - 8 = 10$. Writing equations and using them to solve word problems will be introduced and developed throughout the text.

The Professor Q approach to solving word problems (formally introduced on page 22) is an attempt to provide order and technique into what is often the most frustrating area of mathematics for students. Provide plenty of opportunity for students to identify only, the necessary operation in word problem without the tedium of formally proceeding through the rest of the problem-solving steps (see page 22, Activity 1). Also, you may wish to help students find key words and phrases such as “altogether”, “total”, “how many more . . .”, etc. as clues as to which operation is appropriate for use.

Read or have read aloud all word problems especially if some members of the group have reading difficulties (see also the Vocabulary heading on the individual lesson pages). Finally, for more capable students, have them answer Professor Q’s 4 questions *mentally* before proceeding to problem-solving step number 2.

MATERIALS

abacus
newspapers and magazines
interlocking centimetre cubes (at least 1000)
advertisements from supermarkets and/or department stores
toothpicks
cutouts of ➤
pictures of objects such as fruit, cars, animals, etc.

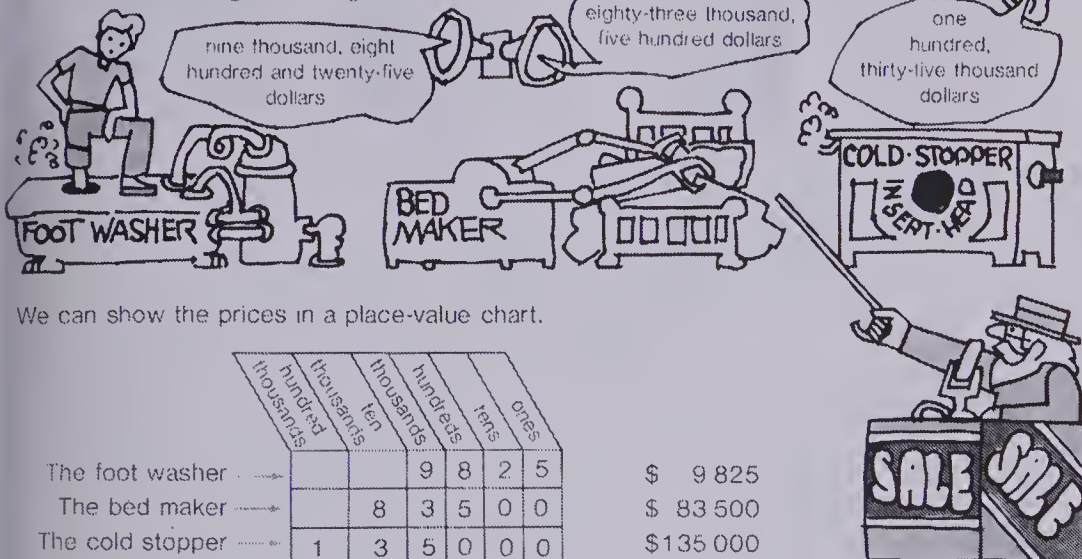
CAREER AWARENESS

Travel Agent [17]

A travel agent provides a service for the travelling public. He/she makes flight reservations, hotel reservations, and car rentals and organizes escorted holidays and cruises. Travel agents supply general information about destinations all around the world. This service is free of charge to the public. A travel agent is paid a commission by airlines, hotels, tour operators, car rental agencies, etc.

Marvelous Machines

Doctor Ivan is selling his strange machines.



We can show the prices in a place-value chart.

	thousands	hundreds	tens	ones	
The foot washer			9	8	25
The bed maker		8	3	5	00
The cold stopper	1	3	5	0	00

\$ 9 825

\$ 83 500

\$135 000

Exercises

1. Read.

- (a) 348 276 (b) 685 203 (c) 97 082 (d) 75 600
 (e) 8270 (f) 956 801 (g) 3657 (h) 60 390
 (i) 84 062 (j) 33 333 (k) 347 852 (l) 4608

2. Write in a place-value chart.

- (a) eighty-eight thousand, four hundred fifty-nine
 (b) five hundred sixty thousand, seven hundred fifty-two
 (c) ninety-one thousand, six hundred twenty
 (d) two thousand, seven hundred thirty-four (e) five thousand, two hundred seven

3. Write the place value of each underlined digit.

- (a) 4765 hundreds (b) 953 hundreds (c) 16 384 ten thousands (d) 54 600 hundreds
 (e) 663 478 thousands (f) 500 362 hundred thousands (g) 18 603 ten thousands (h) 437 052 ten thousands

Place value 1

OBJECTIVE

To review place value from ones to hundred thousands

PACING

Level A All
 Level B All
 Level C All

MATERIALS

abacus

RELATED AIDS

CALC. ACTIVITY MASTERS — 2, 31.

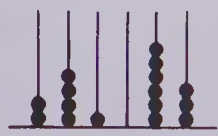
BACKGROUND

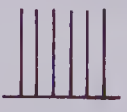
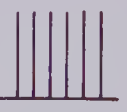
The digits 0, 1, 2, . . . , 9 and their order (or place value) can be used to represent any number. Each successive place-value location has a value ten times greater than the location preceding it (i.e., a "5" in the hundred's location represents a value ten times greater than a "5" in the ten's location, and so on). This holds true throughout the base 10 system as exemplified by the place-value range presented in this chapter (from billions to ten thousandths).


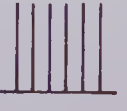
SUGGESTIONS

Initial Activity Draw six-location place-value charts on the chalkboard. Ask the students to make a list of examples where 6-digit numbers might be used, e.g., banks, population, lotteries, etc. Have students suggest appropriate large numbers. Show each number first on the abacus, then write or have it written on the place-value chart. Have the students read each number after it has been written. Ensure that some of the numbers have "0". Review "0" as a placeholder.

Names: (A) _____ (B) _____ (C) _____

 241 063

 _____  _____

 _____  _____

etc.

(continued on page 7)

USING THE BOOK

Read through the display as shown at the top of the student page. Identify the three ways that the prices of Dr. Ivan's Marvellous Machines are represented: in words; in a place-value chart; in numbers.

To generate confidence, read each number in Exercise 1 with the students. Then ask individual students to read each number. Assign Exercises 2 and 3 to all students. You may wish to point out that the answer for Exercise 3(a) is in the back of the text (as are all such labelled exercises).

The review nature of this page makes it an ideal time to lay the foundations by demonstration and/or examples, as to what constitutes the acceptable answer format for your class.

ACTIVITIES

1. Make an odometer using a paper

towel roller and strips of paper numbered 0 to 9. Wrap strips loosely around the roller.



Have students show how the odometer changes from 9 to 10, 99 to 100, etc.

Have students write the number that is one greater than 9999. Show the number on the odometer. Have students write the number that is one less than 100 000. Show the number on the odometer. Combine with several more examples.

2. Prepare and distribute an abacus exchange sheet as shown. Note that in order to be complete, three children contribute to each sheet. Student (A) randomly places dots on each blank abacus. Student (B) labels each now dotted abacus with an appropriate number. Student (C) corrects the work of students (A) and (B).

OBJECTIVE

To review place value to thousandths

PACING

Level A All
Level B All
Level C All

MATERIALS

at least 2 blocks each made up of 1000 cubes as shown at the top of the pupil page

RELATED AIDS

CALC. ACTIVITY MASTERS — 22, 24, 44, 46, 76, 77.

SUGGESTIONS

Initial Activity On the chalkboard, draw a large place-value chart similar to that shown on the pupil page. Show 1 large block. Identify it on the place-value chart by writing 1.000 in the appropriate locations. Repeat for other whole-number examples. Continue this procedure showing block examples for such numbers as 1.568, 2.391, and 1.357. Be sure to (a) have *children* show examples using blocks, (b) have each concrete example identified on the place-value chart, and (c) have each concrete example and its corresponding numerical representation on the chart written as a fraction.

USING THE BOOK

Read through the information shown at the top of the pupil page, consolidating what was demonstrated in the Initial Activity. Complete Exercise 1 orally. You may wish to demonstrate an acceptable answer format using Exercises 2(a), 2(b), and 3(a) before assigning the page.

ACTIVITIES

- See "The P.V. Game" as described in the Activity Reservoir.
- Play "Concentration" as described in the Activity Reservoir. Use matching cards such as:

17.3

$17\frac{3}{10}$

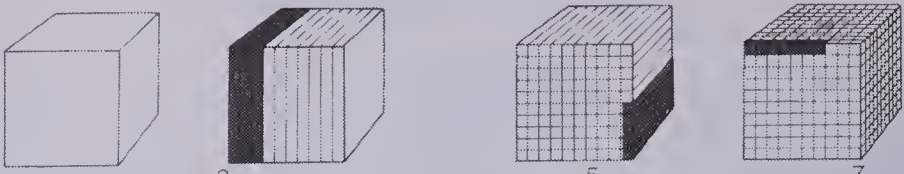
86.05

$86\frac{5}{100}$

etc.

- If you have not already done so, see the abacus exchange sheets

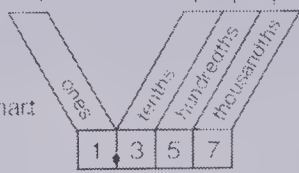
Decimals



$\frac{3}{10}$ or 0.3

$\frac{5}{100}$ or 0.05

$\frac{7}{1000}$ or 0.007



Place-value chart

We say:
one and three hundred fifty-seven thousandths
or
one decimal three five seven.

We write:
 $1\frac{357}{1000}$
or
1.357.

Exercises

- Read each in two ways.

(a) 7.2	(b) 0.3	(c) 4.7	(d) 13.4	(e) 64.9
(f) 35.05	(g) 8.13	(h) 19.38	(i) 55.09	(j) 12.77
(k) 6.002	(l) 4.369	(m) 23.408	(n) 12.042	(o) 69.313
(p) 200.011	(q) 87.413	(r) 346.007	(s) 68.019	(t) 500.845
- Write each as a decimal.

(a) $7\frac{3}{10}$ 7.3	(b) $13\frac{9}{10}$ 13.9	(c) $63\frac{7}{10}$ 63.7	(d) $214\frac{2}{10}$ 214.2	(e) $78\frac{8}{10}$ 78.8
(f) $18\frac{89}{100}$ 18.89	(g) $7\frac{27}{100}$ 7.27	(h) $32\frac{16}{100}$ 32.16	(i) $6\frac{7}{100}$ 6.07	(j) $41\frac{9}{100}$ 41.09
(k) $2\frac{235}{1000}$ 2.235	(l) $5\frac{147}{1000}$ 5.147	(m) $11\frac{26}{1000}$ 11.026	(n) $50\frac{47}{1000}$ 50.047	(o) $16\frac{5}{1000}$ 16.005
- Write the place-value meaning of each underlined digit.

(a) 8. <u>4</u> 32	(b) 3. <u>9</u> 76	(c) 17. <u>2</u> 03	(d) 28. <u>0</u> 32	(e) 88. <u>0</u> 04
(f) 46. <u>1</u> 78	(g) 80. <u>6</u> 7	(h) 10. <u>4</u>	(i) 701. <u>3</u> 82	(j) 19. <u>0</u> 5

2 Place value to thousandths

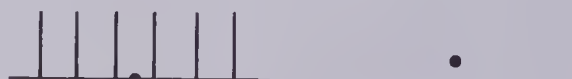
ANSWERS:

Page 1 2.

	hundred thousands	ten thousands	thousands	hundreds	tens	ones
(a)		8	8	4	5	9
(b)	5	6	0	7	5	2
(c)		9	1	6	2	0
(d)			2	7	3	4
(e)			5	2	0	7

Page 2. 3. (a) hundredths; (b) tenths; (c) thousandths; (d) hundredth; (e) thousandths; (f) tenths; (g) hundredths; (h) tenths; (i) thousandths; (j) hundredths

described in Activity 2, page 1. Modify the abacus blanks to include decimals to thousandths by instructing the students to place decimal points accordingly:



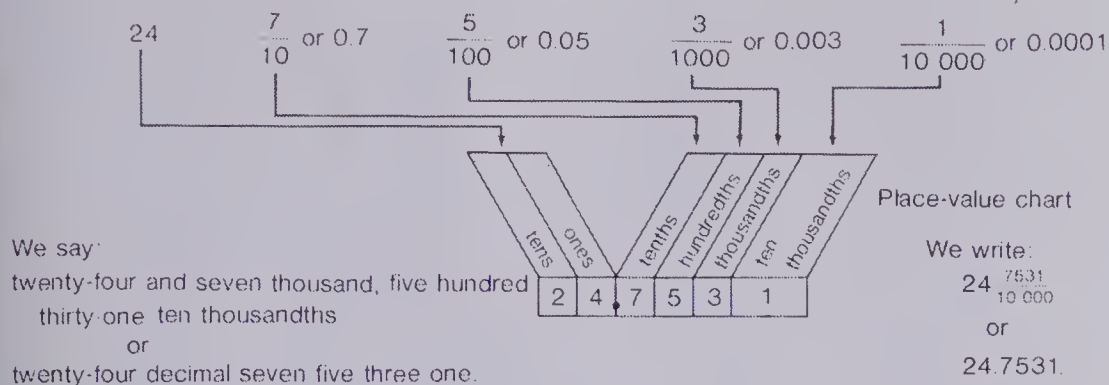
- Ask students to find examples of the use of decimals in everyday life, e.g., money, different forms of measurement expressed in decimal form (e.g., 14.5 kg, 12.85 m).

EXTRA PRACTICE

Write each as a fraction and as a decimal.

- two tenths
- five hundredths
- three thousandths
- forty-five hundredths
- four and seven tenths
- thirty-six and twenty-five hundredths
- eight and two hundred sixty-two thousandths

More Decimals



Exercises

1. Read each in two ways.

- (a) 7.7614 (b) 3 263 (c) 8.68 (d) 10.3 (e) 9.8826
(f) 0.061 (g) 36.54 (h) 28.715 (i) 40 3946 (j) 36.9
(k) 6.0042 (l) 75 102 (m) 11.0001 (n) 83.65 (o) 68.0043

2. Write as a fraction.

- (a) 1.7 $1\frac{7}{10}$ (b) 8.35 $8\frac{35}{100}$ (c) 2 384 $2\frac{384}{1000}$ (d) 17.2371 $17\frac{2371}{10000}$
(e) 36.053 $36\frac{53}{1000}$ (f) 97.0072 $97\frac{72}{10000}$ (g) 43.601 $43\frac{601}{1000}$ (h) 236.7003 $236\frac{7003}{10000}$

3. Write as a decimal.

- (a) $3\frac{487}{1000}$ 3.487 (b) $29\frac{8}{100}$ 29.08 (c) $37\frac{29}{10000}$ 37.0029 (d) $41\frac{447}{1000}$ 41.447
(e) $19\frac{71}{100}$ 19.71 (f) $82\frac{3}{10000}$ 82.0003 (g) $157\frac{43}{1000}$ 157.043 (h) $446\frac{9}{10}$ 446.9

4. Write the place-value meaning for each underlined digit.

- (a) 76.5429 (b) 107.0569 (c) 25.7462 (d) 91.413
thousandths ten thousandths tenths hundredths

5. Write as a decimal.

- (a) twenty-four and thirty-seven hundredths 24.37
(b) seven and nineteen thousandths 7.019 (c) forty and fifty-six ten thousandths 40.0056

Place value to ten thousandths 3

OBJECTIVE

To introduce place value to ten thousandths

PACING

Level A All
Level B All
Level C All

RELATED AIDS

HMS—DM1.

SUGGESTIONS

Initial Activity On the chalkboard, draw a place-value chart similar to the one in the display, but do not include the ten thousandth's column. To establish the symmetry of our counting system, draw in the column next to the tens and ask students to identify it [hundreds]. Next, add the thousand's column and ask students to name it. Then draw the next column and ask students to name it [ten thousands]. Draw attention to the symmetry of the chart on either side of the decimal point, i.e., tens, hundreds, thousands, tenths, hundredths, thousandths. Now draw in the ten thousandth's column and ask students to provide the name for it. Fill in the chart as shown in the display and go over the methods of saying and writing the number. Choose another number and have a student write it on the place-value chart. Establish how it is said and written. Continue with several more examples.

USING THE BOOK

Read through the display together at the top of the pupil page. Discuss (a) the similarity to what was presented on page 2 and (b) the various ways in which 24.7531 is represented. Complete Exercise 1 orally. You may also wish to complete the first few examples of Exercises 2 and 3 orally to establish an acceptable answer format.

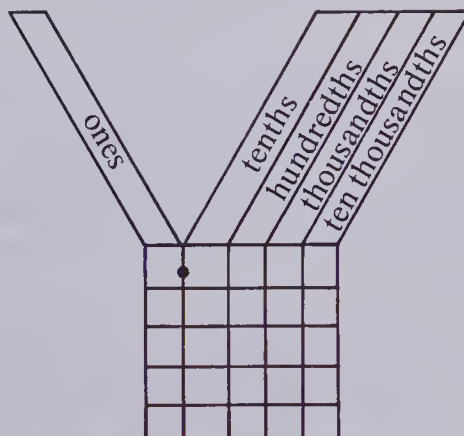
ACTIVITIES

1. See "The P.V. Game" as described in the Activity Reservoir.

2. See "Concentration" as described in the Activity Reservoir. Use matching card sets such as:

19.4251	$19\frac{4251}{10000}$
126.052	$126\frac{52}{1000}$ etc.

3. Provide blank "Place Value Bingo" sheets as shown. Have the children fill in the spaces provided using the numerals from 0 to 9. A leader or caller calls out various numerals and headings in bingo fashion — "under the tenths, six", etc. Regular Bingo rules apply.



EXTRA PRACTICE

Write the decimal form for each.

1. $38\frac{7}{100}$ 2. $134\frac{857}{1000}$
3. $96\frac{4375}{10000}$ 4. $16\frac{37}{1000}$
5. $59\frac{36}{10000}$ 6. $147\frac{99}{100}$
7. $500\frac{1}{10000}$ 8. $43\frac{675}{10000}$

OBJECTIVE

To review addition of whole numbers and dollar amounts

PACING

- Level A All
- Level B All
- Level C All

RELATED AIDS

BFA COMP LAB II — 10, 11, 15.
BFA PROB. SOLVING LAB II — 206.
CALC. ACTIVITY MASTERS — 3, 9, 32, 33.

SUGGESTIONS

Initial Activity Conduct a warm-up activity involving addition facts. Call out a series of numbers. Students mentally compute their sum and write the answer. Begin with small single-digit numerals and a limited number of addends. Say numbers slowly. Increase the number of addends to a maximum of 5, using single-digit numerals. Write the following examples on the chalkboard.

(A) 32
8576
977
+ 1354

Review the addition technique with students. Encourage students to think—11, 17, 19 rather than 4 and 7 is 11, 11 and 6 is 17, 17 and 2 is 19.

(B) \$ 79.07
903.13
+ 275.07

Review addition of money with the students. Remind them that we add money in the same way we add whole numbers. However, we insert the decimal point and the dollar sign in money.

(C) 56 + 307 + 4736 + 5

Review with the students the technique of adding horizontally. You may wish to instruct the students to rewrite all or some of the examples in vertical form before they add.

USING THE BOOK

Before students begin their work, you may wish to have them project how many they think they will get correct in each set. When their work has been marked they can compare their projections and the number they got correct.

Tune Up — Addition

Add

1. (a) 8
6
7
4
3
+6
34

(b) 4
7
8
5
2
+5
31

(c) 9
8
0
4
7
+2
30

(d) 33
85
94
4
+69
285

(e) 80
47
60
38
+76
301
2. (a) 6764
4237
+4609
15 610

(b) 4682
3007
63
+9572
17 324

(c) \$605.75
26.32
+974.37
\$ 1606.44

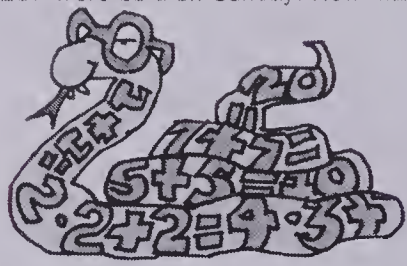
(d) \$917.36
402.95
5.32
+427.70
\$ 1753.33

(e) \$483.81
741.32
400.16
28.38
+768.02
\$ 2421.69
3. (a) 3 + 7 + 6 + 2 + 7 25
(c) 91 + 42 + 34 + 41 208
(e) 13 + 27 + 3 + 70 113

(b) 4 + 6 + 9 + 3 + 0 22
(d) 62 + 4 + 90 + 54 210
4. (a) 676 + 426 + 375 1477
(c) 7483 + 68 + 745 + 774 9070

(b) 403 + 210 + 654 1267
(d) \$8603.17 plus \$4328.20 \$12 931.37
5. During the Rick's Record Shop Super Sale, 489 albums were sold on Friday, 976 were sold on Saturday, and 287 were sold on Sunday. How many albums were sold during the three days? 1752

Are you a super adder,
good adder,
average adder?



Super	18-20
Good	13-17
Average	10-12

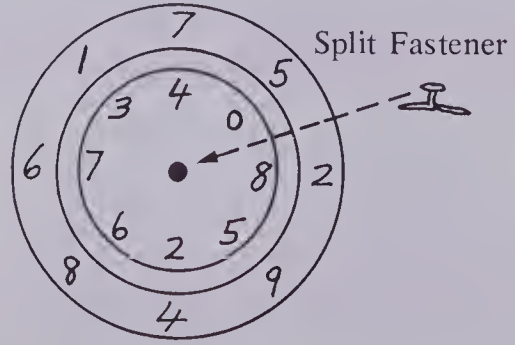
4 Addition

You may wish to assign the work over 2 class periods or in tandem with some of the exercises on page 5.

Review exercises such as these are ideal also in establishing acceptable answer procedures, work habits, or for use as an informal test for diagnostic purposes.

ACTIVITIES

1. To review basic facts and provide practice in the mental process involved in column addition, make two circles labelled as shown and connected split fastener as shown.



The inner wheel is spun and when it stops, the numbers on the inner wheel

are added to the numbers opposite them on the outer wheel. The answers can be oral or written. As an extension, use numbers with more than one digit and/or time the responses.

2. Provide challengers such as: Find the missing digits.

66?
+ 325
??7

372
+ 4?5
?97

315?
2?35
+ 289
?876

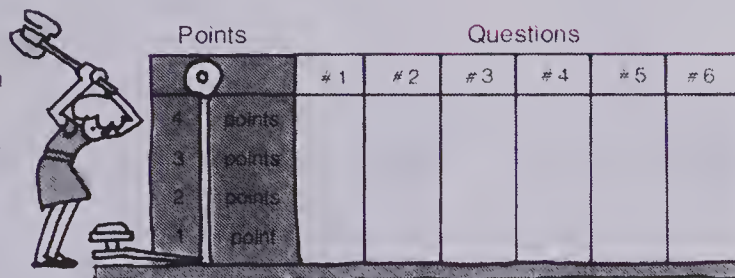
3. Some students might enjoy using calculators to prepare their own "challengers" as shown in Activity 2 above. Have them written on cards (with answers provided on the back) and exchanged with classmates, other groups, or classes in the school.

Tune Up—Subtraction

Subtract

1. (a) $\begin{array}{r} 645 \\ - 128 \\ \hline 517 \end{array}$ (b) $\begin{array}{r} 390 \\ - 149 \\ \hline 241 \end{array}$ (c) $\begin{array}{r} 8207 \\ - 4939 \\ \hline 3268 \end{array}$ (d) $\begin{array}{r} 4700 \\ - 1321 \\ \hline 3379 \end{array}$
2. (a) $\begin{array}{r} 94\ 329 \\ - 25\ 530 \\ \hline 68\ 799 \end{array}$ (b) $\begin{array}{r} 60\ 022 \\ - 7\ 869 \\ \hline 52\ 153 \end{array}$ (c) $\begin{array}{r} 56\ 004 \\ - 28\ 226 \\ \hline 27\ 778 \end{array}$ (d) $\begin{array}{r} 68\ 247 \\ - 4\ 909 \\ \hline 63\ 338 \end{array}$
3. (a) $\begin{array}{r} 588\ 467 \\ - 361\ 187 \\ \hline 227\ 280 \end{array}$ (b) $\begin{array}{r} 207\ 156 \\ - 39\ 078 \\ \hline 168\ 078 \end{array}$ (c) $\begin{array}{r} 800\ 000 \\ - 17\ 876 \\ \hline 782\ 124 \end{array}$ (d) $\begin{array}{r} 294\ 094 \\ - 3\ 685 \\ \hline 290\ 409 \end{array}$
4. (a) $\begin{array}{r} \$219.78 \\ - 27.83 \\ \hline \$191.95 \end{array}$ (b) $\begin{array}{r} \$20.59 \\ - 14.58 \\ \hline \$6.01 \end{array}$ (c) $\begin{array}{r} \$4768.00 \\ - 2176.46 \\ \hline \$2591.54 \end{array}$ (d) $\begin{array}{r} \$56\ 834.16 \\ - 7\ 462.27 \\ \hline \$49\ 371.89 \end{array}$
5. (a) $429 - 267 = 162$ (b) $700 - 287 = 413$ (c) $\$347.76 - \$50.62 = \$297.14$ (d) $\$7624.39 - \$4552.50 = \$3071.89$
6. (a) How much less than 11 000 is 7560? 3440
 (b) What is the difference between 29 362 and 76 840? $47\ 478$
 (c) 68 000 minus 9765 $58\ 235$
 (d) It is 2365 km to Vancouver.
 The Browns have driven 1870 km toward Vancouver.
 How many more kilometres do they have to travel? 495 km

Each correct answer is worth 1 point.
Copy and graph your results.



Drill — subtraction 5

OBJECTIVE

To review subtraction of whole numbers and dollar amounts

PACING

Level A All
Level B All
Level C 1-4 (parts (c) and (d)), 5, 6

RELATED AIDS

BFA COMP LAB II—21, 24-28.
CALC. ACTIVITY MASTERS—9, 32, 33.

SUGGESTIONS

Initial Activity Conduct a drill for subtraction facts using flash cards. Write the following examples on the chalkboard to review subtraction techniques with and without regrouping.

$$\begin{array}{r} 584 \\ - 321 \\ \hline \end{array} \quad \begin{array}{r} 834 \\ - 651 \\ \hline \end{array} \quad \begin{array}{r} 15\ 362 \\ - 3\ 685 \\ \hline \end{array}$$

$$\begin{array}{r} 307 \\ - 154 \\ \hline \end{array} \quad \begin{array}{r} 5000 \\ - 276 \\ \hline \end{array} \quad \begin{array}{r} \$17.48 \\ - 8.07 \\ \hline \end{array}$$

Review the meanings of the terms “difference” and “minus”.

USING THE BOOK

You may wish to use this page as an informal test for diagnostic purposes. Have the students project the number of questions they will have correct before they do the page. Compare their results with their projections. You may wish to (a) assign this page over 2 class periods or (b) assign some exercises in tandem with selections from page 4 or (c) provide graph paper or blank graphs as shown at the bottom of the page for recording results. If you use the suggested Pacing above with Level C students, advise them that, for purposes of graphing, parts (c) and (d) of Exercises 1 to 4 are each worth 2 points.

ACTIVITIES

1. Have each student make up a problem which involves a single subtraction computation, e.g., “My allowance is \$10.00. I spent \$3.75. How much do I have left?” Write 5 or 6 on the chalkboard each day as extra practice.

2. On a stencil, make up subtraction questions which contain some errors in the answers, e.g.,

$$\begin{array}{r} 26 \\ - 9 \\ \hline 18 \end{array}$$

Students first find the errors, then correct them.

3. This is a game for 2 players. Have each pair write the digits 0 to 20 on individual pieces of paper, e.g., $\boxed{0}$ $\boxed{1}$ $\boxed{2}$ etc. They are shuffled and placed face down in a pile. The first player takes the top three cards and arranges them in any order to form a subtraction question, e.g., $\boxed{12}$ $\boxed{7}$ $-$ $\boxed{6}$. This represents the subtraction question $127 - 6$. If the opponent answers correctly, he/she scores points equal to the resulting difference (121). Players exchange roles. Player with the greatest cumulative total when the deck is exhausted is the winner.

OBJECTIVE

To extend the students' understanding of addition of whole numbers to include addition of decimals up to thousandths

PACING

Level A All
Level B All
Level C All

MATERIALS

blocks made up of one thousand interlocking centimetre cubes

RELATED AIDS

BFA COMP LAB II — 92-95.

SUGGESTIONS

Initial Activity Using chalkboard examples similar to the one in the display, review addition of tenths and hundredths. Draw specific attention to the aligning of the decimal point.

Use the interlocking cubes to demonstrate the addition of 35 thousands and 27 thousandths. Have the answer expressed in thousandths. Write the addition example on the board. Do several more examples using the blocks, then record the examples on the board. Be sure to draw attention to the similarity of process to the addition of whole numbers.

USING THE BOOK

Do the first two examples of Exercise 1 with the students. Then assign the rest of the exercises.

In Exercise 2, you may wish to do the first example on the board with the students, reminding them of the importance of aligning the columns accurately.

ACTIVITIES

1. Distribute strips of paper of varying lengths. Students measure the strips precisely, i.e., to nearest millimetre. Ask them to compute the total length of 2 strips, 3 strips, etc., writing their answers in decimal form.

2. Have students find the perimeter of their desks to the nearest centimetre using a metric ruler. Write the answer in decimal form.

3. Have students find the perimeter of the classroom to the nearest centimetre using a metrestick

Adding Decimals I

$3.435 + 0.527 = ?$

$3.435 + 0.527 = 3.962$

Hint
Line up the decimals

Write \rightarrow

3	435
+ 0	527
3	962

Exercises

1. Copy and complete.

Write \rightarrow

2	2
6	4
<hr/>	
8	6

8.6

(b)

Write \rightarrow

1	7
7	4
<hr/>	
8	1

9.1

(d) $13.68 + 8.26 = 21.94$

(i) $7.428 + 9.604 + 4.843 = 21.875$

(e) $26.50 + 14.25 = 40.75$

(j) $8.666 + 4.287 + 16.005 = 28.958$

(f) $37.07 + 11.60 = 48.67$

(k) $5.006 + 0.276 + 36.305 + 42.444 = 84.031$

(g) $48.93 + 19.48 = 68.41$

(l) $0.6125 + 0.4073 + 0.3214 + 0.0786 = 1.4198$

(h) $231.37 + 429.84 = 661.21$

(m) $4.3856 + 7.1074 + 5.4923 + 3.6701 = 20.6554$

2. Write in columns, and then add.

(d) $7.326 + 4.475 = 11.801$

(g) $17.493 + 49.851 = 67.344$

(j) $35.5 + 63.9 + 98.3 = 197.7$

(i) $57.05 + 32.327 = 89.377$

(e) $36.99 + 80.62 = 117.61$

(h) $17.08 + 448.12 = 465.20$

(k) $3.4685 + 7.9126 + 7.3074 = 18.6885$

(l) $36.7264 + 9.4756 + 42.3898 = 88.5918$

(c) $40.5 + 27.3 = 67.8$

(f) $796.52 + 473.06 = 1269.58$

(i) $0.27 + 0.38 + 0.05 = 0.70$

(l) $542.63 + 35.24 + 498.81 = 1076.68$

6 Addition of decimals

ANSWERS:

2. (a) 11.801 (b) 117.61 (c) 67.8 (d) 67.344 (e) 465.20 (f) 1269.58
(g) 197.7 (h) 18.6885 (i) 0.70 (j) 89.377 (k) 88.5918
(l) 1076.68

or metric measuring tape. Write the answer in decimal form.

EXTRA PRACTICE

Add.

<p>1. $0.394 + 5.761 + 37.448 = 43.603$</p>	<p>2. $65.9825 + 37.4681 + 9.0159 = 112.4665$</p>
--	--

3. $357.42 + 900.57 + 63.004 = 1320.994$

4. $158.443 + 840.159 + 682.376 = 1680.978$

Adding Decimals II

Barbara measured 3 lines. Their lengths were 3.7 m, 14.08 m, and 9.4 m. She found the total length by adding.

$$\begin{array}{r} 3.7 \\ 14.08 \\ + 9.4 \\ \hline 27.18 \end{array}$$

Remember, keep the decimal points under each other.



Altogether the lines measured 27.18 m.

Exercises

1. Add

$$\begin{array}{r} 4.68 \\ 3.1 \\ + 24.75 \\ \hline 32.53 \end{array}$$

$$\begin{array}{r} 52.517 \\ 3.46 \\ 957.634 \\ + 4.17 \\ \hline 1017.781 \end{array}$$

$$\begin{array}{r} 36.742 \\ 4.8 \\ + 15.748 \\ \hline 57.290 \end{array}$$

$$\begin{array}{r} 30.0 \\ 57.463 \\ 8.1754 \\ + 56.82 \\ \hline 152.4584 \end{array}$$

$$\begin{array}{r} 104.53 \\ 37.469 \\ + 41.0372 \\ \hline 183.0362 \end{array}$$

$$\begin{array}{r} 8.715 \\ 19.4602 \\ 9.44 \\ + 26.953 \\ \hline 64.5682 \end{array}$$

$$\begin{array}{r} 74.05 \\ 13.176 \\ + 342.9 \\ \hline 430.126 \end{array}$$

$$\begin{array}{r} 3.45 \\ 28.6004 \\ 13.501 \\ + 57.2765 \\ \hline 102.8279 \end{array}$$

2. Write the numbers in columns, and then add

$$4.53 + 9.761 + 6.449 = 20.740$$

$$(c) 142.76 + 37.541 + 3.7 = 184.001$$

$$(e) 645.38 + 39.4 + 4.7621 = 689.5421$$

$$(g) 900.0 + 42.358 + 16.4417 = 958.7997$$

$$(i) 28.49 + 56.7 + 38.985 + 62.51 = 186.685$$

$$26.4 + 13.563 + 48.5 = 88.463$$

$$(d) 59.61 + 347.555 + 98.3 = 505.465$$

$$(f) 3468.9 + 490.76 + 713.459 = 4673.119$$

$$(h) 34.0 + 76.99 + 187.4167 = 298.4067$$

3. Mr. Davis painted 3 lines for schoolyard games. One was 3.45 m, another was 12.4 m, and the last was 8.55 m. How long are these altogether? **24.4 m**

Addition of decimals 7

OBJECTIVE

To provide practice in addition of decimals with gaps in the addends

PACING

Level A All
Level B All
Level C All

RELATED AIDS

BFA COMP LAB II—92-95.
BFA PROB. SOLVING LAB II—137.

SUGGESTIONS

Initial Activity On a piece of paper or in their notebooks, students draw a line measuring precisely 0.105 m. They draw another line measuring precisely 0.06 m. Ask students to find the total length of both lines. Do the example on the board stressing the need to place the decimal points under each other and also the need to align the columns accurately.

$$\begin{array}{r} 0.105 \\ + 0.06 \\ \hline 0.165 \end{array}$$

USING THE BOOK

Read through the display at the top of the pupil page. Be sure to draw attention to the lengths of the three lines that Barbara measured and the computation shown.

You may also wish to (a) complete Exercises 1(a) and 2(a) orally, (b) advise the students to annex a zero where necessary to help keep computations accurate, and (c) provide graph paper on which the students may work.

ACTIVITIES

1. Have students find the perimeters of rectangles, triangles, and/or irregular shapes of various sizes. They should express the answers in decimal form.

2. See "Square It" as described in the Activity Reservoir. Use decimals with varying numbers of decimal places.

3. See "500 Grand" as described in the Activity Reservoir. Adjust the playing board accordingly.

(continued from page 1)

3. See "The P.V. Game" as described in the Activity Reservoir.

EXTRA PRACTICE

1. Write in words.

- (a) 594 000 (b) 65 428
(c) 802 300 (d) 10 012
(e) 76 497 (f) 400 900
(g) 3478 (h) 385 018

2. Write the number that is one more.

- (a) 989 (b) 3769
(c) 89 999 (d) 47 378

3. Write the values of the underlined digits in Exercise 1.

OBJECTIVE

To extend students' understanding of subtraction of decimals to thousandths

PACING

Level A All
Level B All
Level C All

RELATED AIDS

BFA COMP LAB II—99.

SUGGESTIONS

Initial Activity If necessary, review the standard procedure used for subtracting 4- and 5-digit whole numbers with regrouping. When several examples are complete on the chalkboard, repeat using decimal numbers to thousandths. Draw attention to the similarity of process. Point out that the major difference is the presence of the decimal point.

You may also wish to demonstrate using the blocks and cubes described on page 6, what the computations shown on the chalkboard represent.

USING THE BOOK

You may wish to explain to the students that baseball batting averages are calculated like this:

$$\frac{\text{number of hits}}{\text{number of times at bat}} \text{ i.e., } \frac{2}{5} = 0.400$$

Read through the display together at the top of the pupil page, emphasizing the similarity of process to the subtraction of whole numbers.

Complete Exercises 1(a), 1(b), and, if necessary, 2(a) orally before assigning the remaining exercises. You may wish to provide graph paper on which the assignment may be more neatly completed.

ACTIVITIES

1. Write an example of subtraction of decimals on the board, e.g., $12.75 - 6.03$. Have students write a problem involving these numbers and then solve it.

2. Using some baseball statistics available in the local paper, prepare several problems similar to that in the display at the top of the pupil page.

3. Have students measure the length and width of a textbook to the nearest millimetre. Have them

calculate the difference in these measurements and express it in decimal form.

EXTRA PRACTICE

1. Have students measure the length and width of the classroom to the nearest centimetre. Have them calculate the difference and express it in decimal form.
2. Have students measure their height and the height of a classmate to the nearest

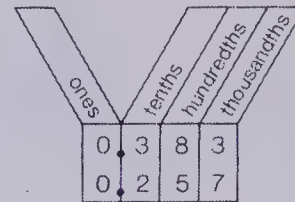
Subtracting Decimals

John's batting average in baseball is 0.383.

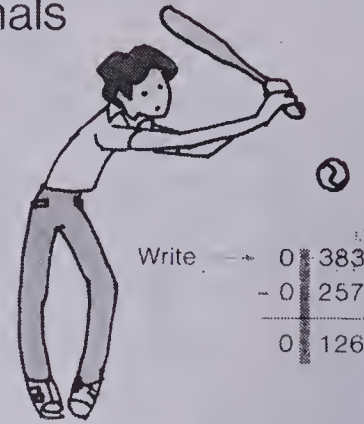
Henry's batting average is 0.257.

How much better is John's batting average than Henry's?

To find out, we must subtract:
 $0.383 - 0.257$



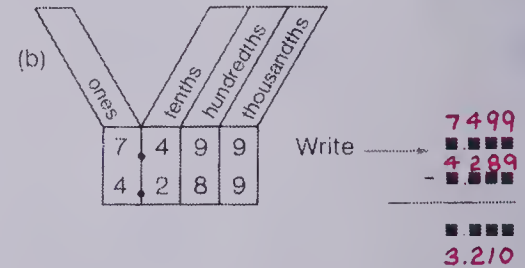
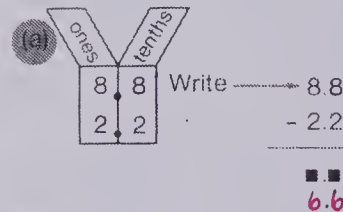
John's batting average is better by 0.126.



Write
$$\begin{array}{r} 0.383 \\ - 0.257 \\ \hline 0.126 \end{array}$$

Exercises

1. Copy and complete.



- (c)
$$\begin{array}{r} 48.844 \\ - 24.379 \\ \hline 24.465 \end{array}$$
 (d)
$$\begin{array}{r} 75.721 \\ - 37.346 \\ \hline 38.375 \end{array}$$
 (e)
$$\begin{array}{r} 53.415 \\ - 35.148 \\ \hline 18.267 \end{array}$$
 (f)
$$\begin{array}{r} 8.5232 \\ - 3.2642 \\ \hline 5.2590 \end{array}$$
 (g)
$$\begin{array}{r} 4.7800 \\ - 2.3489 \\ \hline 2.4311 \end{array}$$

2. Write in columns, and then calculate.

- (a) $8.9 - 3.3$ (b) $59.46 - 32.13$ (c) $340.8 - 70.2$ (d) $46.658 - 18.279$
(e) $95.76 - 31.40$ (f) $7.7006 - 2.6225$ (g) $80.85 - 45.38$ (h) $0.902 - 0.418$
(i) $517.5 - 270.2$ (j) $0.4468 - 0.1859$ (k) $56.91 - 27.65$

3. Janet's batting average is 0.319. Gail's is 0.280.

How much better is Janet's average than Gail's? **0.039**

8 Subtraction of decimals

ANSWERS:

2. (a) 5.6 (b) 27.33 (c) 270.6 (d) 28.379 (e) 64.36 (f) 5.0781
(g) 35.47 (h) 0.484 (i) 247.3 (j) 0.2609 (k) 29.26

centimetre. Have them calculate the difference and express it in decimal form.

3. Jane ran the race in 13.4 s. Sally ran the race in 9.8 s. How much longer did it take Jane than Sally? [3.6 s]
4. One piece of wood measured 5.95 m. Another piece of wood measured 18.75 m. How much longer was one piece than the other? [12.8 m]

OBJECTIVE

To extend students' understanding of place value to include hundred millions

PACING

- Level A All
- Level B All
- Level C All

RELATED AIDS

CALC. ACTIVITY MASTERS — 63.

SUGGESTIONS

Initial Activity On the board draw a place-value chart which shows locations up to hundred thousands. Ask students to suggest numbers to be written in the chart. Do several examples. Say a number which includes millions. Ask students how that number could be shown on a place-value chart. They should be able to suggest continuing the pattern of headings to millions. Have the children suggest other numbers which include millions. Fill them in on the chart. Have the students read the numbers that have been written. Examine the chart drawn in the display. Go over the composition of each number seeing how it is "built-up" as far as hundred millions.

USING THE BOOK

Do Exercise 1 orally with students before asking them to write each number.

Do Exercises 2(a) and 3(a) on the board with the students before assigning the rest of the exercises.

ACTIVITIES

1. Make an odometer, using a paper towel roller and strips of paper numbered 0 to 9. Wrap strips loosely around the roller.



Have students show how the odometer changes from 9 to 10, 99 to 100, etc.

Have students write the number that is one greater than 9999. Show the number on the odometer. Have students write the number that is one less than 100 000. Show the number on the odometer. Continue with several more examples.

2. Provide an assignment card such as:
"You have won a million dollars. You

Millions

hundred millions

ten millions

millions

hundred thousands

ten thousands

thousands

hundreds

tens

ones

A			3	7	6	8	5	4	7
B		5	7	3	0	2	8	3	9
C	8	4	7	3	3	5	2	9	6

A

3 000 000

700 000

60 000

8 000

500

40

7

3 768 547

B

50 000 000

7 000 000

300 000

2 000

800

30

9

57 302 839

C

800 000 000

40 000 000

7 000 000

300 000

30 000

5 000

200

90

6

847 335 296

A

three million, seven hundred sixty-eight thousand, five hundred forty-seven

B

fifty-seven million, three hundred two thousand, eight hundred thirty-nine

C

eight hundred forty-seven million, three hundred thirty-five thousand, two hundred ninety-six

Exercises

1.

Write each number in words

(a)

7 682 487

(b)

9 470 365

(c)

76 043 882

(d)

504 003 905

(e)

89 150 327

(f)

351 004 203

(g)

769 000 200

2.

Write each in a place-value chart.

(a)

three million, five hundred sixty-one thousand, two hundred thirty

(b)

eighty-four million, one hundred five thousand, nine hundred seventy-two

(c)

four hundred thirty-eight million, fifty-six thousand, one hundred nine

(d)

one hundred fifty-two million, nine hundred thousand

(e)

two million, eighty-four thousand, seventy

3.

In the number 685 342 017, write the digit that is in each of these places.

(a)

ten millions

8

(b)

thousands

2

(c)

tens

1

(d)

hundred thousands

3

(e)

ten thousands

4

(f)

hundreds

0

(g)

ones

7

(h)

hundred millions

6

10 Place value millions

ANSWERS:

1. (a) seven million, six hundred eighty-two thousand, four hundred eighty-seven
(b) nine million, four hundred seventy thousand, three hundred sixty-five
(c) seventy-six million, forty-three thousand, eight hundred eighty-two
(d) five hundred four million, three thousand, nine hundred five
(e) eighty-nine million, one hundred fifty thousand, three hundred twenty-seven
(f) three hundred fifty-one million, four thousand, two hundred three
(g) seven hundred sixty-nine million, two hundred.
continued on page 12.

are going to invest it in real estate. Look in your local newspaper and make a list of properties and land you would buy. What can you buy for your million?"

3. See "The P.V. Game" as described in the Activity Reservoir. Extend the game to include hundred millions.

4. Students can find examples of numbers in the millions by researching the solar system. They can find the given distances between the earth, sun, and the planets. They might like to make a bulletin board

with illustrations of the solar system and the distances between the planets.

Research into population figures for countries in the world can also give useful practice in working with numbers in the millions.

EXTRA PRACTICE

Rearrange these cards so that you have different numbers 573 428 105. One way would be 428 105 573. Write the combination that would give you the largest number; the smallest.

Billions

A famous hamburger chain sold twenty-four billion hamburgers.
We can show the number twenty-four billion in a place-value chart.



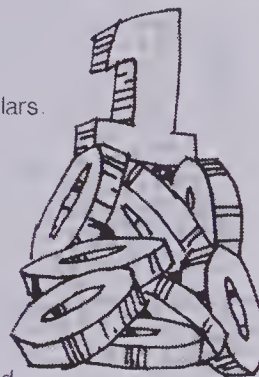
hundred billions	billions	ten billions	hundred millions	millions	ten millions	hundred thousands	thousands	ten thousands	hundreds	tens	ones
	2	4	0	0	0	0	0	0	0	0	0

24 000 000 000 hamburgers were sold.

Exercises

Write each number in a place-value chart.

- A steel company's gross sales in one year was 1 billion, 8 million dollars.
- The steel company's net profit was 120 million dollars.
- An oil company's gross sales in the first quarter of the year was 18 billion, 700 million dollars.
- The oil company's gross sales in the next quarter was 15 billion, 200 million dollars.
- The volume of trading at the stock market on one day was 3 million, 21 thousand shares, valued at 38 million, 92 thousand dollars.
- The volume of trading on the following day was 3 million, 57 thousand shares, valued at 45 million, 42 thousand dollars.
- In Britain, 318 million, 457 thousand prescriptions were written by doctors in one year.
- The total cost of the drugs prescribed was 14 billion dollars.



Billions 11

OBJECTIVE

To extend students' understanding of place value to include hundred billions

PACING

Level A All
Level B All
Level C All

RELATED AIDS

CALC. ACTIVITY MASTERS — 61.

SUGGESTIONS

Initial Activity On the board draw a place-value chart which shows from ones to hundred millions. Ask students to suggest numbers to be written on the chart. They could include some of the numbers they have researched in the previous lesson. Write the numbers on the chart. Have the students read the numbers. Give them the fact shown in the display concerning the 24 billion hamburgers. Ask students how this could be shown on a place-value chart. They should have no difficulty suggesting the addition of a billion's section. Have the students suggest other numbers which include billions. Write them on the chart. Have the children read the numbers on your chart. Write the expanded form of these numbers on the board.

USING THE BOOK

Read each example orally to ensure that your less able students understand the statements.

You may wish to prepare and distribute a blank place-value chart which, when labelled by the students, would extend to hundred billions. Have them use this to write their answers to the exercises.

EXTRA PRACTICE

What is the place value of each underlined digit?

- 764 382 768
- 276 549 387 446
- 983 476 000 000
- 473 509 000 176
- 385 176
- 440 785 276 495

ANSWERS:

hundred billions	billions	ten billions	hundred millions	millions	ten millions	hundred thousands	thousands	ten thousands	hundreds	tens	ones
(a)			1	0	0	8	0	0	0	0	0
(b)				1	2	0	0	0	0	0	0
(c)	1	8	7	0	0	0	0	0	0	0	0
(d)	1	5	2	0	0	0	0	0	0	0	0
(e)					3	0	2	1	0	0	0
(f)					3	8	0	4	2	0	0
(g)					4	5	0	4	2	0	0
(h)	1	4	0	0	0	0	0	0	0	0	0

ACTIVITIES

1. Make an odometer using a paper towel roll and strips of paper numbered 0 to 9, taped around the roll. Have 3 strips to a period and leave a small space between each period. One student says a number, the other students show the number on the roll.

0	1	2	3	4	5	6	7	8	9
---	---	---	---	---	---	---	---	---	---

2	6	7	4	5	8	6	7	2	3	1	5
---	---	---	---	---	---	---	---	---	---	---	---

2. Make sets of cards labelled 0 to 9. On a stencil draw a chart similar to one below.

--	--	--	--	--	--	--	--	--	--	--	--

BILLIONS MILLIONS THOUSANDS ONES
At a given signal each student draws a card from the set and places it in one of the slots. Once placed, it cannot be moved. When all cards have been placed one by one, the students with the largest (or smallest number) wins.

3. Review place value and challenge the students with puzzles such as:

--	--	--

243

6124

?

OBJECTIVE

To review the term rounding and to provide practice in rounding numbers

PACING

Level A All
Level B All
Level C All

RELATED AIDS

HMS—DM3.

SUGGESTIONS

Initial Activity You may wish to discuss the term "approximate" and its use in everyday life. Ask students to suggest situations where an approximate figure is sufficient. Explain that when we "round" a figure we are giving an approximation. Go over the steps in the display at the top of the pupil page with the students. With less able students, you may also wish to do a few examples rounding numbers on a number line.

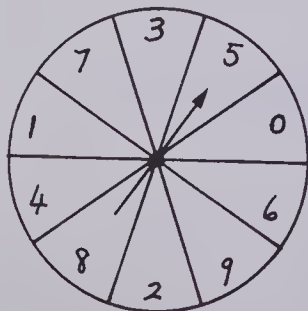
USING THE BOOK

You may wish to do orally a few examples of each exercise with the students before assigning the balance.

Be sure that the children are familiar with an acceptable answer format.

ACTIVITIES

1. Have the students use spinners as shown to make up from 3- to 9-digit numbers and write them on cards. Make up enough so that there are 5 cards per participant. Have them round each of their five numbers to the nearest hundred, hundred thousand, and million and write these on the back of the card.



7 624 341
Front

7 624 300
7 600 000
8 000 000
Back

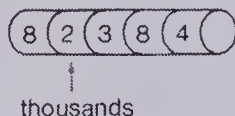
2. On a stencil, make up a series of number lines. Round to the nearest 1000.

Rounding Whole Numbers

Janice wants to round 82 384 to the nearest thousand. Here is how she does it.

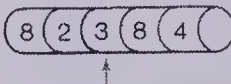
Step 1

She finds the thousand's digit.



Step 2

She finds the next digit to the right.



Step 3

If that digit is 5 or more, Janice makes the thousand's digit one greater.

If that digit is 4 or less, the thousand's digit remains the same.



Janice rounds 82 384 to 82 000. Why?

Exercises

Use Janice's three steps to help you.

1. Round to the nearest hundred.

(a) 412 **400** (b) 8965 **9000** (c) 5278 **5300** (d) 34 834 **34 800** (e) 523 389 **523 400**
(f) 486 205 **486 200** (g) 39 449 **39 400** (h) 6298 **6300** (i) 40 354 **40 400**

2. Round to the nearest ten thousand.

(a) 85 270 **90 000** (b) 17 049 **20 000** (c) 585 938 **590 000** (d) 473 162 **470 000**
(e) 955 420 **960 000** (f) 61 027 **60 000** (g) 457 231 **460 000** (h) 90 235 **90 000**

3. Round to the nearest hundred thousand.

(a) 478 327 **500 000** (b) 934 856 **900 000** (c) 5 278 332 **5 300 000**
(d) 627 345 **600 000** (e) 7 498 287 **7 500 000** (f) 16 337 441 **16 300 000**

4. Round to the nearest million.

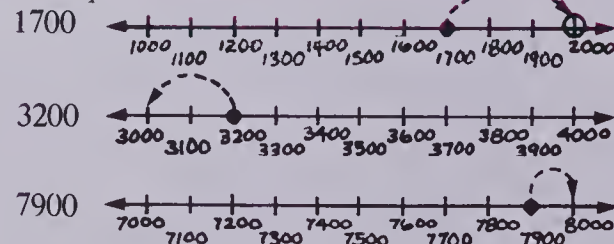
(a) 9 437 238 **9 000 000** (b) 85 848 000 **86 000 000** (c) 485 287 123 **485 000 000**
(d) 95 948 246 **96 000 000** (e) 90 617 745 **91 000 000** (f) 486 592 614 **487 000 000**

12 Rounding

Continued from page 10.

	ones	tens	hundreds	thousands	ten thousands	hundreds of thousands	millions	ten millions	hundreds of millions
(a)	0	3	2	1	6	5	3		
(b)	2	7	9	5	0	1	4	8	
(c)	9	0	1	6	5	0	8	3	4
(d)	0	0	0	0	0	9	2	5	1
(e)	0	7	0	4	8	0	2		

Examples



EXTRA PRACTICE

1. Round to the nearest ten thousand.

(a) 38 476 (b) 523 497
(c) 177 832 (d) 359 276

2. Round to the nearest hundred thousand.

(a) 476 532 (b) 884 735
(c) 37 285 634
(d) 5 328 476

3. Have the students make problems such as the one written below and trade with a classmate.

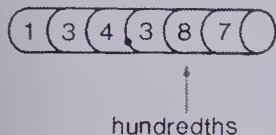
There are 3089 candies in a jar. What is the approximate number?
(a) 4000 (b) 3000 (c) 3842

Rounding Decimal Numbers

Ron wants to round 134.387 to the nearest hundredth.
Here is how he does it.

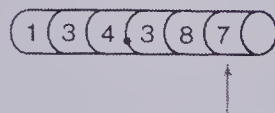
Step 1

He finds the hundredth's digit.



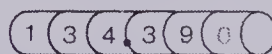
Step 2

He finds the next digit to the right.



Step 3

If that digit is 5 or more, the hundredth's digit becomes one greater. If the digit is 4 or less, the hundredth's digit stays the same.



Ron rounds 134.387 to 134.39. Why?

Exercises

Use Ron's three steps to help you.

1. Round to the nearest tenth.

(a) 7.48 7.5	(b) 8.32 8.3	(c) 26.29 26.3	(d) 48.35 48.4
(e) 385.14 385.1	(f) 49.359 49.4	(g) 16.476 16.5	(h) 500.076 500.1

2. Round to the nearest hundredth.

(a) 13.654 13.65	(b) 4.175 4.18	(c) 37.349 37.35	(d) 3.666 3.67
(e) 68.071 68.07	(f) 400.688 400.69	(g) 32.005 32.01	(h) 428.502 428.50

3. Round to the nearest whole number.

(a) 7.128 7	(b) 28.87 29	(c) 624.8 625	(d) 49.42 49
(e) 48.79 49	(f) 500.92 501	(g) 85.249 85	(h) 948.049 948

4. John cycled 9.48 km to school. About how far did he cycle? Round to the nearest tenth.
9.5 km

Rounding decimal numbers 13

OBJECTIVE

To provide practice in rounding decimal numbers

PACING

Level A All
Level B All
Level C All

MATERIALS

magazine and catalogue clippings

RELATED AIDS

CALC. ACTIVITY MASTERS — 26, 55, 85.

SUGGESTIONS

Initial Activity Display clippings which deal with prices of a variety of items.

Have students practise rounding the prices.

Example

39¢ is rounded to 40¢,

\$5.99 is rounded to \$6.00, etc.

USING THE BOOK

Read through the display at the top of the pupil page together. Emphasize the three steps Ron uses to round decimal numbers. Point out the similarity of what is being done here to what was presented on page 12. You may wish to post the three steps to rounding somewhere in the room for future reference.

Do the first two examples of each exercise with the students. Have them explain the rounding process orally as it has been described.

ACTIVITIES

1. Have students make their own collection of clippings from magazines and catalogues. Ask them to round the prices.

2. Have students also make a collection of examples of decimal numbers in everyday life. These too can be rounded.

3. If you have not already done so, see Activity 1 on page 12. Have the children generate numbers with up to

3 decimal places and round to tenths and hundredths.

EXTRA PRACTICE

1. Round to the nearest tenth.

(a) 53.84	(b) 9.37
(c) 6.58	(d) 14.05
(e) 27.12	

2. Round to the nearest hundredth.

(a) 3.762	(b) 7.465
(c) 16.229	(d) 13.053
(e) 48.989	

OBJECTIVE

To provide practice in estimation by rounding

PACING

Level A All
Level B All
Level C All

MATERIALS

a department store or supermarket advertisement from the local newspaper

RELATED AIDS

HMS—DM4.

BFA COMP LAB II—8, 9, 15.

CALC. ACTIVITY MASTERS—4, 21, 23.

SUGGESTIONS

Initial Activity Show the advertisement to the class so that all prices can be seen clearly. Advise the children that pencils won't be necessary for the first part of this lesson because they will be doing their calculations mentally. The challenge: — They have \$10 to spend. — Who can choose a collection of items closest to \$10 without going over?

Allow several minutes for the selections to be made. Have the children now write their names and selections only (no calculations) on a piece of paper. Have these exchanged with classmates for actual calculation to establish the winners.

When this exercise is complete, ask for suggestions for a simple way to make these "guestimates". Elicit the answer "rounding off the prices". If time permits, give additional practice by varying the sums of money allocated (have the children ignore sales tax for now).

USING THE BOOK

Go over the display at the top of the page step by step.

Conduct a discussion about how the rounding occurred. Reinforce the idea that rounding is a means of getting an approximation.

Note: In these exercises, rounding is not being used as a means of checking for accuracy.

You may wish to complete Exercises 1, 2, 6, 7, and 10 orally at the chalkboard before assigning the

Estimating

An estimate tells "about how many" things are in an answer or set. To estimate, round the actual numbers first and then calculate.

Actual	Estimate	Actual	Estimate	Actual	Estimate
583	600			\$ 63.52	\$ 60
604	600			27.26	30
462	500			34.68	30
387	400	1981	2000	+ 48.43	+ 50
+ 727	+ 700	- 1633	- 1600		
2763	about 2800	348	about 400	\$ 173.89	about \$ 170

Exercises

Estimate, and then check by calculating the actual amount.

<p>1. 748 → 700</p> <p>620 → 600</p> <p>284 → 300</p> <p>702 → 700</p> <p>+ 566 → + 600</p> <p>(2920) about 2900</p>	<p>2. 488 → 500</p> <p>234 → 200</p> <p>709 → 700</p> <p>348 → 300</p> <p>+ 709 → + 700</p> <p>(2488) about 2400</p>	<p>3. 945 → 900</p> <p>279 → 300</p> <p>788 → 800</p> <p>649 → 650</p> <p>+ 541 → + 550</p> <p>3100 (3202)</p>	<p>4. 419 → 400</p> <p>715 → 700</p> <p>466 → 450</p> <p>732 → 700</p> <p>+ 107 → + 100</p> <p>2400 (2439)</p>	<p>5. 613 → 600</p> <p>281 → 300</p> <p>565 → 550</p> <p>119 → 100</p> <p>+ 449 → + 450</p> <p>2000 (2027)</p>
<p>6. 1580 → 1600</p> <p>- 1105 → - 1100</p> <p>(475) about 500</p>	<p>7. 1289 → 1300</p> <p>- 1090 → - 1100</p> <p>(199) about 200</p>	<p>8. 1750 → 1800</p> <p>- 1663 → - 1700</p> <p>100 (87)</p>	<p>9. 1903 → 1900</p> <p>- 1481 → - 1500</p> <p>400 (422)</p>	

Watch the signs.

<p>10. \$ 70.18 → \$ 70</p> <p>81.43 → 80</p> <p>26.24 → 30</p> <p>+ 53.65 → 50</p> <p>(\$ 231.50) \$ 230</p>	<p>11. \$ 41.65 → 40</p> <p>63.41 → 60</p> <p>35.75 → 35</p> <p>+ 41.88 → 40</p> <p>\$ 180 (\$ 182.69)</p>	<p>12. \$ 95.83 → 100</p> <p>- 36.54 → - 40</p> <p>\$ 60 (\$ 59.29)</p>	<p>13. \$ 42.21 → 40</p> <p>+ 67.37 → 70</p> <p>\$ 110 (\$ 109.58)</p>	<p>14. \$ 34.32 → 30</p> <p>58.65 → 60</p> <p>62.84 → 60</p> <p>+ 16.85 → 15</p> <p>\$ 170 (\$ 172.66)</p>	<p>15. \$ 43.71 → 40</p> <p>- 27.09 → - 30</p> <p>\$ 10 (\$ 16.62)</p>
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14 Estimate by rounding

balance of the page.

ACTIVITIES

1. Have students clip items from newspapers and catalogues which have numbers. Ask students to rewrite the item using rounded numbers in place of the actual numbers.

2. On activity cards with sales advertisements, have the students choose a given number of cards and compute the totals of the prices involved using rounded numbers.

3. Make an estimation of the number of students in the school by (a) establishing estimates of the number of students per class and (b) adding these estimates. When all estimates have been made, reveal the actual number.

EXTRA PRACTICE

Estimate only.

<p>1. 336</p> <p>428</p> <p>595</p> <p>+ 607</p> <p>[1966]</p>	<p>2. 934</p> <p>715</p> <p>862</p> <p>+ 359</p> <p>[2870]</p>
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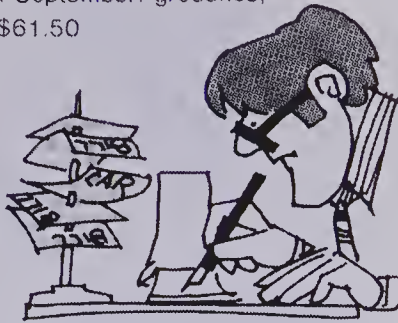
<p>3. 9854</p> <p>- 3766</p> <p>[6088]</p>	<p>4. \$87.42</p> <p>- 41.36</p> <p>[\$46.06]</p>
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Estimation Stories

These are Mr. Periwinkle's major expenses for the month of September: groceries, \$279.84; rent, \$326; hydro, \$52.28; gasoline for his car, \$61.50. About how much money will he need?

Actual		Estimate
\$279.84		\$280
326.00	Round	330
52.28	to	50
+ 61.50		+ 60
\$ ■■■■		\$ 720

He will need about \$720.



Exercises

- At the Maple Lane Community Fair, the bakery stall made \$157.38, the book stall made \$26.13, the toys and games stall made \$30.94, and the home-preserves stall made \$49.85. About how much did these four stalls make?
(Round to the nearest ten.) **\$ 270**
- In Cedarbrae School there are 17 students in Kindergarten, 21 students in Grade 1, 29 students in Grade 2, 32 students in Grade 3, 36 students in Grade 4, 35 students in Grade 5, and 34 students in Grade 6.
Round to the nearest ten to give an estimate of the total number of students in Cedarbrae School. **210**
- On a trip out west, Mr. Schmidt travelled 412 km on the first day, 630 km on the second day, and 580 km on the third day.
Round to the nearest hundred to give an estimate of the total number of kilometres Mr. Schmidt travelled on the three days. **1600 km**
- For Unicef, these collections were made:
John, \$4.79; Jean, \$3.15; Suzette, \$6.22; and Jason, \$5.92.
About how much did these four people collect?
(Round to the nearest dollar.) **\$20**

Word problems — estimation 15

OBJECTIVES

To give further practice in estimating
by using rounded numbers
To solve word problems involving
estimation

PACING

Level A All
Level B All
Level C All

VOCABULARY

stall, home preserves

MATERIALS

classified and advertising sections of
the newspaper, catalogues

RELATED AIDS

BFA PROB. SOLVING LAB II—15,
29.

SUGGESTIONS

Initial Activity Tell the students that they are going to be given one thousand dollars to furnish their own bedroom. Have them use the newspaper and/or catalogues for locating prices. They are to use rounded numbers for estimating rather than actual prices. You may wish to discuss with the students that it would be advisable to round to the higher number so that they would not overspend.

USING THE BOOK

Read through the example of Mr. Periwinkle's expenses together at the top of the page. Clarify how the estimate was arrived at. Calculate the actual expense total. Point out that the \$720 estimate was useful in quickly telling "about how much" would be necessary.

Assign the exercises. Be sure that the children are familiar with an accepted answer format. For less able students, you may wish to read through the problems together.

ACTIVITIES

- Have students present in oral or visual forms their "new" bedrooms. Approximate and actual prices of their "purchases" should be discussed.
- Prepare some Estimation Activities Cards such as:

Circle the best estimate.			
(a)	576 + 312	800	900 1000
(b)	846 732 + 181	1500	1600 1700
(c)	262 + 491 + 902	1700	1800 1900

- Students make a list of world records using approximate figures as well as actual figures.

OBJECTIVE

To give practice in working with numbers in a puzzle-solving situation

PACING

Level A All
Level B All
Level C All

RELATED AIDS

BFA PROB. SOLVING LAB II—208, 212, 214, 218, 223, 228.

USING THE BOOK

Have the students read all the puzzles. Allow the less able and average students to choose the puzzles they wish to attempt to solve. The more able students should attempt them all, but not necessarily in order. Present this page for "fun" and encourage students to work out the answers in whatever informal or formal way they wish. It would be better if they worked out their answers on scratch pads or extra paper, rather than their math notebooks. The scratch-pad approach gives the student more freedom to experiment with different solutions.

ACTIVITIES

1. There are a number of excellent, inexpensive puzzle books on the market. Cut and paste a selection of these puzzles onto index cards for students to do in their spare time.
2. Write a "Puzzle for the Week" on a bulletin board and invite students to write out their solutions and pin them on the board.
3. Your more inventive students should be asked to create their own puzzle and have other students try to solve it.

Exploring Numbers

1. Find a pattern for each. Copy and complete.

(a) 20, 25, 20, 25, 20, 25, 20, 25, 20

(b) 0.1, 0.3, 0.5, 0.7, 0.9, 1.1, 1.3

(c) 1, 10, 19, 28, 37, 46, 55, 64, 73

(d) 4.005, 4.105, 4.205, 4.305, 4.405, 4.505, 4.605

(e) 3, 8, 11, 16, 19, 24, 27, 32, 35

(f) 1, 2, 3, 2, 3, 4, 3, 4, 5, 4, 5, 6, 5, 6, 7

2. Find the number.

I am larger than 20 but smaller than 30.

I am not a multiple of 4.

I am an even number.

My digits are not the same.

What number am I? 26

- 3.

Think of a number.

Multiply it by 2.

Add 6.

Subtract the number you first thought of.

Find half of this number.

Add 7.

Write your answer.

Repeat, using different starting numbers. What answer do you always get? 10

4. Copy and find the unknown numbers.

(a) $\begin{array}{r} \overset{3}{\blacksquare} 948 \\ \quad \overset{7}{\blacksquare} 67 \\ \quad \quad \overset{5}{\blacksquare} 98 \\ + 47\overset{9}{\blacksquare}2 \\ \hline 17\ 492 \end{array}$

(b) $\begin{array}{r} 94\overset{3}{\blacksquare}5 \\ \quad \overset{8}{\blacksquare} 7\overset{2}{\blacksquare} \\ \quad \quad 4\ 305 \\ + \overset{7}{\blacksquare} 826 \\ \hline 28\ 438 \end{array}$

(c) $\begin{array}{r} \overset{3}{\blacksquare} \overset{6}{\blacksquare} 24\overset{6}{\blacksquare} \\ - 59\overset{42}{\blacksquare}\overset{7}{\blacksquare} \\ \hline 23\ 819 \end{array}$

(d) $\begin{array}{r} \overset{4}{\blacksquare} \overset{9}{\blacksquare} 60\overset{2}{\blacksquare} \\ - \overset{36}{\blacksquare}\overset{8}{\blacksquare} 6\overset{2}{\blacksquare} \\ \hline 23\ 747 \end{array}$

5. John found that every even number larger than 2 can be written as the sum of 2 prime numbers: $4 = 2 + 2$ $6 = 3 + 3$ $8 = 3 + 5$


Check John's discovery for all of the even numbers up to 30.



16 Exploring numbers activity

ANSWERS:

5. $10 = 5 + 5$, $12 = 5 + 7$,
 $14 = 7 + 7$, $16 = 3 + 13$,
 $18 = 5 + 13$, $20 = 3 + 17$,
 $22 = 11 + 11$, $24 = 5 + 19$,
 $26 = 13 + 13$, $28 = 5 + 23$,
 $30 = 7 + 23$



The Travel Agent

1. Mr. and Mrs. Bertram went to Nassau for their honeymoon.
 Their air fare plus hotel was \$1018.
 Their meal plan was \$240. They rented a car for \$99.
 How much did they pay the travel agent for their honeymoon? \$1357
2. Mr. Brown flew to Calgary on business once a month for one year.
 The round trip was \$350.
 How much a year did he pay the travel agent for his air fare? \$4200
3. Fifteen girl guides and two leaders went to Disney World for two weeks.
 The cost per person for one week was \$315.
 How much did they pay the travel agent altogether? \$10 710
4. Mr. Samson booked a charter trip to Barbados for his wife and two children. He paid a special rate of \$219 for each child.
 He paid \$359 each for himself and his wife.
 How much did he pay the travel agent for his trip? \$ 1156
5. Miss Simpson saved \$2500 for a three-week holiday in London, England.
 Her charter air fare was \$359. Her hotel was \$950.
 She rented a car for \$289. How much did she pay the travel agent?
 How much money did she have left to spend? \$ 902

\$1598

Word problems — multiple steps 17

OBJECTIVE

To give practice in problem solving

PACING

Level A 1-3

Level B 1-4

Level C All

RELATED AIDS

BFA PROB. SOLVING LAB II—41.

SUGGESTIONS

Initial Activity Display a word problem on the chalkboard such as:
 Mr. Wilkinson flew from Winnipeg to Edmonton for a meeting.
 His return airfare was \$284.
 His hotel room for 1 night cost \$47.
 Food cost \$36.

How much did the trip cost altogether?

Review Professor Q's format for formal problem solving:



What is the main idea? (Trip to Edmonton)



What is being asked? (Total cost?)



What are the important facts? (\$284, \$47, \$36)



What operation(s) should be used? (Addition)

Solve the problem, clarifying the steps and procedures as you continue.

USING THE BOOK

With less able students you may wish to have the problems read orally first to ensure they understand the context of each problem. Often, less able students are unsuccessful in problem solving, not because they can't do the math involved, but because they can't read the problem with understanding.

The word problems become more involved towards the end of the page in that more calculations need to be completed in order to arrive at a correct answer.

ACTIVITIES

1. Invite a travel agent to come and talk to the students about different facets of the job.

2. Have students get up-to-date brochures from an agency. Using the brochures, have them plan a one-week holiday for themselves. Be sure to provide atlases so that planned routes can be shown.

3. Prepare a challenge card such as:

"Pretend you have \$1000 to spend on

a holiday for you and a friend. Plan your holiday and tell how much you will have left over (if any)."

EXTRA PRACTICE

Mr. Clark went to a conference. His travelling expenses were \$129.00. His registration fee was \$150.00. His hotel accommodation was \$135.00. His meals were \$65.00 and his parking fee was \$15.00.

How much did he pay altogether?

OBJECTIVE

To review the concept of a number sentence and give practice in working with number sentences

PACING

Level A 1(a)-(n)
Level B All
Level C All

RELATED AIDS

BFA PROB. SOLVING LAB II — 212, 213, 221.

SUGGESTIONS

Initial Activity Give examples of equalities such as 3 pencils in one hand and 3 pencils in another, one student holding 6 readers and another student holding 6 readers. Review the "equal" symbol and recall with the students that the "equals" symbol means that whatever is placed on the left-hand side of the symbol must equal whatever is placed on the right-hand side of the symbol.

Review the "not equals" symbol. Ask students to demonstrate using objects close at hand, an example of nonequality, e.g., 5 pencils in one hand, 1 pencil in the other, etc.

Present several examples on the chalkboard such as:

$$6 + 17 \bigcirc 23, 3 + 4 + 5 \bigcirc 12, \\ 17 - 11 \bigcirc 6, 46 - 7 \bigcirc 38, \text{ etc.}$$

Have the appropriate symbols inserted in each blank, discussing the "balance-effect", rationale, etc.

Review the Order of Operations rule that has been developed in earlier texts (i.e., Operations in brackets first followed by multiplication, division, addition, and subtraction in the order they appear.) Present examples such as:

$$3 + 4 \times 5 - 6 \div 2 = \bigcirc \text{ and} \\ 3 \times (4 - 2) \div 5 = \bigcirc.$$

Follow this with:

$$3 \times 5 + 2 \bigcirc 3 + 5 \times 2 \text{ and} \\ 6 \div (2 + 4) \bigcirc 6 \div 2 + 4.$$

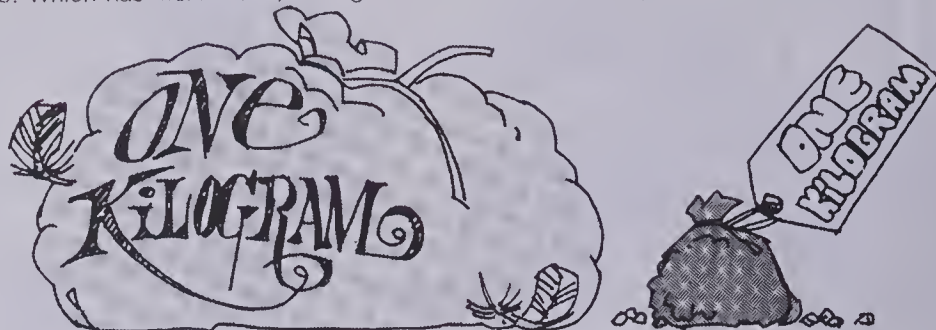
USING THE BOOK

Read through the riddle which is posed in the display at the top of the pupil page. Point out that an "equals" symbol could be used in solving it.

Complete several examples before assigning the page, especially of the sort of number sentence which has an expression on each side of the symbol, i.e., $6 \times 3 \bullet 5 + 5 + 5$.

Number Sentences

Riddle. Which has more mass, a kilogram of feathers or a kilogram of gravel?



Answer: Neither. They are equal because they have the same mass — 1 kg.

The symbol for equal is =.

We can write a number sentence: $8 = 4 + 4$.

We say "8 equals 4 plus 4."

The symbol for not equal is \neq .

We can write a number sentence: $8 \neq 4 + 3$.

We say "8 is not equal to 4 plus 3."

Exercises

1. Use = or \neq to make each number sentence true.

- (a) $7 + 5 \bullet 13 \neq$
- (c) $4 + 4 + 4 \bullet 11 \neq$
- (e) $32 - 10 \bullet 20 \neq$
- (g) $6 \times 0 \bullet 6 + 0 \neq$
- (i) $\$1.00 \bullet 50 \text{ cents} + 50 \text{ cents} =$
- (k) $64 \div 8 \bullet 8 =$
- (m) $27 - 9 \bullet 1 + 1 + 1 =$
- (o) $18 + 3 + 7 \bullet 7 + 3 + 18 =$
- (q) $7 \times 5 + 8 - 5 \bullet 7 \times 13 - 5 \neq$
- (s) $\$10.00 - \$7.25 \bullet \$2.75 =$
- (u) $6 + 5 \times 5 \bullet 6 + 25 =$

- (b) $3 \times 5 \bullet 15 =$
- (d) $9 \times 9 \bullet 81 =$
- (f) $5 + 3 \bullet 3 + 5 =$
- (h) $1 \times 1 \bullet 1 \times 0 \neq$
- (j) $20 + 30 \bullet 40 + 10 =$
- (l) $(2 \times 3) - 5 \bullet 6 - 5 =$
- (n) $100 - 75 \bullet 3 \times 8 \neq$
- (p) $(3 \times 3) + 1 \bullet (3 \times 1) + 3 \neq$
- (r) $3 + 5 \times 4 \bullet 8 \times 4 \neq$
- (t) $999\,999 + 1 \bullet 1\,000\,000 =$
- (v) $56 - 7 - 3 \bullet 56 \div 4 \neq$

Hint:
Do the operation
in brackets
first

ACTIVITIES

1. Have students draw illustrations or diagrams showing both equalities and inequalities.

2. Play "Concentration" as described in the Activity Reservoir. Use equalities only, and present pairs as exemplified by some of the exercises on the pupil page.

3. See "Number Sentence" as described in the Activity Reservoir.

EXTRA PRACTICE

Use = or \neq to make each number sentence true.

- (a) $48 + 5 \bigcirc 5 + 48$
- (b) $100 + 20 \bigcirc 200 - 80$
- (c) $5 \times 6 + 3 \bigcirc (5 \times 6) + 3$
- (d) $347 + 10 \bigcirc 337 - 20$
- (e) $99\,000 \bigcirc 10\,000 - 1$
- (f) $60 \div 10 + 2 \bigcirc 60 \div (10 + 2)$

Solving Equations

Solve these equations by finding the correct number for n .

$$n - 8 = 11$$

You know that

$$11 + 8 = 19.$$

Use 19 in place of n .

$$19 - 8 = 11$$

$$n = 19$$

It works!

$$n + 5 = 12$$

You know that

$$12 - 5 = 7.$$

Use 7 in place of n .

$$7 + 5 = 12$$

$$n = 7$$

It works!



Using the opposite related equation helps solve for n .

Exercises

Solve these equations.

1. (a) $n - 5 = 12$

$$12 + 5 = \blacksquare 17$$

$$17 - 5 = 12$$

$$n = \blacksquare 17$$

2. (a) $n + 4 = 16$

$$16 - 4 = \blacksquare 12$$

$$12 + 4 = 16$$

$$n = \blacksquare 12$$

3. Solve.

$$\blacksquare 2 \quad n + 7 = 9$$

$$(e) \blacksquare 52 \quad n - 35 = 17$$

$$\blacksquare 18 \quad n - 8 = 10$$

$$(f) \blacksquare 34 \quad 29 + n = 63$$

$$(c) \blacksquare 2 \quad n + 13 = 15$$

$$(g) \blacksquare 84 \quad n - 50 = 34$$

$$(d) \blacksquare 26 \quad 26 + n = 52$$

$$(h) \blacksquare 57 \quad n - 16 = 41$$

4. Solve.

$$(a) \$2.50 + n = \$3.50$$

$$(b) \$12.75 + n = \$25.15$$

$$(c) n + \$29.50 = \$100.00$$

$$(d) \blacksquare 16.0 \quad n - 7.3 = 8.7$$

$$(e) \blacksquare 22.0 \quad n - 13.7 = 8.3$$

$$(f) \blacksquare 71.67 \quad n - 45.26 = 26.41$$

$$\star 5. 14 - \blacksquare 8 = 6$$

$$\star 6. 33 - \blacksquare 14 = 19$$

OBJECTIVE

To provide practice in solving equations

PACING

Level A 1-3

Level B 1-4

Level C All

RELATED AIDS

BFA COMP LAB II — 13, 14.

SUGGESTIONS

Initial Activity Write the following word problems on the chalkboard: "John had some marbles. He lost 8. He had 11 marbles left. How many did he have in the beginning?" and "Mary added 5 more stamps to her stamp page. She now had 12 stamps on the page. How many stamps were there in the beginning?"

Solve the first one by writing an equation and "thinking out loud" as you go through the solution process:

$$n - 8 = 11.$$

" n stands for the number of marbles John started with—unknown."

" -8 stands for the 8 that he lost."

"11 stands for what was left."

"What number, represented by ' n ' will make the equation true?"

"We know that $11 + 8 = 19$; let's try 19 for n ."

" $19 - 8 = 11$; it works! John had 19 marbles in the beginning."

Repeat this process for problem 2.

USING THE BOOK

Read through the display at the top of the pupil page. Consolidate the processes as discussed in the Initial Activity.

You may wish to complete Exercises 1(a), 2(a), and 3(a) orally at the chalkboard before assigning the balance.

ACTIVITIES

1. On a bulletin board, display a number of equations for students to solve. Have answers displayed on the reverse side thus making this activity self checking.

2. Ask students to write equations similar to the ones in the display. Other students solve them.

3. See the "Coded Riddles" idea in the Activity Reservoir.

OBJECTIVE

To review the commutative and associative properties of addition

PACING

Level A 1(a)-(n), 2(a)-(d)
Level B 1(a)-(p), 2, 3
Level C 1(g)-(r), 2(d)-(h), 3

MATERIALS

toothpicks

RELATED AIDS

BFA PROB. SOLVING LAB II — 231.

SUGGESTIONS

Initial Activity Distribute toothpicks to each student. Ask them to place 4 toothpicks to the left on their desk. Then ask them to place an additional 5 toothpicks on the right-hand side. Ask: "What is the sum?" Represent this numerically on the chalkboard thus: $4 + 5 = 9$.

Repeat the process, this time placing 5 toothpicks left and 4 to the right. Again elicit the sum and write $5 + 4 = 9$. Point out on the chalkboard that, since both expressions equal nine, then $4 + 5$ must be equal to $5 + 4$.

Do this several times with different numbers. Lead students to the conclusion that $A + B = B + A$ because both are equal to C.

In the same way we can use the toothpicks to demonstrate the associative property of addition: $(A + B) + C = A + (B + C)$. Go over the display at the top of pupil page 20 to reinforce the student's understanding. With more able students you may wish to use the words commutative and associative but do not overly emphasize these special terms at this stage.

USING THE BOOK

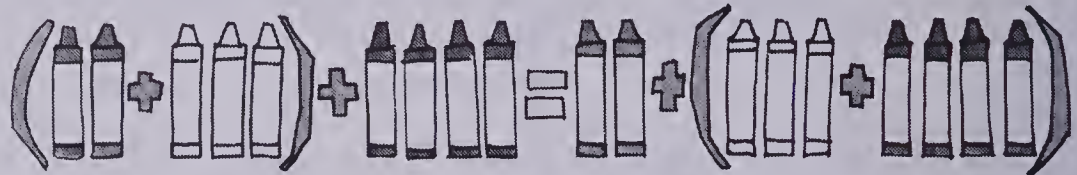
Read through the display at the top of pupil page 20 together. Recap the points which were made during the Initial Activity. Assign the exercises. You may wish to complete 1(a), 1(c), 2(a), and 3(a) orally first. For those assigned to work on Exercise 3, discuss why rearrangement of the addends makes the process simpler (i.e., addition of multiples of ten is quicker). Be sure that all students are familiar with the accepted answer format for use with their workbooks.

Properties of Addition



When you add, you can change the order of the addends.
The sum remains the same:

$$2 + 3 = 3 + 2$$



When you add, you can change the grouping of the addends.
The sum remains the same:

$$(2 + 3) + 4 = 2 + (3 + 4)$$

Exercises

1. Solve the following, using addition properties.

(a) $5 + 6 = 6 + a$ **5**

(b) $7b + 37 = 37 + 7$

(c) $50 + a = 20 + 50$

(d) $a + 16 = 16 + 43$

(e) $14 + 33 = b + 14$

(f) $(5 + 2) + 3 = 5 + (2 + a)$

(g) $(9 + 4) + 5 = 9 + (4 + b)$ **5**

(h) $(8 + 7) + a = 8 + (7 + 4)$

★ (i) $(5 + 14) + a = b + (14 + 3)$

(b) $17 + 19 = 19 + a$ **17**

(d) $5a + 23 = 23 + 5$

(f) $72 + b = 36 + 72$

(h) $22 + b = 51 + 22$

(j) $36 + 12 = b + 12$

(l) $(6 + 2) + 4 = 6 + (2 + b)$ **4**

(n) $(7 + a) + 2 = 7 + (1 + 2)$

(p) $43a + 11 + 6 = 43 + (11 + 6)$

★ (r) $(a + 17) + 1 = 2 + (b + 1)$
2 **17**

Hint:
Do the operations
in brackets
first.

2. Solve the following without computing.

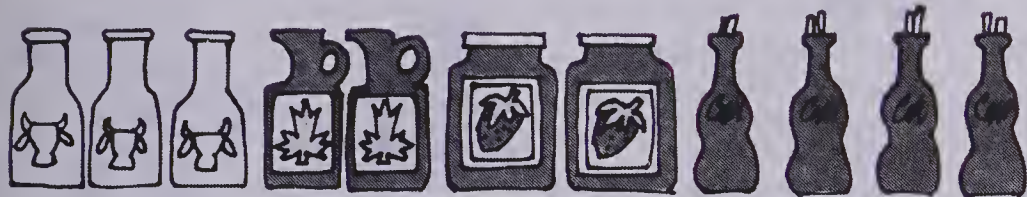
- (a) $(0.3 + 0.7) + 0.4 = 0.3 + (0.7 + a)$ **0.4**
 (b) $(9.1 + 5.0) + 0.7 = 9.1 + (5.0 + b)$ **0.7**
 (c) $(0.121 + 1.3) + 16 = 0.121 + (a + 16)$ **1.3**
 (d) $(\$1.05 + \$1.75) + \$3.20 = \$1.05 + (b + \$3.20)$ **1.75**
 (e) $(\$18.50 + \$3.21) + \$0.90 = \$18.50 + (\$3.21 + a)$ **0.90**
 (f) $(2613 + 114) + 1567 = 2613 + (114 + b)$ **1567**
 (g) $(9 + 19) + 3 = (9 + a) + 19$ **3**
 (h) $13 + (27 + 5) = (13 + b) + 27$ **5**

3. We can change the order to make our adding easier.

$(9 + 5) + 1$ can be changed to $(9 + 1) + 5$.

Change the order to make your adding easier.

- (a) $4 + 20 + 6$ (b) $27 + 8 + 3$ (c) $8 + 13 + 2$
 (d) $5 + 13 + 25$ (e) $7 + 19 + 13$ (f) $12 + 69 + 8$
 (g) $37 + 22 + 3$ (h) $4 + 76 + 16$ (i) $9 + 35 + 41$
 (j) $25 + 27 + 25$ (k) $50 + 98 + 50$ (l) $40 + 63 + 60$



BRAINTICKLER

Each question has 5 blanks. The missing digits are 2, 3, 4, 5, 8.

Find the correct order.

A. $\begin{array}{r} 1332 \\ \times \blacksquare 4 \\ \hline \blacksquare \blacksquare \blacksquare 5328 \end{array}$

B. $\begin{array}{r} \blacksquare \blacksquare \blacksquare \blacksquare 3258 \\ \times \blacksquare 4 \\ \hline 13\ 032 \end{array}$

C. $\begin{array}{r} \blacksquare \blacksquare \blacksquare \blacksquare 5234 \\ \times \blacksquare 8 \\ \hline 41\ 872 \end{array}$

Properties of addition 21

ANSWERS:

- (a) $4 + 6 + 20$ (b) $27 + 3 + 8$ (c) $8 + 2 + 13$ (d) $5 + 25 + 13$ (e) $7 + 13 + 19$
 (f) $12 + 8 + 69$ (g) $37 + 3 + 22$ (h) $4 + 16 + 76$ (i) $9 + 41 + 35$
 (j) $25 + 25 + 27$ (k) $50 + 50 + 98$ (l) $40 + 60 + 63$

EXTRA PRACTICE

1. These 3 numbers are arranged in different ways. Are the sums the same?

$$\begin{array}{r} 4 \quad 6 \quad 9 \quad 4 \quad 6 \quad 9 \\ 6 \quad 9 \quad 4 \quad 9 \quad 4 \quad 6 \\ +9 \quad +4 \quad +6 \quad +6 \quad +9 \quad +4 \end{array}$$

2. Arrange these numbers in any order. Add.

$$\begin{array}{r} 12 \quad 66 \quad 178 \quad \$1.29 \quad 4.98 \\ 3 \quad 72 \quad 476 \quad 3.67 \quad 3.46 \\ 15 \quad 95 \quad 553 \quad 5.28 \quad 6.72 \end{array}$$

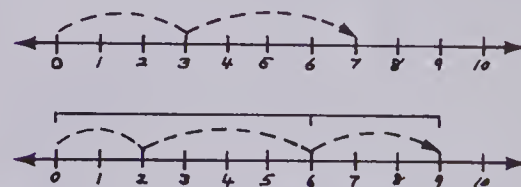
ACTIVITIES

1. Conduct basic facts "drill" games. Give a fact. Students write another fact from the given fact, e.g., given fact $3 + 8 = 11$, students write $8 + 3 = 11$. Extend to include larger numbers, as well as money and decimal numbers.

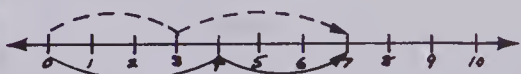
2. Play "Concentration" as described in the Activity Reservoir. Use matching card pairs such as:

$0.3 + 1.7$ $1.7 + 0.3$
 $\$3.50 + \1.75 $\$1.75 + \3.50 etc.

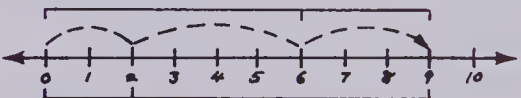
3. Provide a series of number lines such as:



Have the students write the equation suggested [$3 + 4 = 7$, $(2 + 4) + 3 = 9$] and then illustrate (with coloured pencil) and write the related sentences according to the laws of commutativity and associativity:



$$3 + 4 = 4 + 3$$



$$(2 + 4) + 3 = 2 + (4 + 3)$$

etc.

4. Prepare cards and labelled envelopes (or other containers) as shown. Have the students determine which 3 cards go into which container to yield the labelled sum.

$\$3.52$ $\$7.99$ $\$1.99$
 $\$3.85$ $\$6.28$ $\$10.00$
 19.80 () 13.83 () etc.

5. Some children might enjoy making up their own equations which demonstrate commutativity and associativity as exemplified by the exercises on this page. Have them written on cards (with answers on the back) for exchange with other classmates, groups, or classes in the school.

OBJECTIVE

To give practice in problem solving

PACING

Level A 1-4

Level B 1-6

Level C 1-8

VOCABULARY

yield, neighbourhood, decorate, population, discovered, produce

RELATED AIDS

HMS — DM5.

BFA PROB. SOLVING LAB II — 19, 27-39.

SUGGESTIONS

Initial Activity Display a word problem on the chalkboard such as: "In a tree study, Steve counted 198 leaves on a branch, Karen counted 269 on another, and Sharon found 324 on a third. How many leaves were counted altogether?"

Review the steps in the problem-solving technique described on page 17. Demonstrate the process, allowing the students to "eavesdrop" as you "think aloud" through the solution.

USING THE BOOK

Review Professor Q's format for formal problem solving. With less able students you may wish to have the problems read orally first to ensure they understand the context of each problem. Often, less able students are unsuccessful in problem solving, not because they can't do the math involved, but because they can't read the problem with understanding.

Allow students to choose the order they wish to do the problems. Less able students may require more than one period to complete the assignment.

You may wish to display the particular problem-solving technique used in your class on a bulletin board for future reference (or in a "reference section" of the pupils' workbooks).

Problems

The drive from Acadia Park to the Lewis home is 87.4 km.

The family stopped for gas after 36.6 km. How much farther must they go to get home?

Follow Professor Q's steps.

Step 1 Answer these four questions:

Questions	Answers
1. What is the main idea?	Driving home
2. What is being asked?	How much farther to go?
3. What are the facts?	87.4 km in all to go. Have gone 36.6 km.
4. Which operation do I use?	Subtraction

Step 2 Write a number sentence.
 $87.4 - 36.6 = n$

Step 3 Make the sentence true.
 $87.4 - 36.6 = 50.8$

Step 4 Write a final statement.
They must go 50.8 km farther to get home.

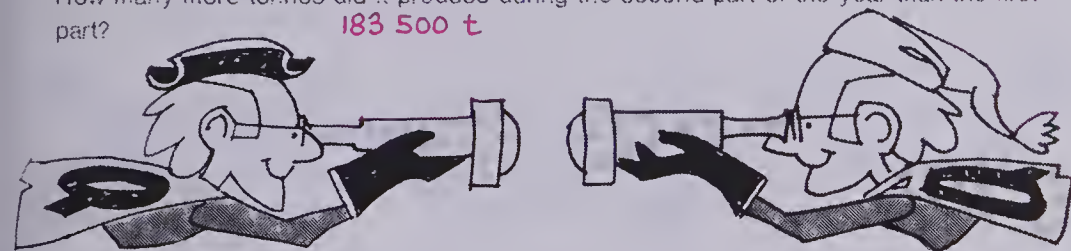


Exercises

Follow Professor Q's steps to solve these problems.

- Manfred's mother asked him to go to the store.
He ran to the store in 6.8 min and walked back from the store in 12.75 min.
How much longer did he take to come back from the store than to go there? **5.95 min**

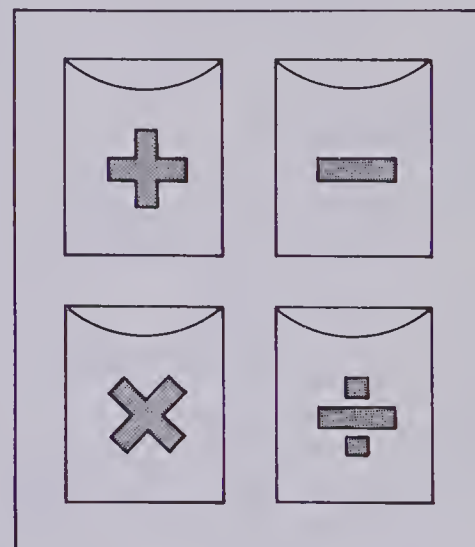
2. Mr. Murphy grew 3 fields of potatoes. The yield in the first field was 3.5 t. The yield in the second field was 2.75 t, and the yield in the third field was 3.25 t.
How many tonnes of potatoes did his fields yield? **9.5 t**
3. Rosa guessed that Luigi's mass was 37 kg. Luigi's mass was actually 34.6 kg.
How many kilograms less was Luigi's mass than Rosa's guess? **2.4 kg**
4. The neighbourhood club got together to make paper chains to decorate the clubhouse for a party. Sam's chain was 546 cm long. Selma's chain was 684 cm long. Ali's chain was 857 cm long. Jennie's chain was 783 cm long. Conrad's chain was 945 cm long.
What was the total length of their chains? **3815 cm**
5. In 1900 the population of Loganville was 1 287 385. In 1980 the population was 3 786 432.
How many more people were there in 1980 than in 1900? **2 499 047**
6. The Grade 6 class in Greenacres School held a penny-collecting drive for charity. Justin collected 468 pennies. Lizzie collected 478 pennies. Tiffany collected 682 pennies. Jim collected 387 pennies. **2015**
How many pennies did they collect altogether? Write this amount in dollars. **\$20.15**
7. Christopher Columbus discovered the New World in 1492.
Jacques Cartier discovered the St. Lawrence River in 1535.
How many years was that after Columbus discovered the New World? **43 a**
8. A steel mill turned out 1 376 000 t of steel during the first part of the year.
Its output during the second part of the year was 1 559 500 t of steel.
How many more tonnes did it produce during the second part of the year than the first part? **183 500 t**



Word problems 23

ACTIVITIES

1. To provide practice in probably the most difficult step in the problem-solving process — “Which operation?” — prepare containers (envelopes, boxes, etc.) labelled as shown.



Write word problems on cards (have the children help) or cut and paste from an old arithmetic text. Children simply read the problem cards and sort them into their appropriate containers. Have a second student check the work of the first.

2. Make up a set of word problems on index cards. Make up a set of answers written on index cards. Students choose an answer card and have to find its problem.

3. Display a “Word Problem of the Day”. The problem should be related to the computational skills being practised on that day. Using relevant data and the familiar names of class and school persons can add motivation and humour to problem-solving situations.

4. On a bulletin board, display a picture that will lend itself to creating problems, e.g., a lady going shopping, cars parked in a parking lot, etc. Ask students to write problems related to the picture. Have other students solve the problems.

EXTRA PRACTICE

1. An ice-cream man sold 536 ice-cream bars on Saturday and 634 ice-cream bars on Sunday. How many did he sell altogether?
2. In Mrs. Simons' class there are 37 students, 14 of whom are boys. How many girls are in the class?
3. Tim had \$150.00. He spent \$89.75. He put \$50.00 in the bank and kept the rest. How much did he keep?

OBJECTIVE

To review and to provide practice in working with inequalities

PACING

Level A 1, 2(a)-(f), 3(a)-(f), 4(a)-(f), 5(a)-(f), 6(a)-(f)

Level B 1, 2, 3(a)-(l), 4(g)-(l), 5, 6

Level C 1, 2, 5-7

MATERIALS

A large cutout of $>$. Cut out pictures of objects such as fruits, cars, animals, etc. A large cutout of $<$.

RELATED AIDS

CALC. ACTIVITY MASTERS — 30.

SUGGESTIONS

Initial Activity Review the meaning and application of $=$ and \neq as presented on page 18.

Review the symbols $<$, $>$ and what each represents. Suggest to students that the small end $\rightarrow <$ points to the smaller number and the large end $\rightarrow >$ points to the larger number. Use your cutouts to show inequalities on the board, e.g., picture of 2 oranges $<$ picture of 5 oranges, etc. Have students build inequalities using the cutouts and the pictures.

Extend the use of the "greater than" or "less than" symbol to separate numbers and numerical expressions:

$4.6 \bigcirc 6.4$ and $4.3 + 5.5 \bigcirc 8.7$.

USING THE BOOK

Read through the display at the top of the pupil page together. Discuss the position of the various balances, why they are tilted (or not), and the symbols which accompany each.

Complete Exercise 1 orally with the class. You may wish to advise the children that the answers for Exercises 3(a) and 4(a) can be checked in the back of the text. Assign the exercises.

Symbols

Using symbols helps save time and space.

<p>3 is equal to 3</p> <p>$3 = 3$</p>	<p>9 is not equal to 13</p> <p>$9 \neq 13$</p>	<p>2.5 is greater than 1.7</p> <p>$2.5 > 1.7$</p>	<p>6 + 3 is less than 19</p> <p>$6 + 3 < 19$</p>
--	---	---	--

Exercises

- Read these statements.

(a) $23 = 23$	(b) $10 + 6 = 16$	(c) $15 - 6 \neq 8$
(d) $1.7 < 7.1$	(e) $1001 > 101$	(f) $\$11.53 \neq \10.99
(g) $0.6 - 0.4 = 0.2$	(h) $\$2.50 + \$2.50 < \$6.00$	(i) $1000 > 0.1000$
(j) $58 + 6 \neq 74$	(k) $58 + 6 < 74$	(l) $121.3 > 12.13$
(m) $12.13 < 121.3$	(n) $\$9.63 + \$2.00 \neq \$10.63$	(o) $\$9.63 + \$2.00 > \$10.63$
- Are these statements correct? Write True or False.

(a) $14 = 14$ <i>True</i>	(b) $6.5 < 5.6$ <i>False</i>	(c) $5 + 6 \neq 11$ <i>False</i>
(d) $15 - 6 > 9$ <i>False</i>	(e) $12.9 + 1.1 > 14.0$ <i>False</i>	(f) $0.8 + 0.1 = 0.9$ <i>True</i>
(g) $6 + 3 = 6.3$ <i>False</i>	(h) $24.7153 > 25.0$ <i>False</i>	(i) $0.319 > 0.289$ <i>True</i>
(j) $23.6 + 1.0 \neq 25.6$ <i>True</i>	(k) $23.6 + 1.0 < 25.6$ <i>True</i>	(l) $23.6 + 1.0 = 25.6$ <i>False</i>

ACTIVITIES

1. Make a deck (about 50-60) of numbered cards. There should be more than one of each number. Make up cards with symbols $<$, $>$, $=$, and \neq . Students play in pairs. First student selects 2 numbered cards, takes a symbol card, and attempts to make the statement true. If correct, the player scores a number of points equal to the value on the *right* side of the equation, whatever that might be. Player with the most points when the deck is exhausted, is the winner.

2. Play the same game, but extend the number of cards to be worked with to 3 and 4.

3. Using the numbered cards and symbol cards, student A makes up 10 number sentences—some true some false. Student B has a given time, e.g., 30 s to rearrange the cards so that all the number sentences are true. Score 1 point for each correct number statement. Players exchange roles. Player with the most points after 3 turns each, is the winner.

4. See "Number Sentence" as described in the Activity Reservoir.

EXTRA PRACTICE

Copy and complete using $=$, $<$, or $>$.

- (a) $76 \bigcirc 32 + 44$ (b) $300 + 297 \bigcirc 448$
 (c) $\$3.95 \bigcirc 395\text{¢}$ (d) $13.7 - 4.9 \bigcirc 10.3$
 (e) $999 + 1 \bigcirc 10\,000$
 (f) $7 + 38 + 42 \bigcirc 38 + 42 + 7$
 (g) $6 + 3 \times 5 \bigcirc (6 \times 3) \times 5$
 (h) $(5 \times 5) \times 5 \bigcirc 5 \times (5 \times 5)$
 (i) $1000 - 998 \bigcirc 3$
 (j) $\$5.00 - \$4.98 \bigcirc \$10.00 - \9.95

3. Copy and complete using $=$ or \neq .

- (a) $11 \bullet 7 \neq$ (b) $18 \bullet 24 \neq$ (c) $27 \bullet 13 \neq$
 (d) $14 \bullet 14 =$ (e) $295 \bullet 360 \neq$ (f) $54 \bullet 72 \neq$
 (g) $66 \bullet 85 \neq$ (h) $90 \bullet 90.0 =$ (i) $102 \bullet 120 \neq$
 (j) $116 \bullet 134 \neq$ (k) $7.9 \bullet 7.90 =$ (l) $\$0.25 \bullet 25\text{¢} =$

4. Copy and complete using $>$ or $<$.

- (a) $18 \bullet 13 >$ (b) $25 \bullet 17 >$ (c) $67 \bullet 68 <$
 (d) $79 \bullet 36 >$ (e) $185 \bullet 851 <$ (f) $127 \bullet 137 <$
 (g) $27 \bullet 2.7 >$ (h) $35 \bullet 0.35 >$ (i) $1127 \bullet 113.0 >$
 (j) $8100 \bullet 81\,000 <$ (k) $\$13.00 \bullet \$15.00 <$ (l) $1.49 \bullet 1.490$ *neither*

5. Use $=$, $>$, or $<$ to make each statement true.

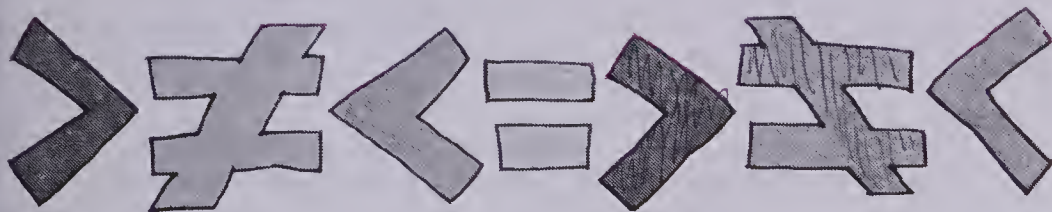
- (a) $36 \bullet 63 <$ (b) $5 + 8 \bullet 13 =$ (c) $8 + 5 \bullet 26 <$
 (d) $181 \bullet 18.10 =$ (e) $10 - 3 \bullet 3 >$ (f) $12 - 6 \bullet 19 <$
 (g) $8 + 8 \bullet 24 <$ (h) $7 + 9 \bullet 27 <$ (i) $5 + 7 \bullet 12 =$

6. Copy and complete using $=$, $>$, or $<$.

- (a) $28 \bullet 27 + 1 =$ (b) $52 \bullet 520 =$ (c) $890 \bullet 891 <$
 (d) $16.5 \bullet 1.5 + 51 >$ (e) $4.7 \bullet 73 - 2.6 =$ (f) $31 \bullet 18 + 12 >$
 (g) $43 \bullet 21.1 + 219 =$ (h) $72 \bullet 100 - 28 =$ (i) $0.26 \bullet 0.2 + 0.05 >$

7. Copy and complete using $=$, $>$, or $<$.

- (a) $20 + 70 \bullet 90 =$ (b) $16 \times 3.0 \bullet 1.92 >$ (c) $\$2.31 + \$2.31 \bullet \$4.63 <$
 (d) $24 - 3 \bullet 19 >$ (e) $2 + 3 + 5 \bullet 10 =$ (f) $7.2 - 1.9 \bullet 7.0 <$
 (g) $9 \times 3 \bullet 27 =$ (h) $111 \bullet 111.1 <$ (i) $3.2 + 0.7 \bullet 3.9 =$



Using $=$, $>$, and $<$ 25

OBJECTIVE

To compare numbers (wholes to wholes, decimals to decimals)

PACING

Level A 1(a)-(l)
Level B 1(a)-(p)
Level C All

RELATED AIDS

HMS — DM6.

CALC. ACTIVITY MASTERS — 20.

SUGGESTIONS

Initial Activity Review first the names of the place-value locations from thousandths to millions. Write two numbers on the chalkboard such as 4251 and 4252. Identify each numeral in each place-value location, i.e., thousands 4, thousands 4; hundreds 2, hundreds 2; tens 5, tens 5; ones 1, ones 2. Point out that, though the thousand's, hundred's and ten's digits are identical, the second number has more ones and is therefore greater, i.e., $4251 < 4252$.

Repeat for other numbers. Be sure to include pairs of whole numbers and pairs of decimal numbers. Also, demonstrate using two numbers which have different numbers of digits, i.e., 925, 8122. Emphasize that, when comparisons are being made, "hundreds" must be compared to "hundreds", and so on. This will help avoid conclusions such as $925 > 8122$ because the first digit of 925 (i.e., 9) is greater than the first digit of 8122.

USING THE BOOK

Read through the examples at the top of the pupil page, consolidating the ideas presented in the Initial Activity.

Complete Exercises 1(a), (b), and (c) orally before assigning the exercises.

ACTIVITIES

1. Have the students make up number pairs for comparison as shown on the pupil page. Have them exchanged with classmates.

2. Provide atlases so that the children can make up lists comparing populations, geographic size, etc. of locations in Canada and the world.

3. See "500 Grand" in the Activity Reservoir. Modify the game so that (a) the playing board has up to 9-

Let's Compare

6824 and 6859

6	8	2	4
↑	↑	↑	
same	same	less	
↓	↓		
6	8	5	9

6824 < 6859

2130 and 916

2	1	3	0
↓			
more			
	9	1	6

2130 > 916

17 685.274 and 17 685.131

1	7	6	8	5	•	2	7	4
↑	↑	↑	↑	↑		↑		
same	same	same	same	same		more		
↓	↓	↓	↓	↓				
1	7	6	8	5	•	1	3	1

17 685.274 > 17 685.131

Exercises

1. Compare. Use =, <, or >.

● 379.48 and 379.67

3	7	9	.	4	8
↑	↑	↑		↑	
3	7	9	.	6	7

379.48 < 379.67

● 2768.5 and 2758.6

2	7	6	8	.	5
		↑			
2	7	5	8	.	6

2768.5 < 2758.6

● 149.37 ● 49.51 >

(e) 884 ● 8837 <

(g) 18 497 ● 13 845 >

(i) 559.26 ● 559.26 =

(k) 16 347 ● 16.343 >

(m) 732 564 ● 732 564 =

★ (o) 767 254.6 ● 945 999 <

(d) 324.68 ● 321.78 >

(f) \$68.75 ● \$68.75 =

(h) 26 483.27 ● 26 417.36 >

(j) \$649.17 ● \$6499 17 <

(l) 5000.165 ● 4000.165 >

(n) 17 643 561 ● 17 654 321 <

★ (p) 9 999 999 ● 10 000 000 <

2. Arrange each set in order from smallest to largest.

(a) 33.9, 34.4, 43.1, 33.2, 32.6

(c) 12.35, 12.37, 12.45, 12.47, 12.37

(b) 167.5, 176.5, 167.7, 157.6, 165.7

(d) 26.375, 26.573, 26.753, 26.357

26 Comparisons

ANSWERS:

2. (a) 32.6, 33.2, 33.9, 34.4, 43.1
 (b) 157.6, 165.7, 167.5, 167.7, 176.5
 (c) 12.35, 12.37, 12.37, 12.45, 12.47
 (d) 26.357, 26.375, 26.573, 26.753

digit numbers; (b) players (2) take turns throwing both dice once to locate a number value; (c) players compare both numbers, the player with the greater number scoring 1 point; and (d) player with the highest point total after 10 turns wins.

EXTRA PRACTICE

Compare using =, <, or >.

- (a) 17 682 ○ 17 692 (b) 324.71 ○ 324.71
 (c) 18 449 ○ 18 349 (d) 77.342 ○ 77.442
 (e) 8000 ○ 8000 (f) \$73.65 ○ \$72.43

Chapter Test

1. (a)
$$\begin{array}{r} 8347 \\ 483 \\ 2768 \\ + 4403 \\ \hline 16001 \end{array}$$
 (b)
$$\begin{array}{r} \$123.95 \\ 43.06 \\ 385.79 \\ + 426.38 \\ \hline \$979.18 \end{array}$$
2. (a)
$$\begin{array}{r} 70\,000 \\ - 38\,564 \\ \hline 31\,436 \end{array}$$
 (b)
$$\begin{array}{r} \$4832.36 \\ - 709.47 \\ \hline \$4122.89 \end{array}$$
3. Write a number sentence and solve it to answer this problem:
Marvin has \$8.70. He wants to buy a skateboard that costs \$19.50.
How much more does he need?
$$\$19.50 - \$8.70 = \$10.80$$
4. (a) $10.7683 + 5.37 + 0.8521 + 2.765$ (b) $302.856 - 0.3401$
5. Write in numerals.
(a) two million, two hundred fifty-five thousand, four hundred ten
(b) ten billion (c) thirty-five billion, seven hundred million
6. Write in words.
(a) 12 700 346 (b) 84 000 000 000 (c) 7 350 000 000
7. Write as decimals.
(a) $17\frac{3}{10}$ (b) $857\frac{19}{1000}$ (a) 300.81 (b) 18.511
8. Write as fractions.
(a) $300\frac{81}{100}$ (b) $18\frac{511}{1000}$
9. Write the place value of each underlined digit.
(a) 3 476.853 (b) 46 842.35 (c) 9 347 852 901 (d) 5.1163
10. Round off as indicated.
(a) 7832 (nearest 1000) (b) 527 (nearest 100) (c) 82 500 (nearest 10 000)
(d) 45.6 (whole number) (e) 63.584 (hundredth) (f) 181.18 (tenth)
11. The town of Hampton elected a new mayor. Mr. Meadows got 4837 votes. Ms. Klein got 3424 votes. Ms. Foley got 341 votes. Mr. Gauthier got 555. Round to the nearest hundred to estimate how many people voted altogether.
12. Copy and complete using =, >, or <.
(a) $28.7 \bullet 28.70$ (b) $19 \bullet 20 - 1$ (c) $7.5 + 9.5 \bullet 13.75$
(d) $\$25.30 \bullet \$12.40 + \$12.90$ (e) $487 \bullet 48.71$ (f) $8\,531\,332 \bullet 8\,532\,332$
13. Solve.
(a) $n + 21 = 36$ (b) $n - 14 = 23$ (c) $n - 7.3 = 10.0$
(d) $(15 + 3) + 12 = 15 + (3 + a)$ (e) $2.3 + 5.10 = 5.10 + a$

Chapter 1 test 27

ANSWERS:

1. (a) twelve million, seven hundred thousand, three hundred forty-six
(b) eighty-four billion
(c) seven billion, three hundred fifty million
2. (a) hundredths (b) thousands (c) hundred millions (d) ten thousandths
3. (a) 8000 (b) 500 (c) 80 000 (d) 46 (e) 63.58 (f) 181.2
4. (a) $n = 15$ (b) $n = 37$ (c) $n = 17.3$ (d) $a = 12$ (e) $a = 2.3$

OBJECTIVE

To evaluate achievement of the chapter objectives

PACING

Level A All
Level B All
Level C All

RELATED AIDS

HMS — DM7.

USING THE BOOK

Each student should do this test independently under supervision. Assistance should be given only when the instructions are not understood. After the work has been corrected, you should provide appropriate remedial work. You may wish to reteach if a large number of students had difficulty with a particular topic or concept.

The following chart will help in this regard. The specific objectives are listed in the Chapter Overview.

An alternate Chapter Test can be found in the Holt Mathematics System Duplicating Masters available for use with this grade level.

Test Item	Objective	Text Page Number
5-9	A	1-3, 10, 11
1, 2, 4	B	4-9
10, 11	C	12-14
12, 13	D	18-21, 24, 25
13(d), 13(e)	E	20, 21
3, 11	F	15, 17, 22

CHAPTER 2 OVERVIEW

This chapter reviews the circle as an introduction to developing the concept of an angle, angle measure using protractors, congruent angles, and angle properties of some polygons. Some geometric designs based on the use of compasses and on straight lines are presented. Finally, polyhedras are constructed and explored.

OBJECTIVES

- A To review the parts of the circle and to recognize the relation between the measure of the radius and diameter
- B To use a protractor to measure an angle, to identify congruent angles, and to name angles according to their measures
- C To use compasses to draw designs with circles and to construct certain polygons in circles; to identify and construct the first six polygons
- D To note and use the fact that the sum of the measures of the angles of a triangle is 180° and that of a quadrilateral is 360°
- E To identify and construct the rectangular prism, cylinder, the tetrahedron, cube, octahedron as well as certain other solids
- F To identify and draw parallel lines and intersecting lines

BACKGROUND

An angle is a pair of rays with a common endpoint. We measure angles using a protractor graduated in degrees. If the measure of an angle is 90° we call it a right angle, if less than 90° we call it acute, and if more than 90° we call it obtuse. A straight angle has a measure of 180° .

Polygons and polyhedra are named according to the number of sides and number of faces, respectively; 3 -tri-, 4 -quad- or -tetra-, 5 -penta-, 6 -hexa-, 7 -hepta- or -septa-, 8 -octa-.

MATERIALS

compasses
protractors
construction paper
tracing paper
scissors
tape
glue
rulers
elastic bands
shoe boxes
large needles
yarn
split fasteners

CAREER AWARENESS

Forestry Workers [52]

Many forestry workers are associated with the timber industry to produce raw materials for lumber, wood products, and wood pulp for paper. The work is strenuous and often dangerous for the novice or careless.

In the larger operations much of the work is mechanized, using power chain saws and large equipment to move trees and logs. In the smaller and private operations there is still a great deal of manual labour.

A major concern to all forestry workers is that of conservation. Complicating the issue is the extended period of time required for rejuvenation and the use of ecologically hazardous chemicals to fight natural parasites such as the Spruce Bud Worm. If decisions are delayed in these areas and forests are depleted without care and effort made for their conservation and renewal, there will be a shortage of trees in the near future.

Forestry Firefighters [53]

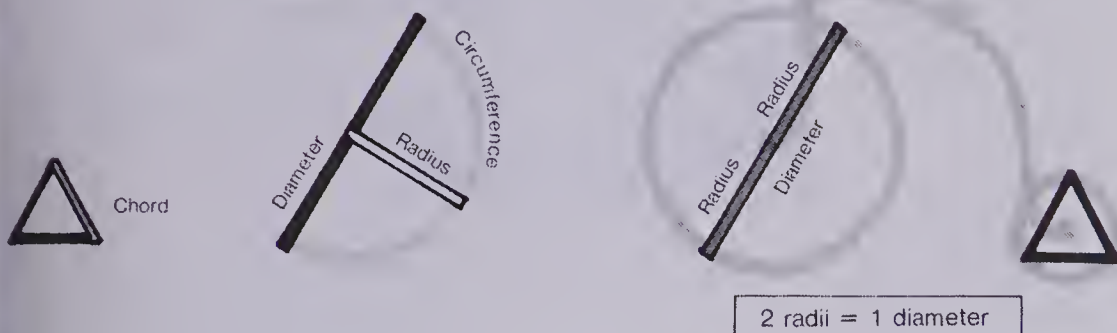
Every spring, summer, and fall there are major outbreaks of forest fires across Canada. Many of these are of natural origin (lightning) but many are caused by man.

While many fires are fought by bulldozers and fire bombers, most are fought to a large extent by man without the aid of big machinery because the fires are often located in inaccessible areas. Given that forest fires can spread literally in leaps and bounds, especially if pushed by strong or gusty winds, fire-fighters must exercise considerable care not to become disoriented and then overtaken or surrounded by fire.

One of the common ways to fight fire is to start a backfire which provides a clean or burned strip over which the uncontrolled fire cannot travel.

Not all forest fires are harmful. Some forest fires are set on purpose. The reason for this is that seeds of some plants must be exposed to extreme heat in order to germinate. Without the fire these trees would never reseed and hence would become extinct in the area. Naturally, these fires are rigidly controlled.

The Circle

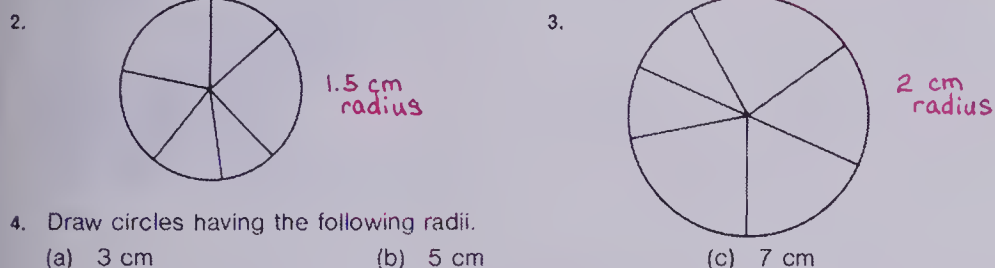


Exercises

1. Draw this figure.
Label: radius, diameter, chord, circumference

Measure each radius in the circles below.

What do you notice about the length of each radius of a circle?



4. Draw circles having the following radii.
(a) 3 cm (b) 5 cm
5. Draw the diameters for each of the above.
6. Compare the length of the radius and the diameter of each circle in Exercise 4.
The radius is half the diameter in each circle.

The circle, radius, diameter 29

OBJECTIVES

To review radius, diameter and introduce chord
To use the relations between the measures of a radius and diameter of a circle

PACING

Level A All
Level B All
Level C All

VOCABULARY

chord, diameter, radius, circumference

MATERIALS

compasses

BACKGROUND

The diameter of a circle may be described as any line segment whose endpoints are on the circle and which passes through the centre of the circle.

The radius is a line segment from the centre of a circle to any point on the circle.

A chord is any segment whose endpoints are on the circle (though not necessarily through the centre of the circle). A diameter is a special chord.

SUGGESTIONS

Initial Activity Ensure that the students can use the compasses to draw circles. Make available scrap paper for the students to use in their spare time. Encourage the students to draw circles and simple circle designs.

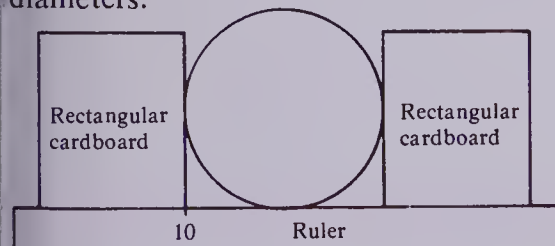
USING THE BOOK

Review the circle by directing the students' attention to the display at the top of the pupil page. Discuss the three terms: radius, diameter, chord. Emphasize that the diameter of a circle is twice as long as the radius.

Ask the students to use the compasses to draw a circle with one diameter, one radius, and one chord. Label. Then ask the students to measure the diameter and the radius in millimetres. Ask: "Is the diameter twice as long as the radius? [Yes]" Then assign the page.

ACTIVITIES

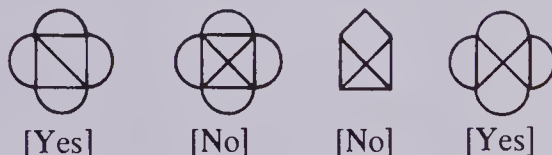
1. Students can bring to class round objects suitable for measuring (e.g., bottle caps, cans, emblems). Have them estimate the radius of each. They will need to guess at the centre first. They then measure the diameter and calculate the radius to check their estimates. A measuring device like this one can be used to measure the diameters.



2. Have students prepare a report and/or bulletin display on the history of the wheel.

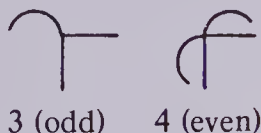
3. Prepare a challenge card such as:

Which of the following figures can you trace without lifting your pencil or retracing?



Challenge the students to make their own puzzles.

Note: A figure is retraceable if it does not have more than 2 odd-number intersections.



(Do not give this solution to the students — let them play with it — some may discover it while many will not.)

OBJECTIVES

- To review the concept of angle as formed by a rotating ray
- To name angles
- To determine congruence by tracing

PACING

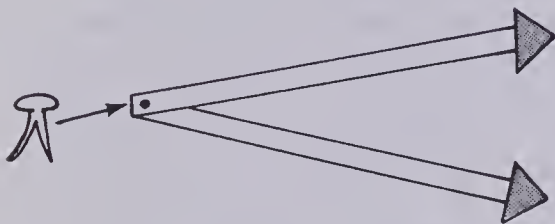
- Level A All
- Level B All
- Level C All

VOCABULARY

vertex, ray, rotating ray, congruent, observers

MATERIALS

a model of an angle: two heavy strips of cardboard with a fastener as illustrated



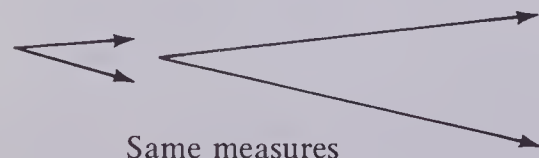
BACKGROUND

The size of an angle is measured by the amount of rotation of a ray from its original position: if it rotates a $\frac{1}{4}$ turn it has a measure of 90° ; if it rotates a $\frac{1}{2}$ turn it has a measure of 180° .

SUGGESTIONS

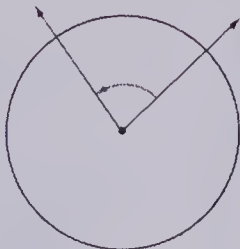
Initial Activity Review the concept of an angle by illustrating with the model that the more the rays "open" the greater is the angle.

Keeping the two rays at the same opening, draw one angle with "short" rays (visible) and one with "longer" rays. Emphasize the sizes of the angles are the same.

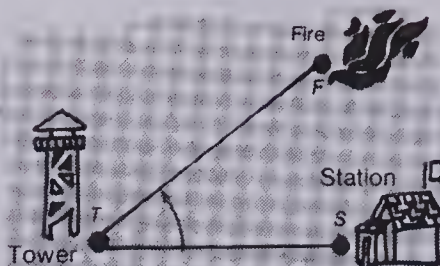


Review the meaning of congruent: having the same measure or having the same size.

Naming Angles



An angle is formed by a rotating ray.

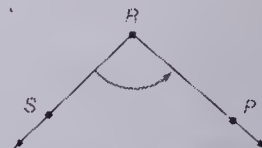


The fire lookout observer sighted the station. He then sighted the fire. The two lines of sight form a model of an angle

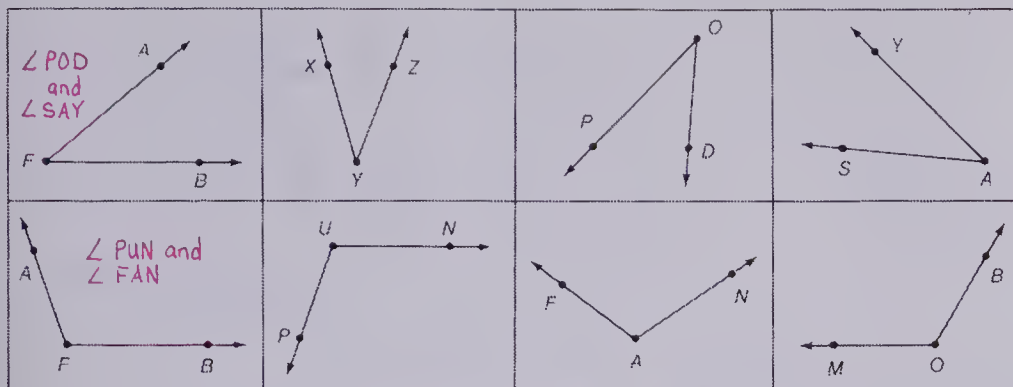
Some angle language: Vertex: point T
Symbol for angle: \angle
Names: $\angle FTS$, $\angle STF$, or $\angle T$
Sides: ray TS and ray TF

Exercises

- Copy this angle.
 - Name the vertex. R
 - Name each ray. ray RS and ray RP
 - Name the angle three ways, using the symbol \angle .
 $\angle SRP$, $\angle PRS$, or $\angle R$
- Draw an angle.
 - Label the vertex M .
 - Mark and label a point N on one ray.
 - Mark and label a point P on the other ray.
 - Name each ray. ray MN and ray MP
 - Name the angle three ways.
 $\angle NMP$, $\angle PMN$, or $\angle M$
- Name each angle three ways.
 - $\angle RAP$, $\angle PAR$, or $\angle A$
 - $\angle TOM$, $\angle MOT$, or $\angle O$
 - $\angle STY$, $\angle YTS$, or $\angle T$

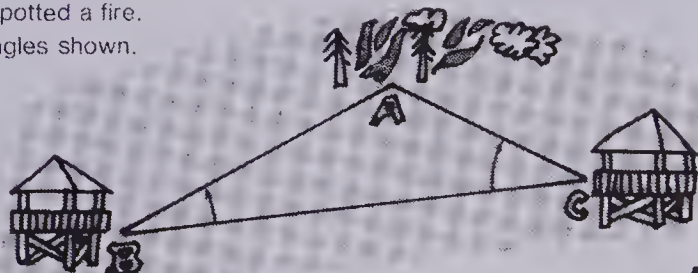


4. Use tracing paper. Which angles are congruent to $\angle AFB$?

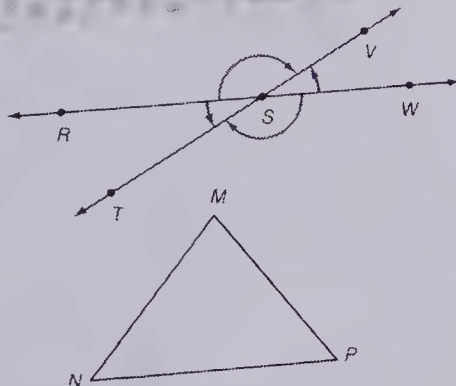


5. Two observers spotted a fire. Name the two angles shown.

$\angle B$ and $\angle C$



6. (a) Why is it not suitable to name $\angle RST$ simply $\angle S$?
 (b) Name each of the four angles shown.
 (c) Which angles are largest?
7. (a) Is it enough to name each angle of this triangle in this manner:
 $\angle M$, $\angle N$, and $\angle P$? Explain.
 (b) Name each angle another way.



Naming angles 31

USING THE BOOK

Discuss the display at the top of the pupil page to review how to name an angle; the terms vertex, vertices, and rays; and the use of the symbol " \angle " to represent "angle".

Assign Exercises 1 and 2. Move about the class to see if there are any difficulties, or do Exercise 1 orally and Exercise 2 on the chalkboard by having individual students do parts (a), (b), and (c). Have all students do parts (d) and (e).

Provide the students with tracing paper for Exercise 4. Emphasize the meaning of both congruent and size of an angle by asking if the tracing "fits" or "matches" one or more of the other angles in the row.

Exercises 6 and 7 should indicate whether or not students understand the concept of naming angles and how certain ways have certain advantages.

ACTIVITIES

1. Have each student make a model of an angle as illustrated in Materials. Under your directions and guidance, have them (a) open the rays slightly and trace a "small" angle; (b) open the rays a little more and draw a larger angle; (c) continue for a series of angles including a $\frac{1}{4}$ turn or right angle, obtuse angles, and straight angle ($\frac{1}{2}$ turn). Have the students label and name each angle.

2. Ask the students to prepare a list of objects or items in the classroom that are models of angles. They can name the type of each angle later (see page 36).

3. Prepare lists of angles for a scavenger hunt. The students are to record the name and location of the item for each angle.

ANSWERS:

6. (a) $\angle S$ could be one of four different angles.

(b) $\angle RSV$, $\angle VSW$, $\angle WST$, and $\angle TSR$

(c) $\angle RSV$, and $\angle WST$

7. (a) Yes

(b) $\angle M - \angle NMP$, $\angle PMN$
 $\angle N - \angle MNP$, $\angle PNM$
 $\angle P - \angle NPM$, $\angle MPN$

OBJECTIVES

- To develop the concept of measure of an angle
- To introduce the degree
- To read a scale in degree measures on a protractor
- To know there are 360° in a circle

PACING

- Level A All
- Level B All
- Level C 1-13

MATERIALS

sets of unit angles made by dividing a circle into 12 parts (30° each), demonstration circular protractor or transparent circular protractor for overhead projector

BACKGROUND

The degree ($^\circ$) is an arbitrary unit accepted by mathematicians around the world as the measure of an angle. The measure of an angle is determined by the amount of turning or rotating of one ray from the position of the other ray. The Babylonians established the practice of dividing the circle into 360 equal parts hence 360° in a complete revolution.

SUGGESTIONS

Initial Activity Provide each group of students with (a) a set of unit angles; (b) a duplicated page of angles each a multiple of the unit angle. Ask the students to find the measure of each angle in "units".

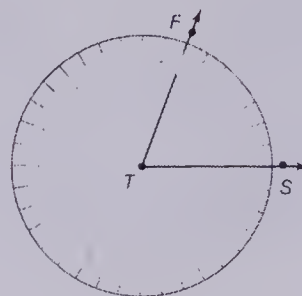
USING THE BOOK

Using the overhead projector and the transparent protractor or the demonstration protractor, instruct carefully step by step how to read the measure of an angle on the protractor.

Exercises 1 to 13 may be done orally or by assignment. Students that had difficulty with Exercise 13 should do Exercise 14.

Measuring Angles I

The measure of $\angle FTS$ is 70° .



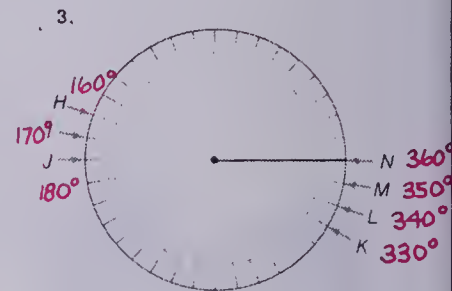
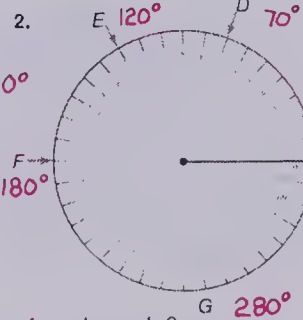
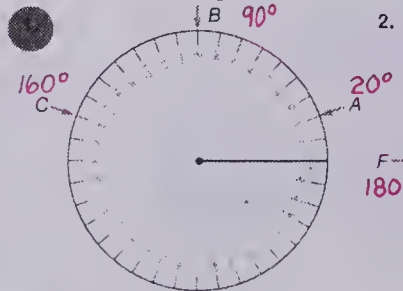
Mathematicians use the **degree** as a unit of measure.

A circle is divided into 360 equal parts. Each part is called one degree.

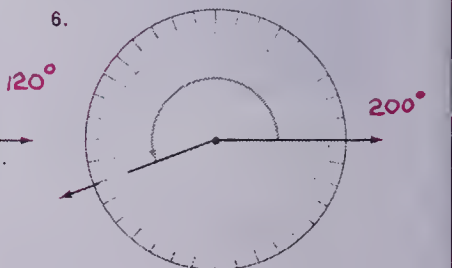
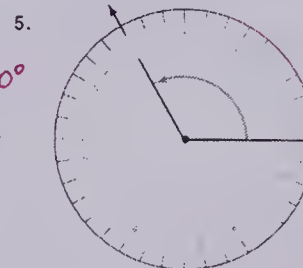
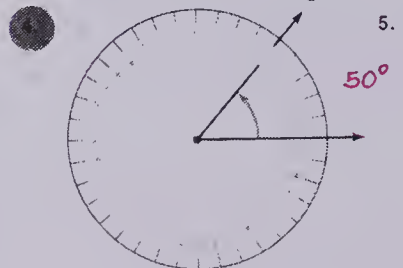
There are 360° (degrees) in one circle.

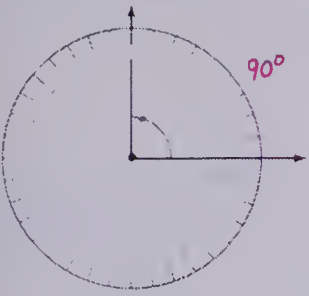
Exercises

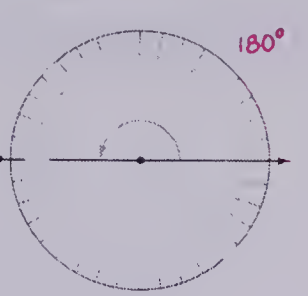
What is the reading at each arrow?

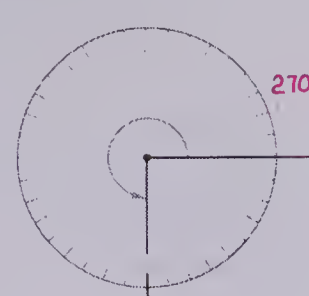


What is the measure in degrees of each angle?



7. 

8. 

9. 

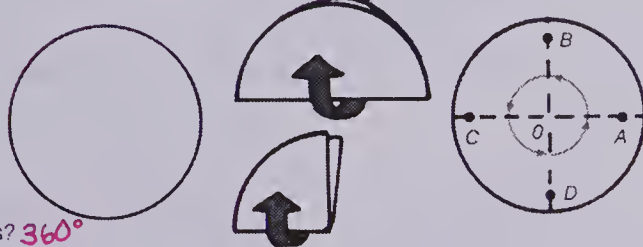
10. A ray rotates $\frac{1}{4}$ of a full turn.
How many degrees? 90°

11. A ray rotates $\frac{1}{2}$ of a full turn.
How many degrees? 180°

12. A ray rotates $\frac{3}{4}$ of a full turn.
How many degrees? 270°

13. A ray rotates one full turn.
How many degrees? 360°

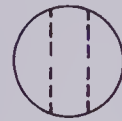
14. Draw and cut out a full circle.
Fold it in half.
Fold in half again.
Open and mark the fold lines.
Label as shown.
What is the measure of $\angle AOB$?
 $\angle BOC$? $\angle COD$? $\angle DOA$?
What is the sum of all four angles? 360°



Measuring angles: degree 33

ACTIVITIES

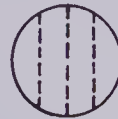
1. Provide students with circular regions. Start by challenging them to divide a region into the fewest number of parts using two line segments; the greatest number of parts using two line segments.



3 pieces (least)

4 pieces (greatest)

Then repeat with 3 and 4 segments (above 4 it gets quite complicated).



Fewest: 4

Greatest: 7



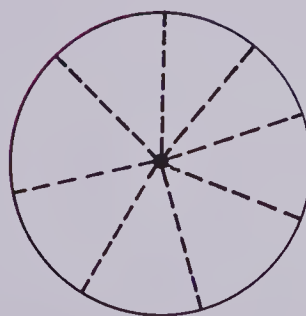
Fewest: 5

Greatest: 11

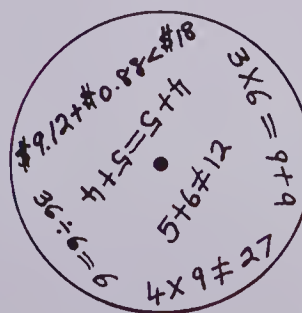
2. By making a chart a student may predict what the numbers are for 5 and more segments.

Number of Line Segments	Least Number of Parts	Greatest Number of Parts
2	3	4
3	4	7
4	5	11
5	?	?
6	?	?

3. Have the children make up Circle Puzzles for use with other classmates, groups, or classes in the school. Students should (a) draw and cut out a full circle (radius about 10 cm) on heavy paper or cardboard; (b) use ruler and pencil to draw on various radii; (c) turn the circle over and write on statements (perhaps from a chalkboard list); (d) turn the circle over again and cut it into pieces along the radii; (e) place the pieces into an envelope (labelled according to the skill being tested: basic facts in \times , \div , $+$, $-$; equivalent fractions; equivalent fraction and decimal amounts; equations and inequations) for later reassembly.



Front



Back

OBJECTIVE

To use a protractor to measure any angle to the nearest multiple of 5°

PACING

Level A All

Level B All

Level C All

VOCABULARY

protractor

MATERIALS

protractor for each student,
demonstration protractor for use on
chalkboard or overhead projector

SUGGESTIONS

Initial Activity There are two scales on a protractor. Discuss each. Using the demonstration protractor discuss how the angle can be measured two ways opening clockwise or opening counterclockwise. Practise reading the scales.

USING THE BOOK

Discuss the displays. Explain that not all protractors look alike and that adjustments may have to be made when using the protractors that are in the school.

Using the demonstration protractor provide a step-by-step explanation of how to use the protractor. One method is given.

- Place centre of protractor on the vertex of the angle.
- Adjust the 0 line of the protractor above one ray.
- Note what way the angle opens.
- Select proper scale by starting at 0.
- Read angle measure on proper scale.

Provide angles drawn on the overhead projector or on the chalkboard. Select students to demonstrate how to measure the angles while you and the class observe. You may ask why the inner (or outer as the case may be) scale was used.

Do Exercises 1 to 4 orally.

Watch the students while they do Exercises 5 to 7 to see that they use the protractors correctly.

Encourage the student to think: Is the angle greater or less than a right angle?

If it is less, then the measure is less than 90° .

If it is greater, then the measure is greater than 90° .

The Protractor

This is a **protractor**.
It is one half of a circle.
It has two scales.
We use a protractor to measure angles.

Step 1

Place the centre of the protractor on the vertex of the angle.

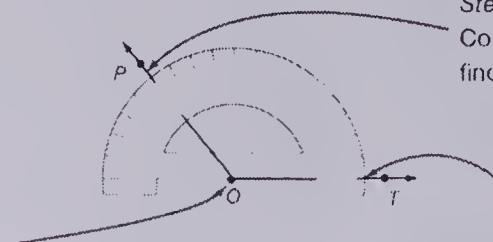
Step 3

Count from 0° to find the angle measure.

$\angle TOP$ measures 130° .
 $\angle TOP = 130^\circ$

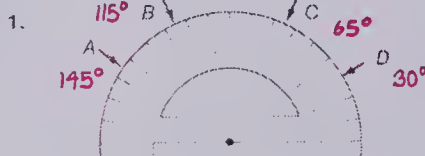
Step 2

Adjust 0° along one ray.

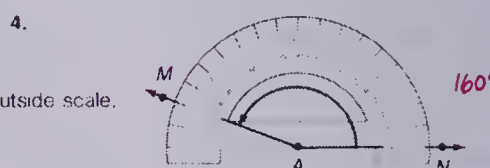
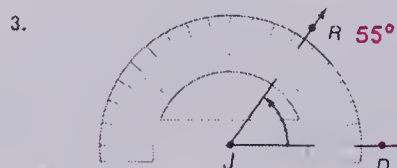


Exercises

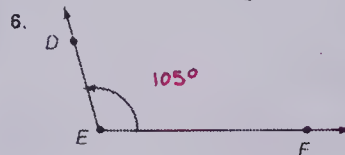
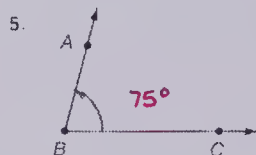
What is the reading at each arrow?



What is the measure of each angle?

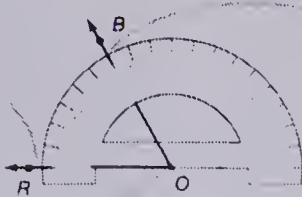


Use your protractor. What is the measure of each angle?



Measuring Angles II

Step 2
Adjust 0° along
one ray.

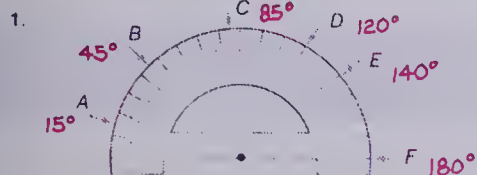


Step 3
Count from 0°
on the inner scale.
 $\angle ROB$ measures 60° .
 $\angle ROB = 60^\circ$

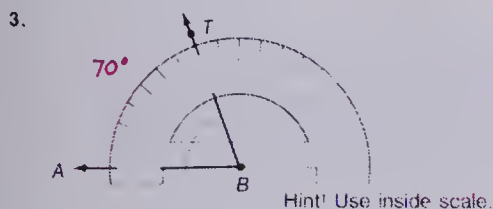
Step 1
Place the centre of the protractor on
the vertex of the angle.

Exercises

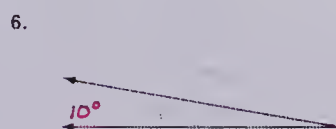
What is the reading at each arrow?



What is the measure of each angle?



Use your protractor. What is the measure of each angle?



Using a protractor 35

ACTIVITIES

1. Draw on the chalkboard or the overhead projector a number of angles suitable for students to measure. Ask students having difficulty to measure the angles while you watch and give assistance.

2. Pair the students. Each student draws 5 angles and records the measure of each. They then exchange angles. Each measures the angles and checks his work with the partner. Some students will measure to the nearest one degree while others will find it desirable to round to the nearest multiple of 5° .

3. "Estimation Game". Draw on a set of 10 cards a number of angles with measures from 0° to 180° . Student A draws a card, estimates its measure, and hands the card to student B who checks using a protractor. Student A scores a point if the estimate is within 10° (or some agreed upon tolerance) of the actual measure. The two students reverse roles. The student with the most points after 10 turns each is the winner.

OBJECTIVES

- To use the protractor to determine whether or not two angles are congruent
- To identify acute, right, obtuse, and straight angles

PACING

- Level A 1-4
- Level B All
- Level C All

VOCABULARY

acute, obtuse, straight angle

MATERIALS

protractor

RELATED AIDS

HMS — DM8.

BACKGROUND

In the previous lesson, the student has measured angles of 90° , less than 90° , and greater than 90° . In this lesson, we name these angles as well as the straight angle. Note we deal with angles with measures of 180° at this time.

SUGGESTIONS

Initial Activity Remind the students how to place the protractor properly to measure an angle. Not all protractors are as shown in this text therefore it may be necessary to give specific instructions for the protractor in use in your classroom.

USING THE BOOK

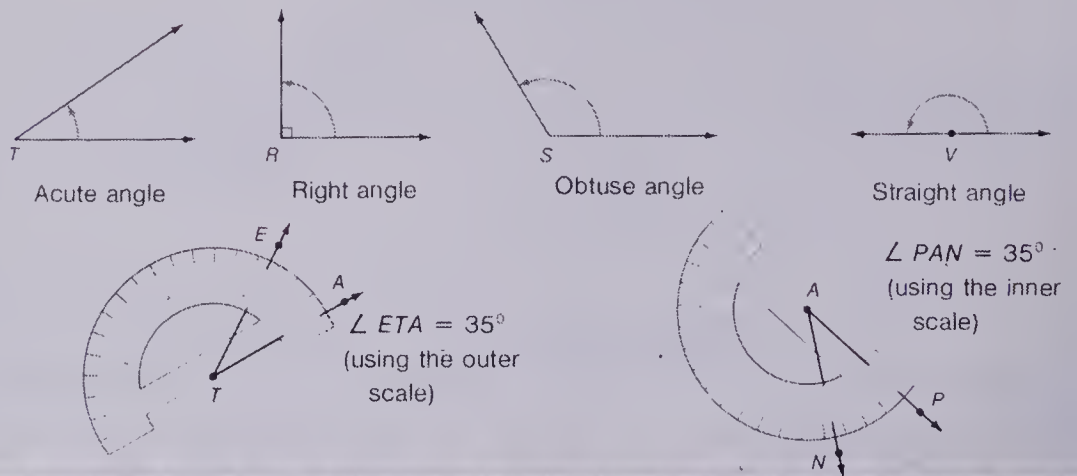
Discuss with the students the four types of angles shown.

Do Exercises 1, 2, and 3 in class with oral responses.

Assign Exercise 4. It may be necessary to show the students how to draw an angle of a certain size using the protractor, although it is not necessary for this exercise. The student only needs to draw an angle less than 90° , more than 90° , etc.

Assign the balance to those capable of completing the exercises.

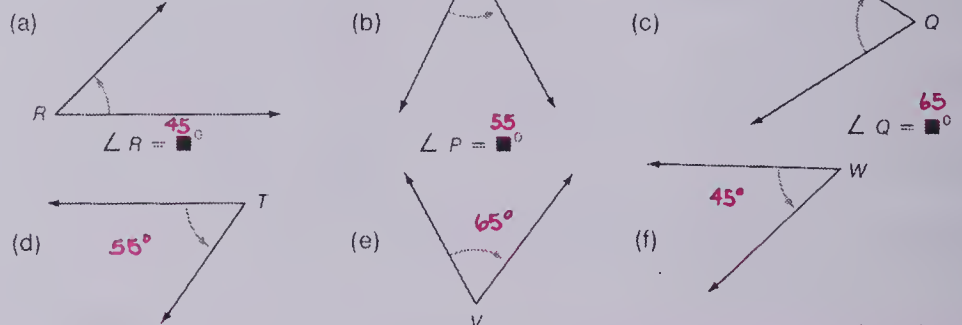
More About Angles



The two angles have the same measures.
The angles are congruent.

Exercises

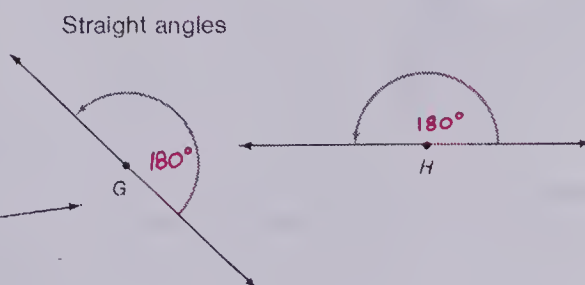
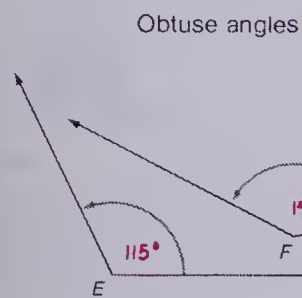
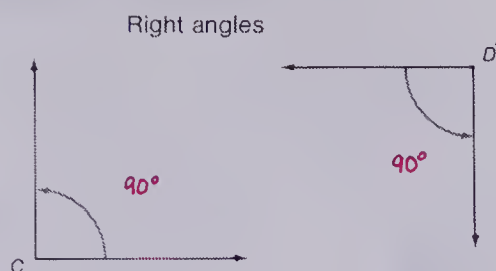
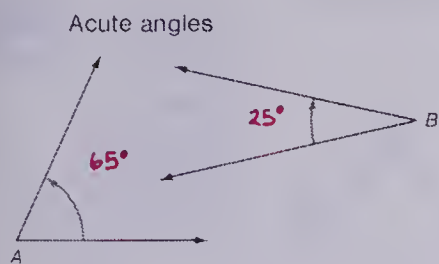
- Measure each angle.



- Which pairs of angles in Exercise 1 are congruent? *(a) and (f), (b) and (d), (c) and (e) are congruent pairs of angles.*

36 Congruent angles using measurement

3. Measure each angle.



4. Draw two angles of each kind: acute, right, straight, obtuse. Label and name each.

5. Copy and complete.

- (a) Angles with measures less than 90° are acute angles.
- (b) Angles with measures of 90° are right angles.
- (c) Angles with measures greater than 90° and less than 180° are obtuse angles.
- (d) Angles with measures of 180° are straight angles.

6. Copy and complete.

- (a) The measure of any straight angle is 180° .
- (b) The measure of any right angle is 90° .
- (c) The measure of any obtuse angle is greater than 90° and less than 180° .
- (d) The measure of any acute angle is less than 90° .

Types of angles 37

ACTIVITIES

1. To improve the students' vocabulary, both in mathematical and non-mathematical areas, ask them to give the meanings of some mathematical words which have nonmathematical meanings also:

volume, count, odd, prime, difference, mean, power, product, square, obtuse, right, acute, plus, ruler, measure, operation.

Variation: Since definitions are often very difficult (and might simply be copied from a dictionary), ask the students to write sentences using the term in different ways to illustrate the different meanings.

Example

The numbers 3, 5, 7, 11, 13 are prime numbers.

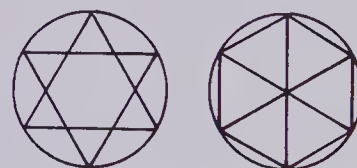
The most often viewed television program is often shown during prime time.

The water pump needs to be primed before it will work.

2. Direct the student to identify places where angles of the four types are used.

3. Students could use their compasses and protractors to make patterns or designs based on the circle. They can investigate the measures of the basic angles in the design.

These are based on angles of 60° (and/or 120°).



This figure has angles of 72° at the centre.



4. Have the students play "Estimation Game", Activity 3, pages 34 and 35.

OBJECTIVE

To draw circles using compasses given the radius

PACING

Level A All

Level B All

Level C All

MATERIALS

compasses (one for each student), rulers

RELATED AIDS

HMS — DM9.

BFA COMP LAB II — 4, 5, 92-95, 100

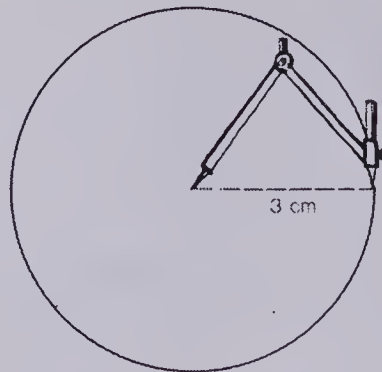
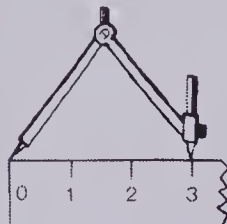
USING THE BOOK

Demonstrate using chalkboard compasses the steps shown in the display at the top of the pupil page. Emphasize how to set compasses for a given radius. Students will need to know how to read centimetres and millimetres on a ruler.

Before assigning the Tune Up, you may wish to have each student set a personal goal for each type of question.

Using Compasses

Draw a circle with a radius of 3 cm



Exercises

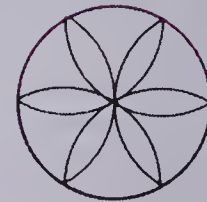
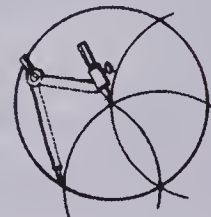
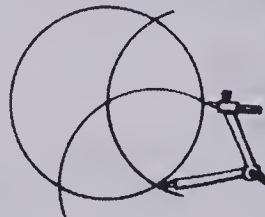
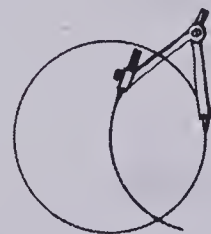
Draw circles using each of these as the radius.

1. A _____ B 2. C _____ D 3. E _____ F

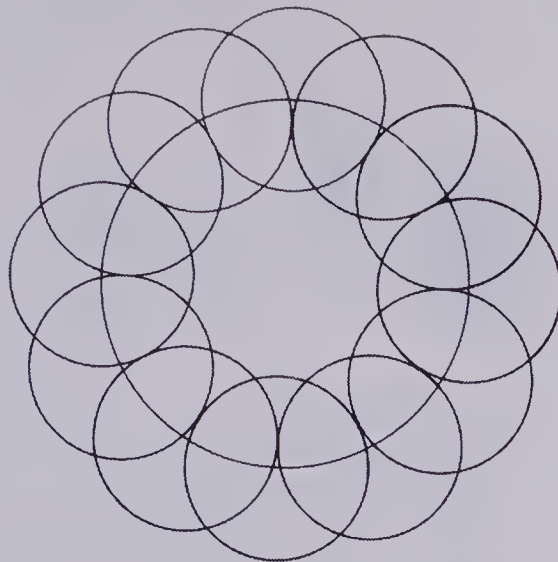
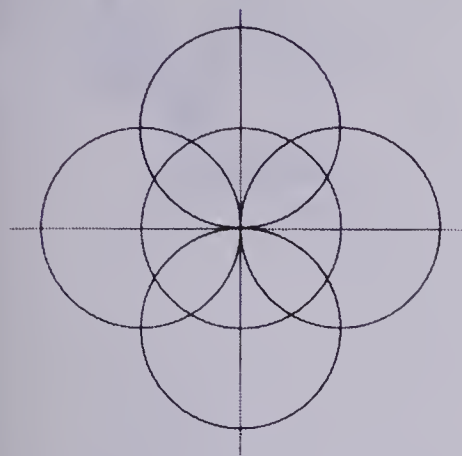
4. Measure the radius of each circle to the nearest tenth of a centimetre. (1) 2.9 cm (2) 4.2 cm (3) 4.6 cm

5. Draw a circle with:
(a) radius 2.8 cm (b) radius 33 mm (c) radius 52 mm

6. Measure the diameter of each of the three circles in Exercise 5. (a) 5.6 cm (b) 66 mm (c) 104 mm



8. Copy and colour each design.



9. Make and colour your own design.

Tune Up

1. (a)	72	(b)	36	(c)	155	(d)	137	(e)	88
	+ 108		+ 144		+ 25		+ 43		+ 92
	<u>180</u>		<u>180</u>		<u>180</u>		<u>180</u>		<u>180</u>
2. (a)	180	(b)	180	(c)	180	(d)	180	(e)	180
	- 37		- 109		- 56		- 45		- 163
	<u>143</u>		<u>71</u>		<u>124</u>		<u>135</u>		<u>17</u>
3. (a)	72	(b)	90	(c)	105	(d)	98	(e)	61
	72		43		33		31		59
	+ 36		+ 47		+ 42		+ 51		+ 60
	<u>180</u>		<u>180</u>		<u>180</u>		<u>180</u>		<u>180</u>
4. (a)	\$33.25 + \$6.97		(b)	7.3 - 2.43		(c)	41.4 + 0.532		
	<u>\$40.22</u>			<u>4.87</u>			<u>41.932</u>		

Using compasses: practice 39

ACTIVITIES

1. Challenge the students to draw a circle for each:

- Draw 3 chords so that a triangle is formed.
- Draw 4 chords so that a square is formed.
- Draw 4 chords so that a rectangle that is not a square is formed.
- Draw 4 chords so that a trapezoid is formed.
- Draw 4 chords so that a parallelogram that is not a rectangle is formed. Is it possible? [No!]

2. Challenge the students to use circles only to draw cartoons similar to this one.



3. Have students follow these

steps:

- Write your house number.
- Double your house number.
- Add 5.
- Multiply by 50.
- Add your age.
- Add the number of days in a normal year.
- Subtract 615.
- Put in a decimal to show dollars and cents.

Then tell them: "The dollars part is your house number; the cents part is your age."

4. See "Football" as described in the Activity Reservoir.

OBJECTIVES

To discover the sum of the angles of
(a) triangles and (b) quadrilaterals
To review scalene, isosceles, and
equiangular triangles

PACING

Level A All
Level B All
Level C All

MATERIALS

construction paper, scissors, protractors

SUGGESTIONS

Initial Activity Review scalene triangle, isosceles triangle, and equilateral triangle. Review how a rectangle is different than a quadrilateral. [Any four-sided plane shape is a quadrilateral but a rectangle is a special quadrilateral that has opposite sides congruent and the four corners are square.]

USING THE BOOK

Direct the students to do each activity on page 40 as indicated by the instructions and pictures.

Discuss the results.

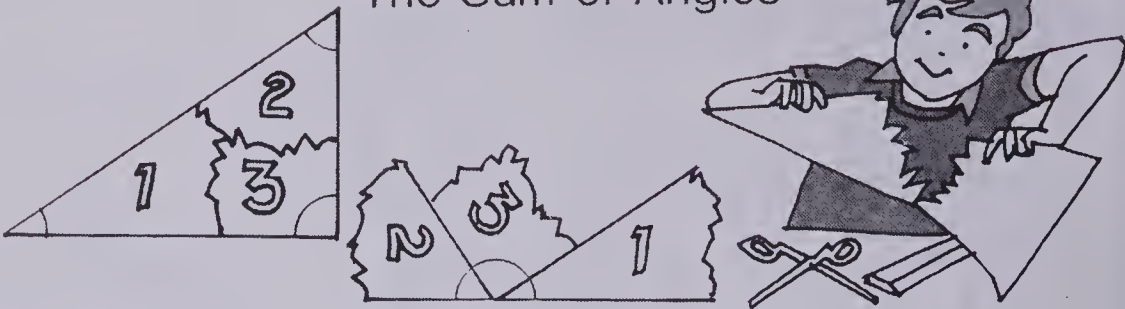
Encourage students to tell what they observe and conclude. Expressing the observations in words is an important learning activity. The students may check their results by measuring each angle and finding the sum in each case.

The discussions that follow the exercises on page 41 should allow students to put into their own words what they observe.

It may be necessary to remind students that measurements are approximate therefore it is quite possible totals may be 179° or 181° — slightly more or less than 180° — for a triangle. Also point out that the more accurately they measure, the closer their answers will be to 180° . The sum of the angles of a quadrilateral is 360° .

The sum of the measures of the angles of a triangle property is stated in the next lesson.

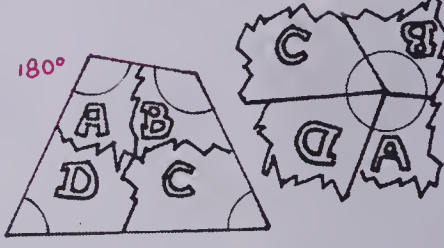

The Sum of Angles



Activity

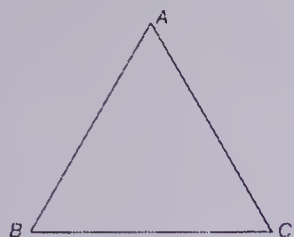
Refer to the display.

1. Draw a scalene triangle. Cut it out.
Label and colour each angle as shown.
Tear off the corners.
Place the angles together as shown. Paste in your book
What is the sum of the angles? 180°
2. Draw an isosceles triangle.
Repeat Activity 1 again. What is the sum of the angles? 180°
3. Draw 2 other triangles.
(a) What is the sum of their angles? 180°
(b) What is the sum of the angles of any triangle? 180°
4. Draw a quadrilateral (4-sided shape). Cut it out.
Label and colour the angles as shown
Tear off the corners.
Place the angles together as shown.
What is the sum of the angles? 360°
5. Draw another quadrilateral.
Repeat Activity 4 again. What is the sum of the 4 angles in any quadrilateral? 360°



Exercises

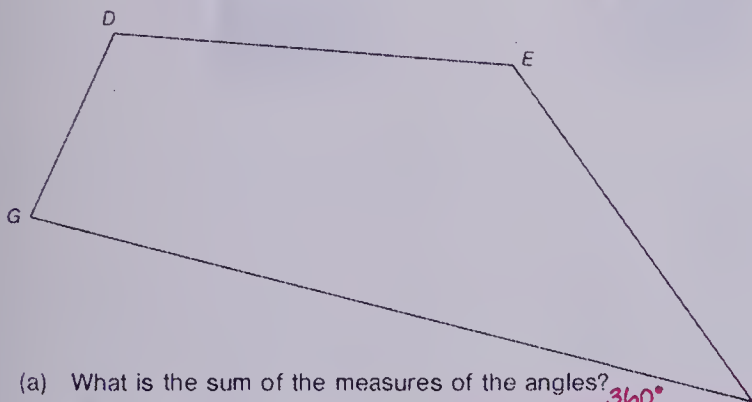
1. Measure the angles. Record the data in the table.



Angle	Measure
A	60°
B	60°
C	60°
Sum	180°

- (a) What is the sum of the measures of the angles? 180°
- (b) Is it what you expected? Explain. Yes. The sum of the angles of any triangle is 180°.
- (c) What do you know about the lengths of the sides of this triangle? What is the name of this special triangle? equilateral triangle. They are congruent.
- (d) What do you notice about the measures of the angles? They are the same.

2. Measure the angles. Record the data in the table.



Angle	Measure
D	110°
E	130°
F	40°
G	80°
Sum	360°

- (a) What is the sum of the measures of the angles? 360°
- (b) Is it what you expected? Explain. Yes. The sum of the angles of any quadrilateral is 360°.

3. Draw a rectangle.

What is the measure of each angle of a rectangle? 90°

What is the sum of the measures of the angles of a rectangle? 360°

Activity sum of angles of a triangle 41

ACTIVITIES

1. Have students draw 3- and 4-sided figures of their own and measure the angles. They should record their results in charts as in Exercises 1 and 2 on page 41.

2. Provide sets of the tangram pieces described in the Activity Reservoir. Students may have made patterns with tangram pieces on previous occasions. Pose these problems:

- (a) Which tangram pieces are congruent?
- (b) How many small triangles (A, E) are needed to cover square B?
- (c) How many small triangles (A, E) are needed to cover parallelogram F?
- (d) How many small triangles (A, E) are needed to cover triangle C?
- (e) Use $>$, $<$, or $=$ to show the relationship between the areas of these pieces:
 - (i) A, E
 - (ii) D, G
 - (iii) A, B
 - (iv) C, G
 - (v) F, E
- (f) Form a square using pieces A, B, C, F, and E. Sketch the square and the way the pieces fit together.
- (g) How many small triangles the size of A or E are needed to cover the square you made in part (f)?

OBJECTIVE

To calculate the measure of the third angle of a triangle given the measures of the other two

PACING

Level A 1-15
Level B 1-20
Level C 1-5, 11-20

MATERIALS

protractors (one for each student)

RELATED AIDS

HMS — DM10.

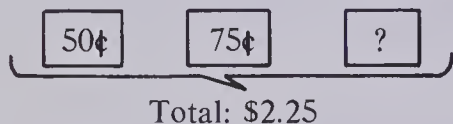
BACKGROUND

We state the rule formally here. Then we use the rule to calculate the third angle of a triangle given the measures of two angles.

There is a basic problem-solving technique involved in these pages. Also there is a lot of addition and subtraction practice.

SUGGESTIONS

Initial Activity Review the basic problem-solving technique before starting the lesson. Suggested approach: Illustrate using the chalkboard. There are 3 boxes containing money. How much is in the third box?



Process:

Add the two amounts.

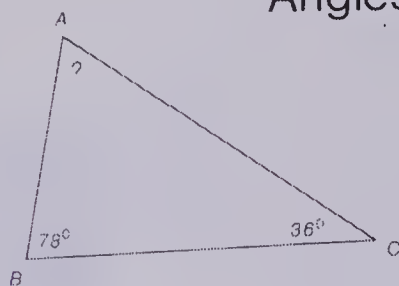
Subtract sum from total.

Difference is the amount in the third box.

Review the two ways previously presented for classifying triangles:

- (a) by angles — acute angle
 - right angle
 - obtuse angle;
- (b) by length of sides — scalene
 - isosceles
 - equilateral.

Angles of a Triangle



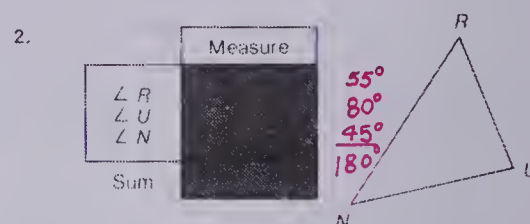
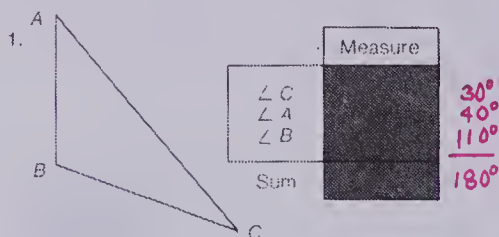
The sum of the measures of the angles of a triangle is 180° .

$$\begin{array}{r} \angle B = 78^\circ \\ \angle C = 36^\circ \\ \hline 114 \end{array} \qquad \begin{array}{r} 180^\circ \\ - 114 \\ \hline \angle A = 66^\circ \end{array}$$

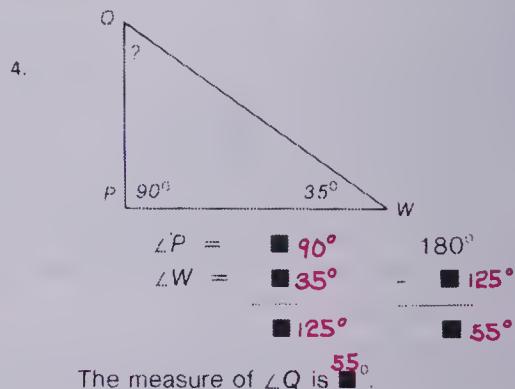
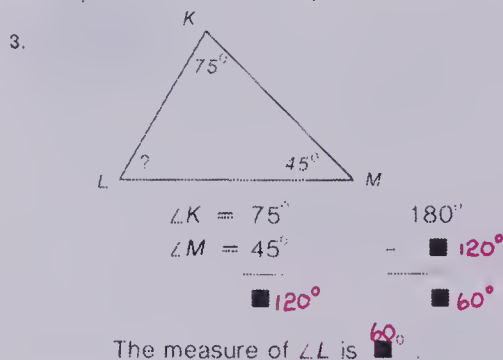
The measure of $\angle A$ is 66° .

Exercises

Use a protractor. Complete the charts.

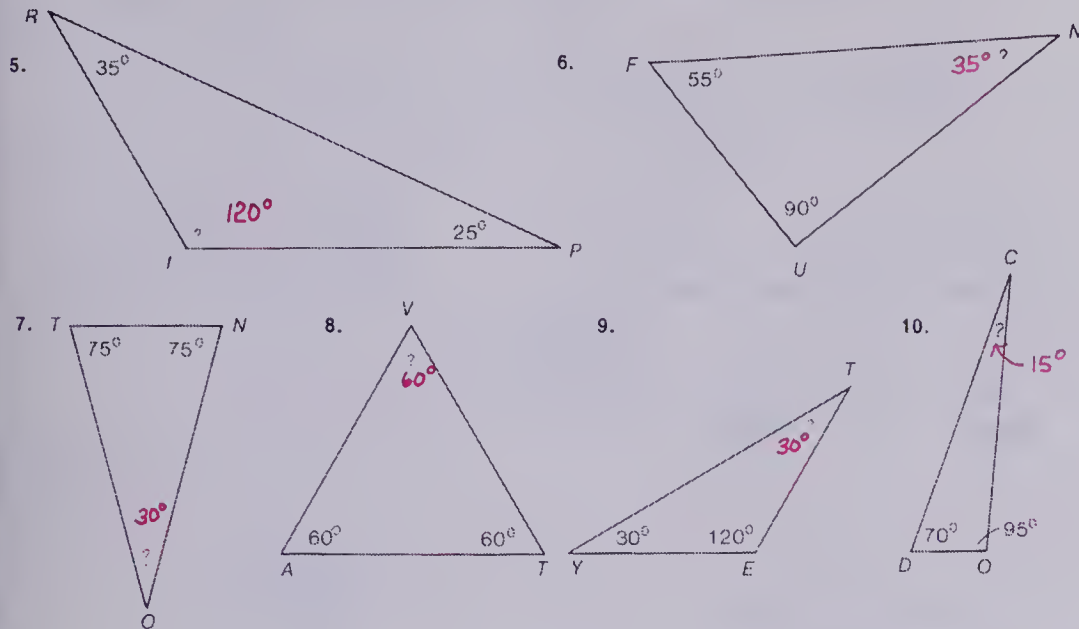


Calculate the measure of the third angle. Copy and complete the calculations. Check your answers with a protractor.



42 Using sums of angles of a triangle rule

Calculate the measure of the third angle.
Check your answer with a protractor.



Find the measure of the missing angle. Copy and complete the chart.

	$\angle A$	$\angle T$	$\angle E$	Angle Sum
11.	27°	112°	41°	180°
12.	53°	59°	68°	180°
13.	72°	45°	63°	180°
14.	25°	110°	45°	180°
15.	88°	64°	28°	180°
16.	128°	35°	17°	180°
17.	95°	49°	36°	180°
18.	right angle	45°	45°	180°
19.	30°	60°	right angle	180°

BRAINTICKLER

Twin primes are prime numbers that differ by 2. There are 16 twin primes between 0 and 100. How many can you find? Example: 11 and 13. How many triplet primes can you find? Example: 3, 5, and 7.

Using sums of angles of a triangle rule 43

ANSWERS:

Braintickler

3 and 5
5 and 7
11 and 13
17 and 19
29 and 31
41 and 43
59 and 61
71 and 73
No others.

USING THE BOOK

Explain the display shown at the top of the pupil page. Emphasize the 180° rule.

Exercises 1 to 3 emphasize the 180° rule. Omit these if you feel your students are confident with this rule.

Assign Exercise 4. Observe each student. Ask a student to tell what steps were followed to get the answer.

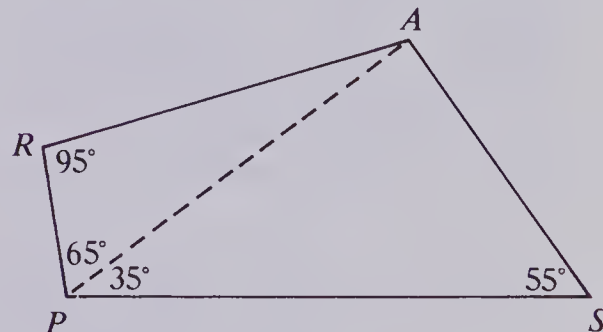
Assign the balance as indicated above.

ACTIVITIES

1. Refer the student to the library to read *The Franklin Mathematics Series*. Denholm, R.A. *Mathematics: Man's Key to Progress Book A*, Pages 1-16.

2. Make up a series of challenge cards:

- Can you draw a triangle whose angles measure 90° , 90° , and 5° ? Try. Can you explain your answer?
- One angle of a triangle is half of a right angle. Its measure is equal to the second angle. The third angle is a right angle. What are the measures of the three angles?
- One angle of a triangle is one third of a right angle. The second angle is twice as large as the first. Find the measures of the three angles.
- What is the measure of $\angle RAS$?



- One angle of a triangle is 80° and the other two are equal in size. What is the measure of each angle?
- One angle of a triangle is 90° and the other two are equal in size. What is the measure of each?
- One angle of a triangle is 60° . The second angle is three times the third. What are the measures of the second and third angles?

3. Have the children make up their own chart as shown for Exercises 11 to 19. They can choose their own angle letters (perhaps their initials) and exchange their charts for completion by classmates.

OBJECTIVES

- To identify intersecting lines as two lines that cross
- To identify perpendicular lines as two lines that intersect at right angles
- To identify parallel lines that never intersect

PACING

- Level A 1-8
- Level B 1-8
- Level C All

VOCABULARY

perpendicular, parallel, intersecting

SUGGESTIONS

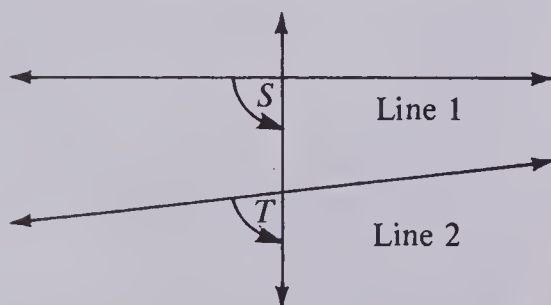
Initial Activity 1. Review briefly intersecting lines and perpendicular lines. Ask students to point out lines in the class (or elsewhere) that are perpendicular and others that intersect.

2. Repeat for parallel lines.

Remind the students that lines (vs. line segments) go on and on and on and ...

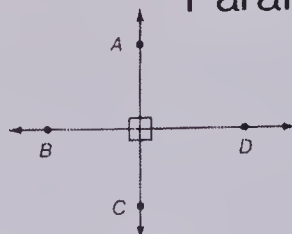
USING THE BOOK

Complete Exercises 1 to 7 orally with the group. Though the instructions for Exercise 8 require paper and pencil, you may wish to complete it orally as well. Discuss Exercises 9 and 10 thoroughly to get the student to generalize that "the angles are equal" when the two lines are parallel. It may be necessary to use two lines that are not parallel to illustrate that this property is true for parallel lines only.

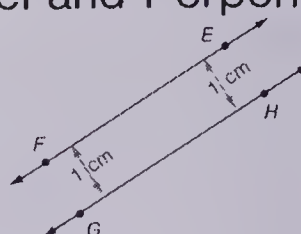


Do not introduce the word "corresponding", unless the students are ready for it.

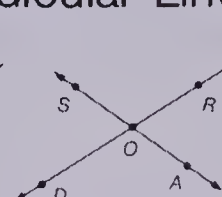
Parallel and Perpendicular Lines



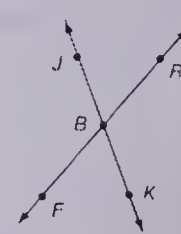
AC is perpendicular to BD. The angles at the intersection are *right angles*.



FE is parallel to GH. They will never intersect.

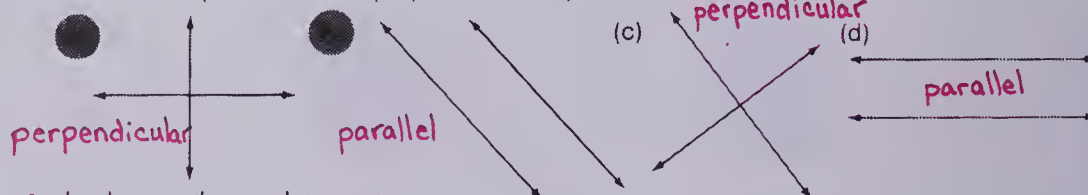


Intersecting lines cross. The angles at the intersection are *not necessarily* right angles.



Exercises

1. Name each pair of lines as perpendicular or parallel.



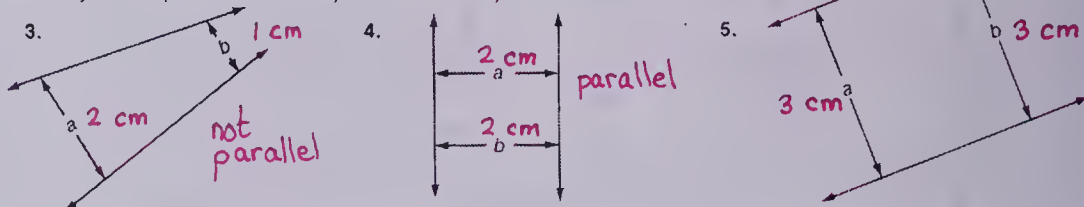
2. Look around your classroom.

Find three examples of each.

- (a) parallel lines
- (b) perpendicular lines
- (c) intersecting lines

How far apart are the lines at *a* and at *b*?

Identify each pair of lines as *parallel* or *not parallel*.



6. Will the lines in Exercise 3 ever intersect? in Exercise 4? in Exercise 5? **Yes; No; No**
7. Will parallel lines ever intersect? **No**
8. Copy and complete.
 (a) Two lines that intersect at right angles are **perpendicular** lines.
 (b) Two lines that never intersect no matter how far they are drawn are **parallel** lines.

- ★ 9. Tom and Fred, two forestry workers, headed the same direction.

They travelled on parallel trails.

They crossed a third trail, CD.

Measure $\angle COT$ and $\angle OAF$.

What do you notice about the measure of these angles?

They are both the same.

	Measure
$\angle COT$	72°
$\angle OAF$	72°

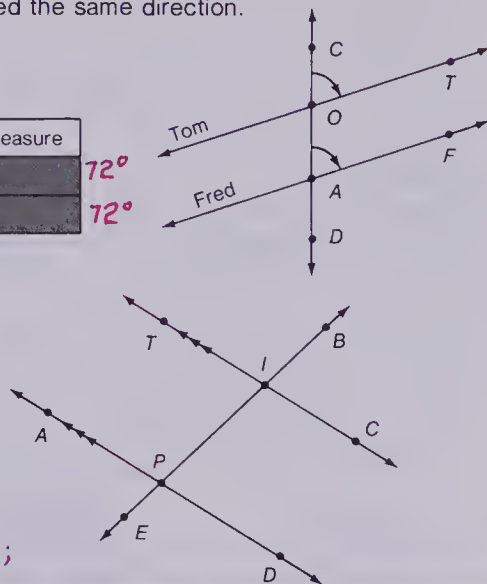
Use your protractor.

10. TC is parallel to AD.

(a)

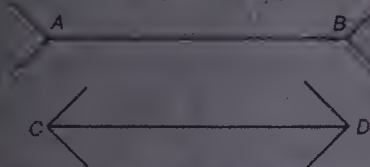
	Measure
$\angle TIP$	75°
$\angle APE$	75°

- ★ (b) Find other pairs of equal angles.
 **$\angle CIP$ and $\angle DPE$; $\angle TIB$ and $\angle API$;
 $\angle IPD$ and $\angle BIC$**



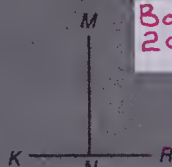
BRAINTICKLER

Which is the longer line segment?



Both are 5cm long.

Which segment is longer, MN or KR?

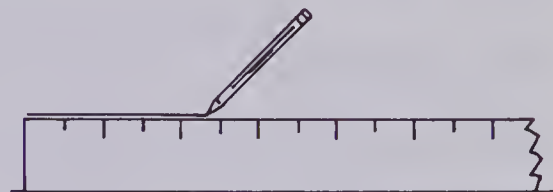


Both are 2cm long.

Angle properties of parallel and perpendicular lines 45

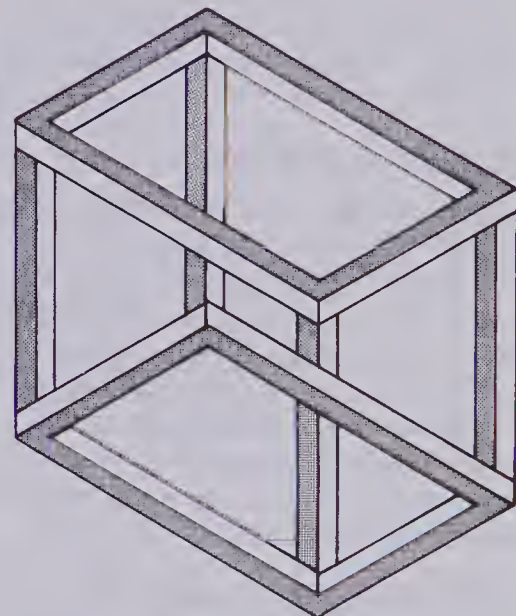
ACTIVITIES

1. (a) Ask students to draw five lines in random positions on the page. Then ask them to draw a perpendicular line to each. (They may use the protractor.)
 (b) Repeat part (a) but ask them to draw a parallel line to each. (They may use a ruler.)

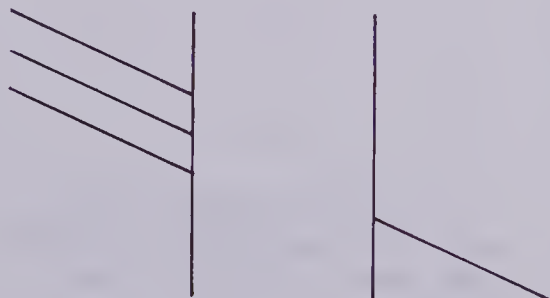


2. Prepare a challenge card such as:

Could this ever be a real model?
Why?



3. Prepare other optical illusion challenges such as:
Which heavy line is longer?



OBJECTIVES

To recognize the names of the polygons up to octagon

To observe the pattern in the number of diagonals in the polygons

PACING

Level A Optional

Level B All

Level C All

VOCABULARY

nonadjacent, polygons, diagonals, pentagon, hexagon, heptagon, octagon

MATERIALS

tracing paper

BACKGROUND

A polygon is a closed plane figure with 3 or more sides. The heptagon is sometimes called a septagon. A regular polygon is one in which all sides are congruent and all angles are congruent.

SUGGESTIONS

Initial Activity Review with the class the definition for "polygon" as mentioned in Background above. You may wish to include in the discussion the meaning of the various Greek prefixes — quad-, penta-, hexa-, etc.

Draw on the chalkboard (or overhead projector) a pentagon. Explaining what a diagonal is, draw in each of the diagonals.

USING THE BOOK

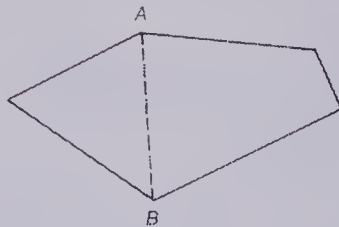
Assign the exercises. If the students do not need the practice of tracing, provide the students with a duplicated page of the six polygons and chart so that they can proceed directly to completing the exercises.

Some students will need assistance analyzing the data in the chart to extrapolate for the nine-, ten-, and twelve-sided polygons.

ACTIVITIES

1. Have the students unscramble these words for the polygons.
gantrile [triangle]
tephagno [heptagon]
draquilalreta [quadrilateral]
gahoxen [hexagon]
taocgon [octagon]
tanpenog [pentagon]

Polygons and Diagonals



A **diagonal** joins two nonadjacent vertices of a polygon.

AB is a diagonal.

How many diagonals can be drawn in a pentagon? **5**

Exercises

Trace each polygon.

In each, draw in all the diagonals.

Record your data in a chart.

Name of Polygon	Number of Sides	Number of Diagonals
Triangle	3	0
Quadrilateral	4	2

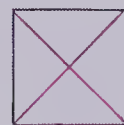
1.



Triangle

3 sides
0 diagonals

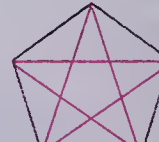
2.



Quadrilateral (square)

4 sides
2 diagonals

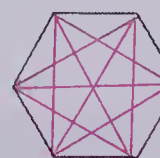
3.



Pentagon

5 sides
5 diagonals

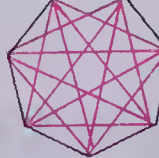
4.



Hexagon

6 sides
9 diagonals

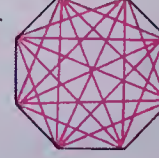
5.



Heptagon

7 sides
14 diagonals

6.



Octagon

8 sides
20 diagonals

7. Predict how many diagonals are in:

(a) a nine-sided polygon, **27**

(b) a ten-sided polygon, **35**

(c) a twelve-sided polygon, **54**

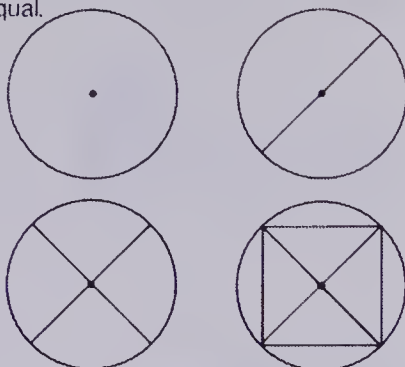
(d) Draw each figure and its diagonals to check your predictions.

Polygons in Circles

A **regular polygon** has all sides equal and all angles equal.

1. Let's draw a regular polygon in a circle.

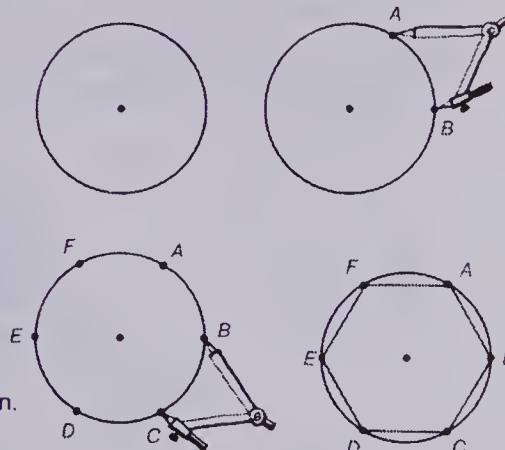
- Draw a circle
- Draw a diameter of the circle.
- Draw another diameter *perpendicular* to the first diameter.
- Draw coloured segments to join the ends of the diameters.
- Name the four-sided figure formed by the coloured segments. *a square*



2. Make an octagon (eight-sided figure), using the above method.

3. Let's draw another polygon in a circle.

- Draw a circle. **Keep the same radius for the rest of this exercise.**
- Mark a point *A* on the circle.
- Place the compasses on *A* and mark another point *B* on the circle.
- Repeat this to mark points *C*, *D*, *E*, and *F* on the circle.
- Draw coloured segments to join the points in order.
- This coloured shape is a regular hexagon. Use your dictionary. What does *hex-* mean? *six*



4. Repeat Steps (a) through (d) in Exercise 3.

Then draw coloured segments to join the points *A*, *C*, and *E*. Name this regular polygon. *an equilateral triangle*

OBJECTIVE

To draw inscribed polygons: square, regular octagon, equilateral triangle, and regular hexagon

PACING

Level A 1-3

Level B 1-3

Level C All

MATERIALS

compasses (one for each student), rulers

USING THE BOOK

Students with average to high reading ability can work on their own.

With students who have reading problems, it is advisable to work through each activity, reading or asking individual students to read the instructions aloud. Each student should make his or her own diagram.

In Exercise 1, discuss how they can make one diameter perpendicular to the other (use a square corner, use the protractor, etc.). Review what perpendicular means.

In Exercises 1 to 3, discuss the features (properties) of each polygon constructed.

ACTIVITIES

1. Direct students to use these constructions to draw more patterns using circles.

2. Have students work in pairs to draw and cut out large models of the polygons, each from a different colour of construction paper. Each polygon is cut into 2, 3, or 4 pieces. Each set is put into an envelope. Students exchange sets and are to reconstruct the polygons.

3. Some students may wish to make kites in the shapes of the 6 polygons.

4. See Activity 3 on pages 32 and 33. Have the children use shapes other than the circle.

OBJECTIVE

To draw nets for and to construct rectangular prism, cylinder, and the three regular polyhedra: tetrahedron, cube, and octahedron

PACING

Level A All
Level B All
Level C All

VOCABULARY

polyhedron, polyhedra, nets

MATERIALS

construction paper or tag, rules, compasses, scissors, glue, tape

BACKGROUND

The names of the three polyhedra are the cube, the tetrahedron, and the octahedron.

USING THE BOOK

You may wish to draw and duplicate the 5 nets for some of your students. The Level C students should be challenged to draw their own. The students will have to plan the lengths of the sides. Specifically, the length of the rectangle in the net for the cylinder is equal to the circumference of the circles — with a little extra for gluing.

The triangles in the tetrahedron and octahedron are equilateral triangles. Once a student has drawn one, the student can trace it for the others.

ACTIVITIES

1. Have the students bring models of the shapes and others that are of interest. Different commercial products are often marketed in interesting shapes. Discuss each and label.

2.



Make these patterns on heavy tag. Each broken line should be 6 cm, centre to centre. Cut out the patterns. Use a small paper punch to punch the holes at the vertices. Score, then crease along the dotted lines. Use elastic bands, glue, or staples to fasten two edges together to form polyhedra. The five regular solids (called Platonic

Drawing Nets

A. Make your own nets to build these shapes.

What edges must have the same measures? *Parallel edges must have the same measures.*

How long must AB be? *AB must be the same length as the circumference of the circles.*

B. Make your own nets to build these three regular polyhedra.

A regular shape is one in which all edges are the same length and all angles have the same measure.

Tetrahedron

Cube
Hexahedron

Octahedron

48 Drawing nets

solids after Plato) can be assembled.

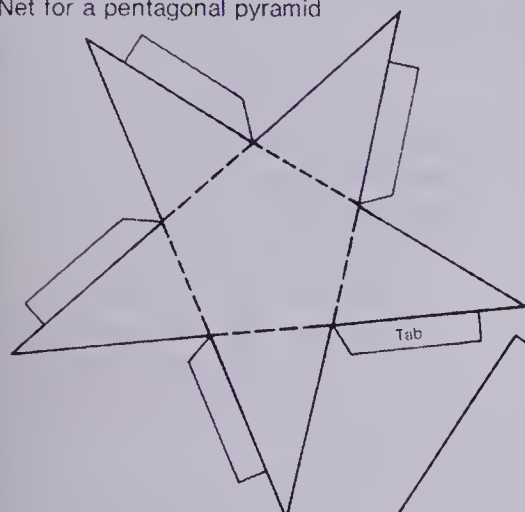
1. tetrahedron (4 triangles)
2. hexahedron (6 squares)
3. octahedron (8 triangles)
4. dodecahedron (12 pentagons)
5. icosahedron (20 triangles)

You may wish to spray paint the models and suspend them with their names from the ceiling.

3. Challenge the better students to find the surface area of each solid made from the nets on page 48. (*Hint: the students may work with the nets.*)

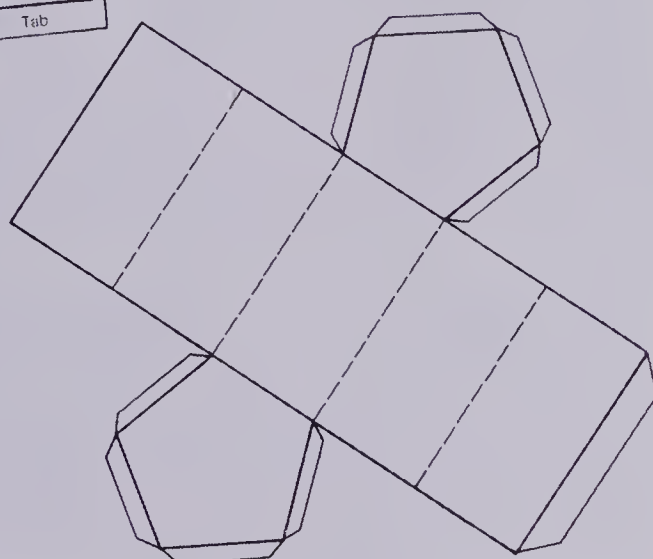
Pentagonal Pyramid and Pentagonal Prism

Net for a pentagonal pyramid



1. Trace each shape onto heavy cardboard. Cut out and assemble to form the shapes.

2. How does a pyramid differ from a prism?



Net for a pentagonal prism

OBJECTIVE

To construct a pentagonal pyramid and a pentagonal prism

PACING

Level A All
Level B All
Level C All

VOCABULARY

pentagonal, prism, pyramid

MATERIALS

tracing paper, heavy cardboard or tag, scissors, glue, tape

RELATED AIDS

HMS — DM11.

BACKGROUND

This exercise, besides introducing two new polyhedra, emphasizes the difference between a prism and a pyramid and reinforces the point that a prism and a pyramid may have different polygons as a base.

USING THE BOOK

You may wish to provide some students with the patterns already duplicated on heavy tag.

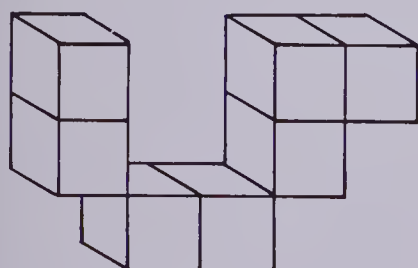
Discuss the differences between pyramids and prisms. Include in your set of prisms and pyramids those made in the exercise on page 48.

Suspend the models from the ceiling.

Pentagonal prism and pyramid 49

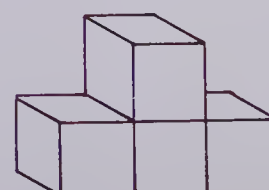
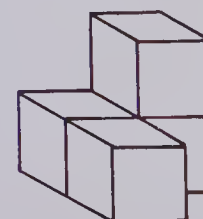
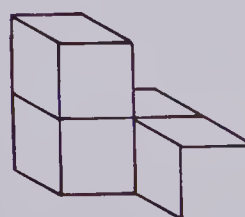
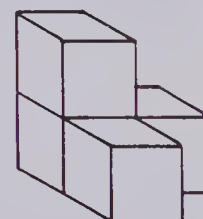
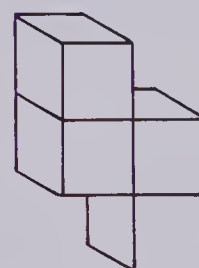
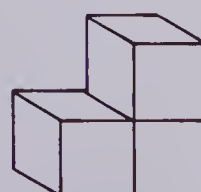
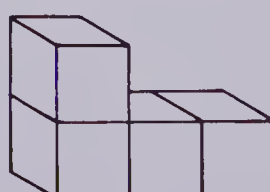
ACTIVITIES

1. Use an overhead projector and squared paper to illustrate an easy way to draw cubes and polycubes. Then have the students design modular buildings based on the cube as the module.



Pieces	Polygons				
	rectangle	square	pentagon	isosceles triangle	...
A AEF	X			X	

3. Glue cubes together in the formations shown to play "Polycubes". In order for 2 persons to play, 2 sets will be needed. Have the students race to see who can assemble the seven pieces to form a cube. There are over 1 105 920 different ways to solve the puzzle.



2. Provide sets of the tangram pieces described in the Activity Reservoir. Assign two exercises:
 - a) How many different polygons can the students make using only the tangram pieces? They should trace each polygon they make and label with the letters of the pieces used.
 - b) Record in a chart like the one shown, the polygons and the pieces needed to make them.

OBJECTIVE

To make a model of a dodecahedron

PACING

Level A Optional
Level B Optional
Level C All

VOCABULARY

dodecahedron

MATERIALS

tracing paper, cardboard, elastic bands, scissors

RELATED AIDS

HMS — DM12.

BACKGROUND

A dodecahedron has 12 faces.

USING THE BOOK

Students may be challenged to do this page when they have their other work done.

ACTIVITIES

1. Ask the students to read about flexagons and to construct several.

2. Students may wish to use drinking straws, pipe cleaners, or thread to make skeleton models of various polyhedra.

3. Construct one or more of the challenging models in the book *Polyhedra Models for the Classroom* by Wenminger (published by National Council of Teachers of Mathematics, 1975).

Pop-up Dodecahedron

Trace this pattern onto cardboard. Make two patterns

Fold lightly along the dotted lines.

Place the two patterns together as shown.

Attach a rubber band over and under the points, alternately.

Toss the model in the air.

It will form a dodecahedron.

(If you are not successful the first time, use a smaller rubber band and/or crease the folds.)

How many faces does a dodecahedron have? 12

Use a dictionary:

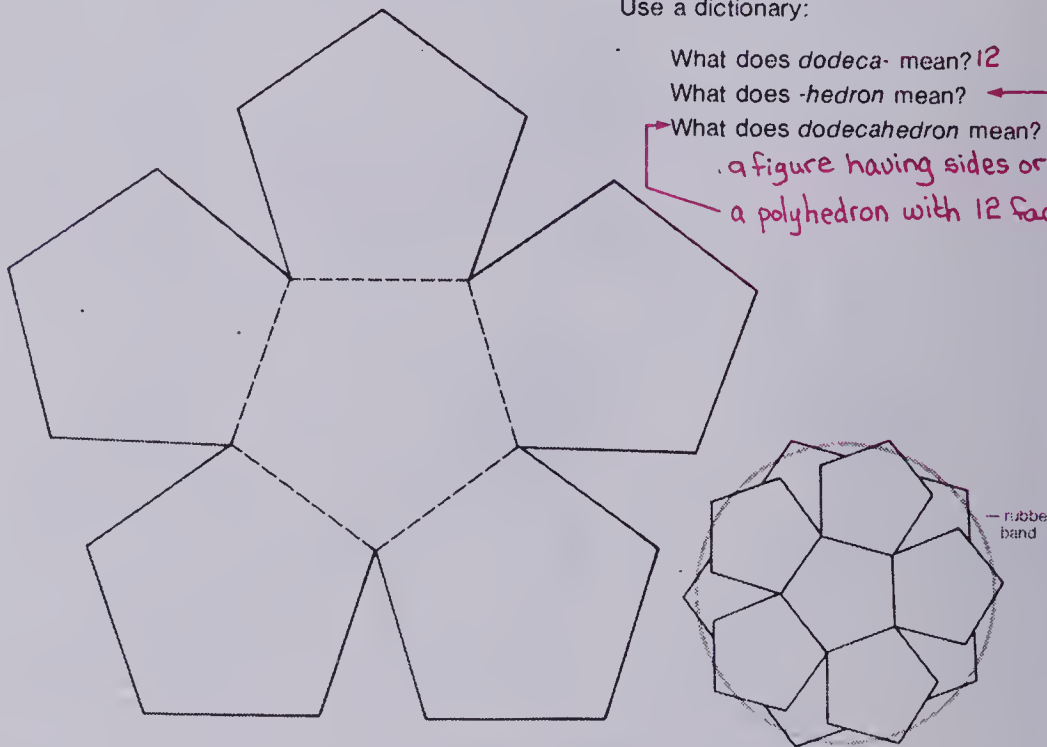
What does *dodeca-* mean? 12

What does *-hedron* mean?

What does *dodecahedron* mean?

a figure having sides or faces

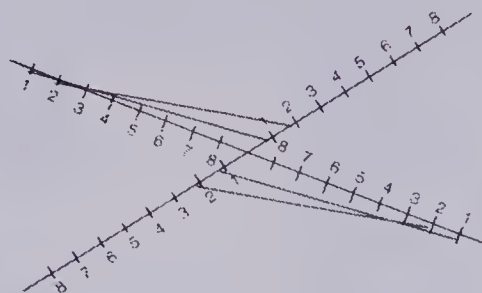
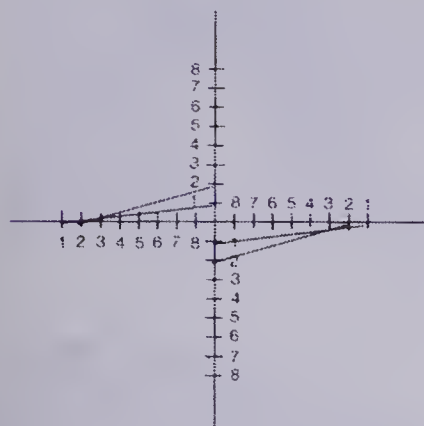
a polyhedron with 12 faces



50 Activity: make a model of a dodecahedron

Making Patterns with Straight Lines

- A. Trace each pattern. Draw lines from 1 → 1
2 → 2, etc.



- B. Make other patterns of your own.

- C. Use a shoe box.

Cut out one side as shown.

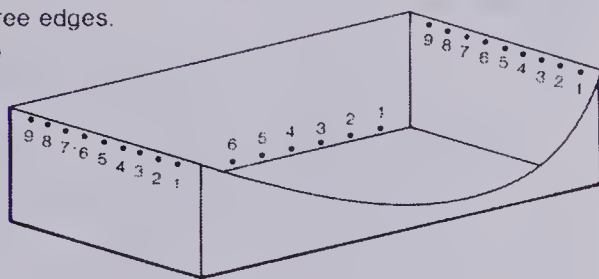
Mark off and label as shown on three edges.

Use a needle and coloured yarn to

stitch from 1 → 1

2 → 2, etc.

Use two colours of yarn.



- D. Use another box. Make your own pattern.

Curve stitching 51

OBJECTIVE

To make curved line patterns using straight lines

PACING

Level A A, C

Level B All

Level C All

MATERIALS

rulers, rectangular (shoe) boxes, large needles, yarn

RELATED AIDS

HMS — DM13 and DM14.

USING THE BOOK

You may wish students to do this page as "Recreational Mathematics".

ACTIVITIES

1. Curved line patterns using straight lines (often called curve stitching) are a common type of craft in which nails are driven into a wood base and thread woven around them. Refer to your library for additional information. Students often enjoy making their own patterns or copying others.

2. Some students may wish to tackle the Polycube Game again as described in Activity 3, page 49. This time, you may wish to add this variation: Student A makes a shape and challenges student B to duplicate it within a given time period (3 min); students reverse roles; winner is the player who succeeds most often after 5 challenges.

OBJECTIVE

To identify the operation necessary to solve a word problem

PACING

Level A All
Level B All
Level C All

RELATED AIDS

BFA PROB. SOLVING LAB II —
27-39, 42-44, 141.

CALC. ACTIVITY MASTERS — 64.

BACKGROUND

Written computation can too often become a process of mechanically following some vaguely understood rule or trick. To avoid this hazard, activities which will encourage meaningful use of written procedures for getting answers should be provided.

(i) Mental computation should precede and accompany written computation. When children compute mentally they are forced to operate with understanding of numbers.

(ii) The ability to make up story problems to match number sentences as well as to write number sentences for given problems is an indication of understanding.

SUGGESTIONS

Initial Activity Discuss forestry workers. See the Career Awareness notes in the Chapter Overview, page 28. The page may be left and integrated with a social studies lesson. Review Professor Q's steps in problem solving (see page 22).

USING THE BOOK

Remind the students that they are to identify the operation(s) needed to get the answer. They should also tell you what numbers need to be added, subtracted, multiplied, or divided.

Discuss each problem using Professor Q's questions. Ask students to "tell" the steps involved.

ACTIVITIES

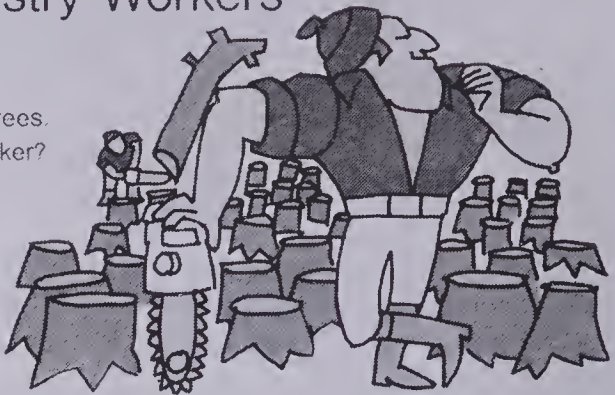
1. Students may be directed to solve the problems.

2. Have the students make up one problem each, with answers. The best of these may be put on cards and placed in a "Problem Box". Students

Forestry Workers

A team of 28 workers cut 1008 trees.
Each worker cut the same number of trees.
How many trees were cut by each worker?


Think: A team of 28.
1008 trees cut.
How many trees each?
Operation: Division.



Exercises

Decide what operation to use: addition, subtraction, multiplication, division.
Do not solve.

- Each truck hauls 11 logs.
There are 126 truckloads.
How many logs? **Multiplication**
- The numbers of logs that arrived in 5 d were 35, 412, 218, 298, and 341.
How many logs in all? **Addition**
- In a 12-month period, 1128 flatcars of lumber were shipped.
What was the average number of flatcars of lumber shipped per month? **Division**
- In the first year, a planer mill shipped 1726 boxcars and 1184 truckloads of shavings.
In the second year, 2394 boxcars and 798 truckloads were shipped.
How many more boxcars were shipped in the second year than in the first year? **subtraction**
- Each tree-feller receives \$1247 per month.
There are 24 tree-fellers and 18 machine operators.
What is the total of the monthly salaries of the tree-fellers? **Multiplication**



52 Word problems: mixed operations

may use these when they have time.

3. See the "choose-the-appropriate-operation" idea listed in Activity 1 for pages 22 and 23.

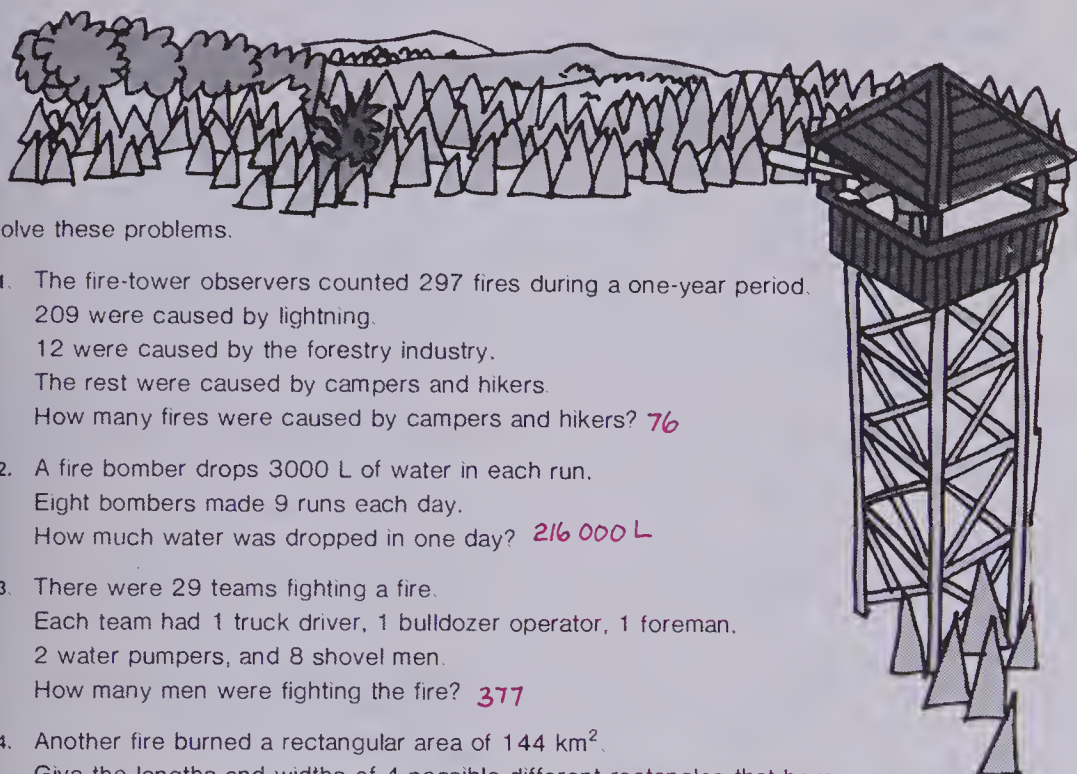
EXTRA PRACTICE

Decide which operations to use.

- Each chain saw costs \$237.95.
How much would 4 chain saws cost?
- One of the logging trucks had to be driven to Ottawa.
The round trip was 612 km.
Another truck was driven to Montreal and back.
The round trip was 397 km.
How much farther did the first truck travel?
- In one day, 154 trees were cut.
Each truck carries 11 trees.
How many truckloads were cut?

- The cooks at camp provide lots of food.
There is a total of 88 people working in the camp.
Each person eats 3 meals per day.
How many meals do the cooks prepare each day?
- A log boom has 720 logs.
It will keep a planing mill busy for 30 h.
How many logs per hour does the mill use?
- The forestry camp has 8 logging crews.
Each crew has a foreman, 4 chain saw operators, and 3 cat operators.
How many men are in the logging crews?

Forestry Firefighters



Solve these problems.

- The fire-tower observers counted 297 fires during a one-year period.
209 were caused by lightning.
12 were caused by the forestry industry.
The rest were caused by campers and hikers.
How many fires were caused by campers and hikers? **76**
- A fire bomber drops 3000 L of water in each run.
Eight bombers made 9 runs each day.
How much water was dropped in one day? **216 000 L**
- There were 29 teams fighting a fire.
Each team had 1 truck driver, 1 bulldozer operator, 1 foreman,
2 water pumpers, and 8 shovel men.
How many men were fighting the fire? **377**
- Another fire burned a rectangular area of 144 km^2 .
Give the lengths and widths of 4 possible different rectangles that have
an area of 144 km^2 .
- One fire burned the area as shown.
How many square kilometres of forest
were burned? **364 km^2**
- In every hour that a forest fire raged, enough timber was burned to make 100 houses.
The fire raged 18 d.
How many houses could have been built with the timber that burned? **43 200**



Problem solving 53

ANSWERS:

12 km x 12 km, 8 km x 18 km, 6 km x 24 km, 4 km x 36 km. Other possible answers.

OBJECTIVE

To solve word problems involving more than one step

PACING

Level A 1-4
Level B All
Level C All

RELATED AIDS

HMS — DM15.

BFA PROB. SOLVING LAB II — 41.

SUGGESTIONS

Initial Activity Discuss the work of forestry firefighters. See the Career Awareness notes in the Chapter Overview, page 28.

USING THE BOOK

Point out that these problems require more than one step or more than one operation. In using Professor Q's questions, add the questions "Do we need to find something before we can calculate the final answer?" and "What do we need to find in order to solve the question we are asked?" You may wish to be more specific and say "Before we can find — — — — —, what do we need to know?"

Students often have difficulty identifying the operations they are to perform. You may, if necessary, direct specific questions to help students identify the operations.

ACTIVITIES

- See the "Choose-the-correct-operation" activity described in Activity 1 for pages 22 and 23.
- Students may write a fiction story or draw a picture to illustrate fighting a forest fire.
- Students may research the numbers and causes of forest fires in Canada. They may also wish to include for some fires the size, duration, and damage caused.

OBJECTIVE

To evaluate achievement of the chapter objectives

PACING

- Level A All
- Level B All
- Level C All

RELATED AIDS

HMS — DM16.

USING THE BOOK

Each student should do this test independently under supervision. Assistance should be given only when the instructions are not understood. After the work has been corrected, you should provide appropriate remedial work. You may wish to reteach if a large number of students had difficulty with a particular topic or concept.

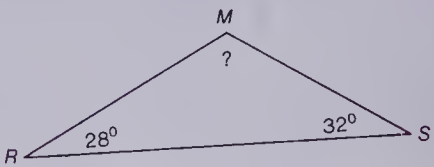
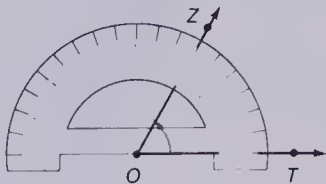
The following chart will help in this regard. The specific objectives are listed in the Chapter Overview (see page 28).

An alternate Chapter Test can be found in the Holt Mathematics System Duplicating Masters available for use with this grade level.

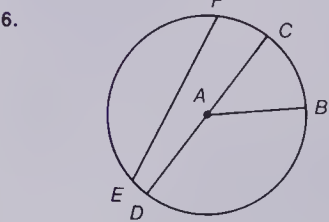
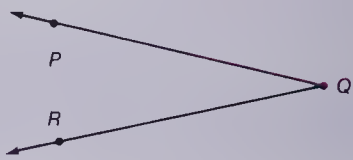
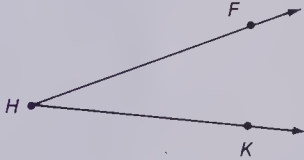
Test Item	Objective	Text Page Number
1, 6	A	29
2, 3, 5	B	32-37
7, 8	C	47
4	D	40-43
9	E	46
10	F	44, 45

Chapter Test

- The radius of a circle is 25 mm.
What is the length of the diameter? *50 mm*
- What is the measure in degrees of a right angle? *90°*
- What is the measure of $\angle ZOT$? *60°*
- What is the measure of $\angle M$? *120°*



5. Use your protractor. Are $\angle FHK$ and $\angle PQR$ congruent? Explain. *Yes Both angles measure 25°.*



6. Which line segment names each?
(a) radius *AB* (b) chord *EF* (c) diameter *CD*

7. Use your compasses to draw a triangle with all sides equal in a circle.
8. Match:
- | | | |
|-------------------|----|--|
| (a) quadrilateral | 1. | |
| (b) pentagon | 2. | |
| (c) octagon | 3. | |
| (d) hexagon | 4. | |

9. State the number of faces on a:
(a) tetrahedron *4* (b) octahedron *8*
10. Draw a pair of parallel lines. *(Two horizontal lines drawn)*

Cumulative Review

1. (a)	$\begin{array}{r} 34.567 \\ + 8.261 \\ \hline 42.828 \end{array}$	(b)	$\begin{array}{r} 456.6 \\ 861.8 \\ + 813.222 \\ \hline 2131.622 \end{array}$	2. (a)	$\begin{array}{r} 64.32 \\ - 8.919 \\ \hline 55.401 \end{array}$	(b)	$\begin{array}{r} \$ 347.16 \\ - 11.63 \\ \hline \$ 335.53 \end{array}$
--------	---	-----	---	--------	--	-----	---

3. Write in figures.

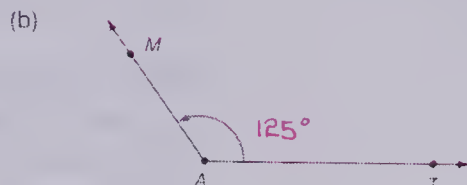
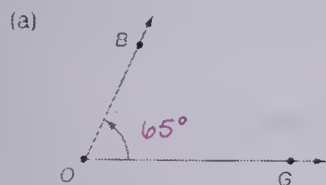
- (a) sixty-two billion, seven hundred million, sixty-five **62 700 000 065**
 (b) nine million, six hundred thousand, three hundred five **9 600 305**

4. In the number 6 315 402 791, what is the place value of:

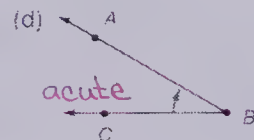
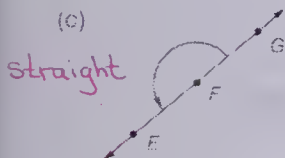
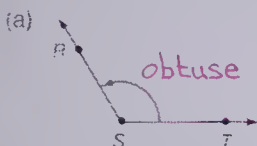
- (a) 6? **billions** (b) 5? **millions** (c) 2? **thousands** (d) 3? **hundred millions** (e) 4? **hundred thousands**

5. Use your protractor.

What is the measure of each angle?

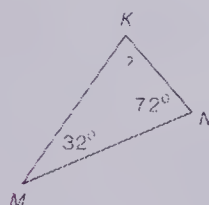


6. Name each angle as acute, right, obtuse, or straight.



7. Find the measure of $\angle K$.

76°



OBJECTIVE

To review and test selected concepts and skills previously covered

PACING

Level A All
 Level B All
 Level C All

USING THE BOOK

This page may be used for diagnostic and remedial as well as review purposes. Students should check their work, correct any errors, and review the pages that contain any problems of the type they missed. Some students can do this on their own while others may need help. If a large number of students have a particular problem incorrect, you may want to reteach that topic to the groups, then assign a duplicated worksheet to reinforce that topic or, refer to an appropriate skill card in the BFA Computational Skills Kit II or BFA Problem Solving Kit II.

Test Item	Text Page Number
1	7
2	9
3, 4	10, 11
5	34, 35
6	36, 37
7	42, 43

CHAPTER 3 OVERVIEW

This chapter reviews and develops the concepts and properties of multiplication of whole numbers by whole numbers, whole numbers by decimal numbers, decimal numbers by whole numbers, and decimal numbers by decimal numbers. The concepts of rounding, estimating, and working with equations are presented. Word problems are also included.

OBJECTIVES

- A To review and develop the multiplication of: whole numbers by whole numbers; decimals by decimals; whole numbers by decimals; decimals by whole numbers
- B To introduce the commutative, associative, and distributive properties of multiplication
- C To review and use rounding and estimation in multiplication
- D To introduce the concept of equations in multiplication
- E To solve word problems; to write equations to solve word problems

BACKGROUND

This chapter, like any concerned with the mastery of some arithmetic operation, is heavily laden with computation and drill exercises. These pupil pages alone contain over 750 different exercises, not counting the extra practice in the back of the book, the Teacher's Edition extra practice, and the duplicating masters. In chapters of this sort, it is important to assign the exercises judiciously; use concrete examples and demonstrations when the situation will allow it; and above all, guard against computation becoming an end unto itself.

The lessons concerned with the actual multiplication algorithm are sequenced so that the latter pages deal with greater number values and numbers with several decimal places. In this regard, emphasize (a) the similarity of process, no matter the number of decimal places; (b) the decimal place rule; (c) decimal place value to give the factors and products meaning; (d) rounding and estimation of factors and products to encourage "number reasonableness"; and (e) real-world applications and examples.

In this chapter, children are presented with two types of equations: $8 \times 7 = n$, $7 + 9 = R \times 4$. In Chapter 1 (see the Chapter 1 Overview; see also page

19) the equations involved addition and subtraction only. Also, it was suggested there that some equations could be solved using related facts. In this chapter, however, because of the two equation types, the size of some numbers, and the use of equations to help solve word problems, confine the discussion to "What number would make this equation true?" It should suffice here to dwell upon the use of letters to stand for unknowns and the need to "balance" an equation so that both sides are equal. The use of related multiplication and division facts to solve for unknowns will be dealt with more fully later (Chapter 9).

MATERIALS

counters (or straws or popsicle sticks)
multiplication flash cards
graph paper
2 overhead transparencies, 10 cm \times 10 cm showing



100 pennies (optional)
stories and articles pertaining to sports facts

CAREER AWARENESS

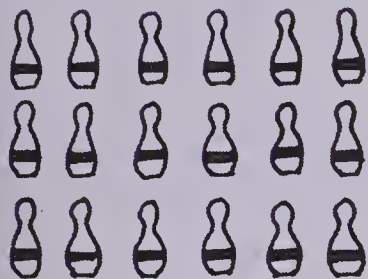
Waitress [88, 89]

A waitress (or waiter) serves food to customers in restaurants and dining rooms. She (he) must be congenial and helpful, fully cognizant of the menu, possess an excellent memory and be in good physical condition to endure long hours, most of which are spent on the feet. Needless to say, she (he) must have a tidy appearance and manner. She (he) is responsible for taking a customer's order, serving the food, and making out a bill for the items ordered. Obviously, ease and familiarity with arithmetic skills are essential, though modern cash registers do remove some of the mental burden. Salaries are usually a basic wage plus tips.

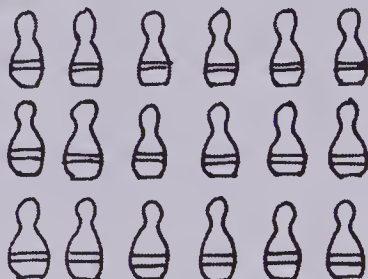
Multiplication



Bobby bowled 3 balls.
He knocked down 6 pins with each ball.
How many pins did he knock down altogether?



6 pins + 6 pins + 6 pins = 18 pins
or
 $6 + 6 + 6 = 18$



3 groups of 6 pins = 18 pins
or
 $3 \times 6 = 18$
Factor Factor Product

Exercises

Solve by adding. Solve by multiplying. Which is quicker?

- There are 6 rows of exercises.
Each row has 5 questions.
How many questions? **30**
- There are 19 cows.
They have 4 legs each.
How many legs? **76**
- There are 6 boys.
They have 25 marbles each.
How many marbles? **150**
- There are 12 eggs in 1 carton.
There are 9 cartons.
How many eggs? **108**
- There are 24 batteries in each carton.
There are 8 cartons.
How many batteries? **192**
- There are 42 chocolates in a box.
There are 7 boxes.
How many chocolates? **294**

Meaning of multiplication 57

OBJECTIVE

To review the meaning of multiplication

PACING

Level A All
Level B All
Level C All

VOCABULARY

factor, product

MATERIALS

counters or straws or popsicle sticks,
etc.

RELATED AIDS

BFA PROB. SOLVING LAB II — 45,
49, 53.

CALC. ACTIVITY MASTERS — 5,
37.

SUGGESTIONS

Initial Activity Present problems
such as:

- You have 4 rows of counters with 5
counters in each row. How many
counters altogether?
- Arrange 16 counters in equal
groups. How many different ways can
you do this?

Have the children show the
answers to these problems using
counters to make arrays on their
desks. Once the arrays are made, have
both addition and multiplication
statements written which describe the
arrays.

USING THE BOOK

Use the display at the top of the page
to review the concept that when there
are several rows of objects with the
same number of objects in each row
we can find the total number of
objects by either (a) adding or
(b) multiplying. Lead the students to
realize that multiplying is the quicker
method.

Have students draw an array for
Exercise 1. You may wish to do the
rest of the questions orally.

ACTIVITIES

1. Have the children draw arrays
showing the exact number of objects
given:

(a) 36 (b) 24 (c) 42.

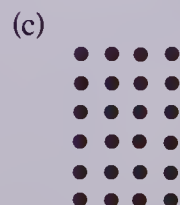
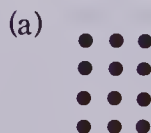
2. Have students complete a
diagram such as the following.

Number of rows	4		5		8	9
Number in each row	7	6		9	4	
Total	48	35	18			27

3. Students develop their own
arrays. Make a bulletin-board display
of these.

EXTRA PRACTICE

- How many objects would you need
to make the following arrays?
(a) 3×5 (b) 9×4
(c) 8×9 (d) 7×3
(e) 5×7 (f) 4×6
- Write a multiplication fact for each.



OBJECTIVE

To provide practice in multiplying by a single digit

PACING

Level A All

Level B All

Level C 2, 4, 6 (score 2 points each)

MATERIALS

flash cards with multiplication facts

RELATED AIDS

BFA COMP LAB II — 29-35.

BFA PROB. SOLVING LAB II — 61, 65, 69.

CALC. ACTIVITY MASTERS — 11.

SUGGESTIONS

Initial Activity Conduct a drill with the flash cards. Show each flash card and instruct the students to write the product. Alternatively, the students could say the product as each card is shown.

Do several examples on the board to review the regrouping process that is necessary in many of the questions in Exercises 3 to 6.

USING THE BOOK

You may wish to have students write the answers only for most of these questions. Your above-average students should be able to write the products only for the whole page. Your less able students may require the extra activity of writing down the examples which require regrouping before solving them.

ACTIVITIES

1. Students can draw a grid with the numbers 1 to 80.

1	2	3	4	5	6	7	8	9	10
11	12	etc.							

Students can discover patterns by colouring certain products, e.g., colour all multiples of 4 red, etc.


2. "Snap". Two sets of cards are needed: one set has multiplication facts; the other set has corresponding answers. Two players play. Cards are shuffled together and dealt face down so that each player has half the deck. Players take turns playing one card at a time. When two corresponding cards are turned up, i.e., fact and answer,

Tune Up

Calculate.

1. (a) $\begin{array}{r} 7 \\ \times 6 \\ \hline 42 \end{array}$	(b) $\begin{array}{r} 8 \\ \times 4 \\ \hline 32 \end{array}$	(c) $\begin{array}{r} 9 \\ \times 7 \\ \hline 63 \end{array}$	(d) $\begin{array}{r} 7 \\ \times 5 \\ \hline 35 \end{array}$	(e) $\begin{array}{r} 8 \\ \times 9 \\ \hline 72 \end{array}$
2. (a) $\begin{array}{r} 6 \\ \times 5 \\ \hline 30 \end{array}$	(b) $\begin{array}{r} 2 \\ \times 9 \\ \hline 18 \end{array}$	(c) $\begin{array}{r} 4 \\ \times 6 \\ \hline 24 \end{array}$	(d) $\begin{array}{r} 3 \\ \times 8 \\ \hline 24 \end{array}$	(e) $\begin{array}{r} 7 \\ \times 7 \\ \hline 49 \end{array}$
3. (a) $\begin{array}{r} 23 \\ \times 3 \\ \hline 69 \end{array}$	(b) $\begin{array}{r} 32 \\ \times 4 \\ \hline 128 \end{array}$	(c) $\begin{array}{r} 50 \\ \times 7 \\ \hline 350 \end{array}$	(d) $\begin{array}{r} 41 \\ \times 6 \\ \hline 246 \end{array}$	(e) $\begin{array}{r} 91 \\ \times 8 \\ \hline 728 \end{array}$
4. (a) $\begin{array}{r} 16 \\ \times 6 \\ \hline 96 \end{array}$	(b) $\begin{array}{r} 35 \\ \times 7 \\ \hline 245 \end{array}$	(c) $\begin{array}{r} 14 \\ \times 9 \\ \hline 126 \end{array}$	(d) $\begin{array}{r} 85 \\ \times 8 \\ \hline 680 \end{array}$	(e) $\begin{array}{r} 78 \\ \times 7 \\ \hline 546 \end{array}$
5. (a) $\begin{array}{r} 243 \\ \times 2 \\ \hline 486 \end{array}$	(b) $\begin{array}{r} 172 \\ \times 3 \\ \hline 516 \end{array}$	(c) $\begin{array}{r} 283 \\ \times 4 \\ \hline 1132 \end{array}$	(d) $\begin{array}{r} 859 \\ \times 7 \\ \hline 6013 \end{array}$	(e) $\begin{array}{r} 813 \\ \times 5 \\ \hline 4065 \end{array}$
6. (a) $\begin{array}{r} 4250 \\ \times 6 \\ \hline 25500 \end{array}$	(b) $\begin{array}{r} 5173 \\ \times 4 \\ \hline 20692 \end{array}$	(c) $\begin{array}{r} 1634 \\ \times 9 \\ \hline 14706 \end{array}$	(d) $\begin{array}{r} 2408 \\ \times 7 \\ \hline 16856 \end{array}$	(e) $\begin{array}{r} 7234 \\ \times 3 \\ \hline 21702 \end{array}$

Tune Up Score Card	
	Points
Super	30-27
Good	26-21
Average	20-15



58 Multiplying by a single digit with and without regrouping

the first player to call "Snap" picks up the pile. The player with the most cards at the end of the game wins.

3. Play "Bingo" as described in the Activity Reservoir. On the grid, have the players write numbers between 2 and 100.

EXTRA PRACTICE

1. (a) $\begin{array}{r} 5 \\ \times 9 \\ \hline 45 \end{array}$	(b) $\begin{array}{r} 7 \\ \times 6 \\ \hline 42 \end{array}$	(c) $\begin{array}{r} 12 \\ \times 5 \\ \hline 60 \end{array}$
--	---	--

(d) $\begin{array}{r} 25 \\ \times 5 \\ \hline 125 \end{array}$	(e) $\begin{array}{r} 36 \\ \times 6 \\ \hline 216 \end{array}$	(f) $\begin{array}{r} 83 \\ \times 9 \\ \hline 747 \end{array}$
---	---	---

2. (a) $\begin{array}{r} 347 \\ \times 5 \\ \hline 1735 \end{array}$	(b) $\begin{array}{r} 982 \\ \times 6 \\ \hline 5892 \end{array}$	(c) $\begin{array}{r} 583 \\ \times 9 \\ \hline 5247 \end{array}$
--	---	---

(d) $\begin{array}{r} 662 \\ \times 8 \\ \hline 5296 \end{array}$	(e) $\begin{array}{r} 705 \\ \times 3 \\ \hline 2115 \end{array}$	(f) $\begin{array}{r} 8537 \\ \times 6 \\ \hline 51222 \end{array}$
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Numeration Systems

Here are some numerals used by early civilizations

	1	10	100		
Egypt	I	∩	9	23	∩ ∩ III
Babylon	Y	<	Y>	142	Y> <<<< Y Y
Greece	α	ι	ρ	221	ρ ρ ι α
Rome	I	X	C	230	CC XXX

In modern times, Canada and most other countries use the Hindu-Arabic numeration system. This system has 10 basic symbols that are called digits. These digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. We use place value. Place value was not used in the ancient numeral systems shown.

Exercises

1. Write the Hindu-Arabic numeral for each ancient Egyptian numeral

● ∩ ∩ ∩ III (b) ∩ ∩ ∩ ∩ II (c) 9 9 ∩ III (d) 9 I
33 42 213 101

2. Write the Hindu-Arabic numeral for each Babylonian numeral

● < Y Y (b) < < < (c) Y> Y (d) Y> Y> Y> < Y
12 30 101 311

3. Write the Hindu-Arabic numeral for each ancient Greek numeral

● α α α (b) ι ι (c) ρ ρ α (d) ρ ι
3 20 201 110

Write ancient Egyptian numerals.

4. 35 5. 459

Write Babylonian numerals

6. 288 7. 602

Write ancient Greek numerals.

8. 209 9. 56

Write Roman numerals.

10. 230 11. 304

Numeration Systems 59

OBJECTIVE

To introduce other numeration systems

PACING

Level A Optional

Level B All

Level C All

VOCABULARY

Hindu-Arabic, numeration system, civilizations

SUGGESTIONS

Initial Activity On the board write the following:

IV X XX C.

Elicit from the students that they are Roman numerals and that they correspond to our numerals 4, 10, 20, and 100. Have students provide other Roman numerals that they know, reviewing the system for decoding Roman numerals. Draw the students' attention to some of the differences between the Roman system and our Hindu-Arabic format. Emphasize especially that we use place value whereas Roman numerals are an "addition type" system (i.e., need to be computed each time they are read: XXXVIII = 10 + 10 + 10 + 5 + 1 + 1 + 1).

USING THE BOOK

Use the display at the top of the pupil page to show some other numeration systems and discuss them. You may wish to do the questions on the board with your less able students. Also remind the students that the answers for Exercises 1(a), 2(a), and 3(a) may be found in the back of the text.

ACTIVITIES

1. Have students do some research on numeration systems and present their findings.

2. Students make up a numeration system of their own.

3. Students work out a numeration system using 5 as a base instead of 10.

ANSWERS:

1. ∩ ∩ ∩ III
2. Y> Y> Y> Y> Y> Y> Y Y
3. ρ ρ α α α α α α α α
4. CC XXX
5. 9999 ∩ ∩ ∩ ∩ III III III III
6. Y> Y> <<<<<<<< Y Y Y Y Y Y Y
7. 110
8. 209
9. 56
10. 230
11. 304

OBJECTIVE

To review and provide practice in multiplying by 1 and 10

PACING

Level A 1-6
Level B 2-9
Level C 2-9

MATERIALS

counters or bottle caps or straws

RELATED AIDS

BFA COMP LAB II — 36.

BFA PROB. SOLVING LAB II — 97.

SUGGESTIONS

Initial Activity Instruct students to arrange 1 group of 4 objects. Ask how many objects they have arranged. Have someone write a multiplication statement for the array. Do this several times using different numbers. Lead the children to the conclusion that the product of one times a number is the number itself. Instruct students to arrange a group of 3 objects. Instruct them to repeat this grouping until they have 10 groups. Ask them to count the number of objects. Again, have a multiplication statement written on the chalkboard. Do this several times using different numbers and ask students to suggest a fast method of multiplying a number by 10.

USING THE BOOK

You may wish to have the students write the products only for Exercises 1 to 4. Your more able students can attempt to write the products only for the whole page. Your less able students may need to write out questions in Exercises 5 and 6 before solving.

ACTIVITIES

1. Play "Simon Says" with multiplication facts dealing with 1 and 10. Students put their hands up if Simon Says 1×17 is 17. Students do not put up their hands if the fact is not prefaced by Simon Says.

2. Have the students find the product of your school's name. The value of each consonant is 5. The value of each vowel is 1.


Ones and Tens

1 group of 6 golf balls = 6 golf balls. $1 \times 6 = 6$

$$\begin{array}{r} 6 \\ \times 1 \\ \hline 6 \end{array}$$

1 packet of tees has 25 tees. $10 \times 25 = 250$ tees.

$$\begin{array}{r} 25 \\ \times 10 \\ \hline 250 \end{array}$$



Exercises

1. (a) 1×8 8	(b) 1×2 2	(c) 1×9 9	(d) 1×5 5	(e) 1×7 7
2. (a) $\begin{array}{r} 12 \\ \times 1 \\ \hline 12 \end{array}$	(b) $\begin{array}{r} 28 \\ \times 1 \\ \hline 28 \end{array}$	(c) $\begin{array}{r} 49 \\ \times 1 \\ \hline 49 \end{array}$	(d) $\begin{array}{r} 85 \\ \times 1 \\ \hline 85 \end{array}$	(e) $\begin{array}{r} 97 \\ \times 1 \\ \hline 97 \end{array}$
3. (a) 432×1 432	(b) 768×1 768	(c) 579×1 579	(d) 685×1 685	(e) 927×1 927
4. (a) 10×4 40	(b) 10×9 90	(c) 10×5 50	(d) 10×3 30	(e) 10×8 80
5. (a) $\begin{array}{r} 27 \\ \times 10 \\ \hline 270 \end{array}$	(b) $\begin{array}{r} 35 \\ \times 10 \\ \hline 350 \end{array}$	(c) $\begin{array}{r} 52 \\ \times 10 \\ \hline 520 \end{array}$	(d) $\begin{array}{r} 36 \\ \times 10 \\ \hline 360 \end{array}$	(e) $\begin{array}{r} 85 \\ \times 10 \\ \hline 850 \end{array}$
6. (a) $\begin{array}{r} 482 \\ \times 10 \\ \hline 4820 \end{array}$	(b) $\begin{array}{r} 853 \\ \times 10 \\ \hline 8530 \end{array}$	(c) $\begin{array}{r} 768 \\ \times 10 \\ \hline 7680 \end{array}$	(d) $\begin{array}{r} 589 \\ \times 10 \\ \hline 5890 \end{array}$	(e) $\begin{array}{r} 417 \\ \times 10 \\ \hline 4170 \end{array}$
7. (a) $\begin{array}{r} 403 \\ \times 1 \\ \hline 403 \end{array}$	(b) $\begin{array}{r} 450 \\ \times 10 \\ \hline 4500 \end{array}$	(c) $\begin{array}{r} 700 \\ \times 1 \\ \hline 700 \end{array}$	(d) $\begin{array}{r} 900 \\ \times 10 \\ \hline 9000 \end{array}$	(e) $\begin{array}{r} 23 \\ \times 10 \\ \hline 230 \end{array}$

8. Write a rule for multiplying by 1. 9. Write a rule for multiplying by 10.

60 Multiplying by 1 and 10

ANSWERS:

8. The answer is the number you are multiplying by 1.

9. To multiply a number by 10 move the figures in the numeral 1 place to the left and put a 0 in the one's place.

Example

GLENWOOD =

$5 \times 5 \times 1 \times 5 \times 5 \times 1 \times 1 \times 5 = 3125$

3. Have students assign different values to consonants. Make 10 the value of each vowel. Have them find the "values" of their names.

EXTRA PRACTICE

1. $\begin{array}{r} 705 \\ \times 1 \\ \hline [705] \end{array}$	2. $\begin{array}{r} 832 \\ \times 10 \\ \hline [8320] \end{array}$	3. $\begin{array}{r} 800 \\ \times 10 \\ \hline [8000] \end{array}$
4. $\begin{array}{r} 111 \\ \times 1 \\ \hline [111] \end{array}$	5. $\begin{array}{r} 19 \\ \times 10 \\ \hline [190] \end{array}$	6. $\begin{array}{r} 483 \\ \times 1 \\ \hline [483] \end{array}$

Shirts and Gloves

There are 24 shirts in a box.

In 100 boxes there are $100 \times 24 = 2400$ shirts.

24

$\times 100$

2400

There are 144 gloves in a box.

In 1000 boxes there are $1000 \times 144 = 144\,000$ gloves.

144

$\times 1000$

144 000



Exercises

1. (a) 100×8 (b) 100×4 (c) 100×27 (d) 100×94 (e) 100×38
2. (a) $137 \times 100 = 13\,700$ (b) $285 \times 100 = 28\,500$ (c) $396 \times 100 = 39\,600$ (d) $474 \times 100 = 47\,400$ (e) $912 \times 100 = 91\,200$
3. (a) $100 \times 4384 = 438\,400$ (b) $100 \times 7963 = 796\,300$ (c) $100 \times 5038 = 503\,800$
(d) $100 \times 6456 = 645\,600$ (e) $100 \times 9124 = 912\,400$
4. (a) $1000 \times 7 = 7\,000$ (b) $1000 \times 5 = 5\,000$ (c) $1000 \times 23 = 23\,000$
(d) $1000 \times 94 = 94\,000$ (e) $1000 \times 45 = 45\,000$
5. (a) $286 \times 1000 = 286\,000$ (b) $687 \times 1000 = 687\,000$ (c) $594 \times 1000 = 594\,000$ (d) $857 \times 1000 = 857\,000$ (e) $718 \times 1000 = 718\,000$
6. (a) $4000 \times 1000 = 4\,000\,000$ (b) $350 \times 100 = 35\,000$ (c) $42 \times 10 = 420$ (d) $5697 \times 100 = 569\,700$ (e) $6324 \times 10 = 63\,240$
7. (a) $3256 \times 1 = 3256$ (b) $4000 \times 10 = 40\,000$ (c) $42 \times 1000 = 42\,000$ (d) $563 \times 100 = 56\,300$ (e) $0 \times 1 = 0$
8. Write a rule for multiplying by: (a) 100 (b) 1000

OBJECTIVE

To review and provide practice in multiplying by 100 and 1000

PACING

Level A 1-6

Level B All

Level C 1-7, parts (a), (c), and (e); 8

RELATED AIDS

BFA COMP LAB II — 36.

BFA PROB. SOLVING LAB II — 101.

CALC. ACTIVITY MASTERS — 18.

SUGGESTIONS

Initial Activity Review the patterns that were evident when multiplying by 1 and 10. Repeat the procedure, this time showing several examples of multiplication of 1-, 2-, 3-, and 4-digit numbers by 100. Elicit from the students a fast way of multiplying by 100. Repeat this procedure for multiplication by 1000.

USING THE BOOK

Your more able students should write the products only. Your less able students may feel more confident if they write out the question first before solving it.

Multiplying by 100 and 1000 61

ANSWERS:

- (a) To multiply a number by 100 move the figures in the numeral 2 places to the left and put a 0 in the one's and ten's places.
- (b) To multiply a number by 1000 move the figures in the numeral 3 places to the left and put a 0 in the one's, ten's, and hundred's places.

ACTIVITIES

Repeat the "find the values" activities described in Activities 2 and 3 on page 60. Use 100 and 1000 as the values for each vowel.

2. Play "Concentration" as described in the Activity Reservoir. Use cards such as:

426×100	42 600
1000×87	87 000

EXTRA PRACTICE

1. (a) $378 \times 100 = [37\,800]$ (b) $9442 \times 10 = [94\,420]$ (c) $857 \times 1000 = [857\,000]$
- (d) $661 \times 100 = [66\,100]$ (e) $20 \times 1000 = [20\,000]$ (f) $111 \times 100 = [11\,100]$
2. (a) $9442 \times 100 = [944\,200]$ (b) $378 \times 1000 = [378\,000]$ (c) $111 \times 10 = [1110]$
- (d) $6722 \times 1 = [6722]$ (e) $99 \times 100 = [9900]$ (f) $857 \times 10 = [8570]$

3. See the "Scramble" idea in the Activity Reservoir. Use cards which show multiplication by 1, 10, 100, and 1000.

OBJECTIVES

- To show a model that illustrates the commutative property
- To provide practice in using the commutative property in multiplication
- To practise multiplying by multiples of ten

PACING

- Level A All
- Level B All
- Level C All

MATERIALS

counters, bottle caps, etc.

RELATED AIDS

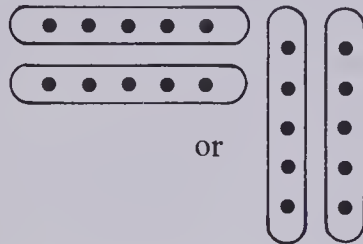
BFA PROB. SOLVING LAB II—77, 81.

CALC. ACTIVITY MASTERS—50.

SUGGESTIONS

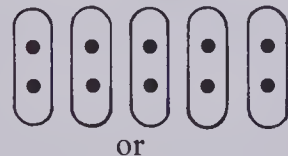
Initial Activity Distribute counters to each student. Ask them to make an array which illustrates the multiplication fact 2×5 .

Example



Ask them to make an array that illustrates the multiplication fact 5×2 .

Example



Write $2 \times 5 =$ and $5 \times 2 =$ on the chalkboard. Have students supply the product. Follow this procedure several times, using different factors. Have the students study the written multiplication sentences on the board and ask them to arrive at a conclusion regarding the factors and their respective products.

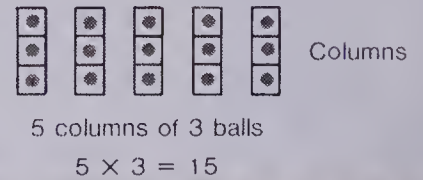
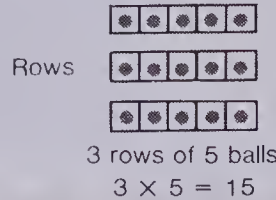
USING THE BOOK

Study the display at the top of the page to reinforce the conclusions reached during the Initial Activity. With more able students, you may wish to refer to the commutative property by name.

You may wish to assign the first

Rows and Columns

Here are 2 ways to show an array of balls.



In multiplication, the order of the factors does not change the product.

Exercises

Copy and complete.

- (a) $9 \times 8 = 72$ (b) $8 \times 9 = \blacksquare 72$ (c) $24 \times 6 = 144$ (d) $6 \times 24 = \blacksquare 144$
- (a) $9 \times 13 = 117$ (b) $13 \times 9 = \blacksquare 117$ (c) $2 \times 48 = 96$ (d) $48 \times 2 = \blacksquare 96$
- (a) $33 \times 3 = \blacksquare 99$ (b) $3 \times 33 = \blacksquare 99$ (c) $143 \times 73 = \blacksquare$ (d) $73 \times 143 = \blacksquare$
- (a) $\$3.60 \times 24 = \blacksquare$ (b) $24 \times \$3.60 = \blacksquare$ (c) $409 \times 70 = \blacksquare$ (d) $70 \times \blacksquare = \blacksquare$
- (a) $\$3.62 \times 201 = \blacksquare$ (b) $\blacksquare \times \$3.62 = \blacksquare$ (c) $921 \times 1000 = \blacksquare$ (d) $1000 \times \blacksquare = \blacksquare$

Tune Up

Calculate.

- (a) $\begin{array}{r} 37 \\ \times 20 \\ \hline 740 \end{array}$ (b) $\begin{array}{r} 54 \\ \times 30 \\ \hline 1620 \end{array}$ (c) $\begin{array}{r} 48 \\ \times 40 \\ \hline 1920 \end{array}$ (d) $\begin{array}{r} 56 \\ \times 50 \\ \hline 2800 \end{array}$ (e) $\begin{array}{r} 72 \\ \times 60 \\ \hline 4320 \end{array}$
- (a) $\begin{array}{r} 131 \\ \times 40 \\ \hline 5240 \end{array}$ (b) $\begin{array}{r} 106 \\ \times 80 \\ \hline 8480 \end{array}$ (c) $\begin{array}{r} 156 \\ \times 20 \\ \hline 3120 \end{array}$ (d) $\begin{array}{r} 162 \\ \times 20 \\ \hline 3240 \end{array}$ (e) $\begin{array}{r} 175 \\ \times 30 \\ \hline 5250 \end{array}$
- (a) $\begin{array}{r} 236 \\ \times 50 \\ \hline 11800 \end{array}$ (b) $\begin{array}{r} 351 \\ \times 60 \\ \hline 21060 \end{array}$ (c) $\begin{array}{r} 516 \\ \times 70 \\ \hline 36120 \end{array}$ (d) $\begin{array}{r} 723 \\ \times 20 \\ \hline 14460 \end{array}$ (e) $\begin{array}{r} 743 \\ \times 80 \\ \hline 59440 \end{array}$

62 Commutative property of multiplication

column of questions to half the class and the corresponding questions in column 2 to the other half. Then answers can be compared.

Before assigning the "Tune Up" exercises, you may wish to review the similarities and number patterns evident in examples such as:

$$\begin{array}{r} 37 \\ \times 2 \\ \hline 74 \end{array} \quad \begin{array}{r} 37 \\ \times 20 \\ \hline 740 \end{array} \quad \begin{array}{r} 18 \\ \times 4 \\ \hline 72 \end{array} \quad \begin{array}{r} 18 \\ \times 40 \\ \hline 720 \end{array}$$

ACTIVITIES

1. Play "Concentration" as described in the Activity Reservoir. Use cards which show the commutative property of multiplication.

$$\begin{array}{|c|} \hline 37 \times 5 \\ \hline \end{array} \quad \begin{array}{|c|} \hline 5 \times 37 \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline 612 \times 20 \\ \hline \end{array} \quad \begin{array}{|c|} \hline 20 \times 612 \\ \hline \end{array}$$

2. Introduce cards which show the products of the multiplication examples in Activity 1. Use the three cards per set (i.e., 37×5 , 5×37 , 185)

to play "Triple Concentration".

3. Prepare assignment cards such as:

- Make an array to show that 6 groups of 2 is the same as 2 groups of 6.
- Use strips of paper to demonstrate that 4 strips each measuring 7 cm is the same length as 7 strips each measuring 4 cm.
- Draw a diagram to show that 5 lines crossing 3 lines and 3 lines crossing 5 lines give the same number of intersections.

Answer:



EXTRA PRACTICE

- (a) $315 \times 80 = \blacksquare [25\ 200]$
(b) $80 \times \blacksquare = \blacksquare [315, 25\ 200]$
- (a) $\$2.76 \times 18 = \blacksquare [\$49.68]$
(b) $18 \times \$2.76 = \blacksquare [\$49.68]$
- (a) $759 \times 1000 = \blacksquare [759\ 000]$
(b) $1000 \times \blacksquare = \blacksquare [759, 759\ 000]$

Multiplying in Different Ways

Multiply.

$$\begin{aligned}(3 \times 4) \times 5 \\ = 12 \times 5 \\ = 60\end{aligned}$$

$$\begin{aligned}3 \times (4 \times 5) \\ = 3 \times 20 \\ = 60\end{aligned}$$

$$\begin{aligned}(3 \times 5) \times 4 \\ = 15 \times 4 \\ = 60\end{aligned}$$

The grouping of the factors does not change the product.

Exercises

Compute

- | | | |
|---|--|--|
| 1. (a) $(2 \times 5) \times 7$ 70 | (b) $2 \times (5 \times 7)$ 70 | (c) $(2 \times 7) \times 5$ 70 |
| 2. (a) $(5 \times 5) \times 6$ 150 | (b) $5 \times (5 \times 6)$ 150 | (c) $(5 \times 6) \times 5$ 150 |
| 3. (a) $(1 \times 3) \times 4$ 12 | (b) $1 \times (3 \times 4)$ 12 | (c) $(1 \times 4) \times 3$ 12 |
| 4. (a) $(7 \times 1) \times 4$ 28 | (b) $7 \times (1 \times 4)$ 28 | (c) $(7 \times 4) \times 1$ 28 |

Copy and complete

- | | |
|---|--|
| 5. (a) $(4 \times 2) \times 6 = \blacksquare \times (2 \times 6)$ | (b) $5 \times (8 \times 3) = (5 \times \blacksquare) \times 3$ |
| 6. (a) $8 \times (4 \times 6) = (8 \times 4) \times \blacksquare$ | (b) $9 \times (3 \times 7) = (9 \times 3) \times \blacksquare$ |
| 7. (a) $(41 \times 6) \times 3 = 41 \times (6 \times \blacksquare)$ | (b) $84 \times (32 \times 6) = (84 \times 32) \times \blacksquare$ |

Compute

- | | | | |
|---|---------------------------------------|---|-------------------------------------|
| 8. (a) 74×30 2220 | 30×74 2220 | (b) 91×3 273 | 3×91 273 |
| 9. (a) 7×5 35 | 5×7 35 | (b) $(3 \times 21) \times 4$ 252 | $3 \times (21 \times 4)$ 252 |
| 10. (a) $(16 \times 3) \times 8$ 384 | $16 \times (3 \times 8)$ 384 | (b) $(51 \times 3) \times 4$ 612 | $(51 \times 4) \times 3$ 612 |
| 11. (a) $(3 \times 401) \times 2$ 2406 | $(2 \times 3) \times 401$ 2406 | (b) $(4 \times 15) \times 3$ 180 | $4 \times (15 \times 3)$ 180 |

Associative property of multiplication 63

OBJECTIVE

To introduce the associative property in multiplication

PACING

Level A 1-8
Level B All
Level C 3-11

SUGGESTIONS

Initial Activity Have the students study the display at the top of the page. Go over each example reminding students that the factors within the brackets are multiplied together first. Their product is multiplied by the third factor. Draw the students' attention to the different bracketing arrangements and the different order of the factors. Lead them to the conclusion that the order of the grouping does not change the product. Remind the students that they are to do the operations with brackets first.

USING THE BOOK

Complete Exercises 1, 5, and 8 orally, discussing the various products and stressing the associative property of multiplication. For Exercises 5, 6, and 7 you may wish to mention the "balance" feature of the expressions, i.e., "What number should replace each \blacksquare to make both sides of the equals symbol the same value?"

ACTIVITIES

1. Have the students prepare 5 "missing-factor" exercises as exemplified by Exercises 5 to 7 on the student page. Have them exchange these with other groups or classmates.

2. To review the Order of Operations rules, prepare and distribute an exercise such as: **MATCH EXPRESSIONS OF EQUAL VALUE.**

Remember — Bless My Dear Aunt Sally

i.e., Brackets first; Multiplication, Division, Addition, Subtraction in the order they appear).

- | | |
|-----------------------|-----------------|
| 1. $3 \times (7 + 1)$ | A $(5 + 5) + 5$ |
| 2. $11 + 11$ | B $100 \div 20$ |

- | | |
|------------------------------|----------------------------|
| 3. $13 - 10 \times 4 \div 5$ | C $3 \times 7 + 1$ |
| 4. $5 \times (5 - 2)$ | D $12 \times (3 \times 1)$ |
| 5. $(16 + 16) \div 8$ | E $2 \times 2 \times 6$ |
| 6. $9 \times 9 \div 9$ | F $3 \times 3 \div 1$ |
| 7. $(6 \times 6) \times 1$ | G $1 \times 1 + 1 + 1 + 1$ |
| 8. $24 - 12 \div 3$ | H $5 \times (2 \times 2)$ |
- [Answers: 1-E; 2-C; 3-B; 4-A; 5-G; 6-F; 7-D; 8-H]

3. See "Number Sentence" as described in the Activity Reservoir.

EXTRA PRACTICE

Rewrite each changing the position of the brackets.

- $(8 \times 5) \times 2 = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$
- $40 \times (3 \times 71) = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$
- $7 \times (9 \times 3) = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$
- $347 \times (42 \times 8) = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

OBJECTIVE

To review and provide practice in multiplying 2 digits by 2 digits

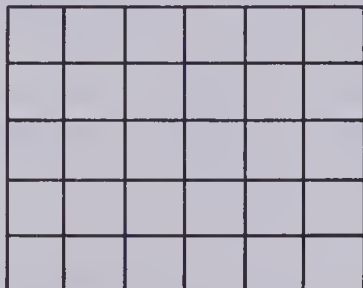
PACING

Level A All
Level B All
Level C All

MATERIALS

flash cards with multiplication facts, "multo" grids

M U L T O



RELATED AIDS

BFA COMP LAB II — 39.
BFA PROB. SOLVING LAB II — 77, 81.
CALC. ACTIVITY MASTERS — 49.

SUGGESTIONS

Initial Activity Distribute "Multo" grids. Instruct students to write any numbers ranging from 1 to 50 in the squares. Show the flash cards one at a time. Students draw a line through the product if it appears on their grid. The first person to get 5 numbers in a row with lines through them wins and calls out "Multo". (This is a variation of the game Bingo.) This activity provides a multiplication "warm up".

On the board do several examples of multiplication by 2 digits. Draw particular attention to the multiplication by tens. Students will perhaps have to be reminded to write a zero as a placeholder before multiplying by the ten's figure. Use coloured chalk to emphasize this point.

USING THE BOOK

Read through the display at the top of the pupil page, discussing the origin of the factors (i.e., 73 stands for the number of papers in one day, 25 stands for the number of days) and why multiplication is the appropriate operation. Emphasize that 365 is the product of 5×73 and that 1460 is the product of 20×73 .

Newspaper Route

Jim delivered 73 newspapers a day.
How many newspapers did he deliver in 25 d?

$$\begin{array}{r} \text{Multiply} \quad 73 \\ \times 25 \\ \hline 365 \leftarrow 5 \times 73 = 365 \\ 1460 \leftarrow 20 \times 73 = 1460 \\ \hline 1825 \end{array}$$

He delivered 1825 newspapers in 25 d.



Exercises

Copy and complete.

$$\begin{array}{r} 51 \\ \times 19 \\ \hline 459 \quad (9 \times 51) \\ 510 \quad (10 \times 51) \\ \hline 969 \end{array}$$

$$\begin{array}{r} 37 \\ \times 43 \\ \hline 111 \quad (3 \times 37) \\ 1480 \quad (40 \times 37) \\ \hline 1591 \end{array}$$

4. (a) $\begin{array}{r} 56 \\ \times 84 \\ \hline 4704 \end{array}$	(b) $\begin{array}{r} 48 \\ \times 37 \\ \hline 1776 \end{array}$	(c) $\begin{array}{r} 72 \\ \times 59 \\ \hline 4248 \end{array}$	(d) $\begin{array}{r} 64 \\ \times 73 \\ \hline 4672 \end{array}$	(e) $\begin{array}{r} 68 \\ \times 25 \\ \hline 1700 \end{array}$
5. (a) $\begin{array}{r} 40 \\ \times 37 \\ \hline 1480 \end{array}$	(b) $\begin{array}{r} 70 \\ \times 23 \\ \hline 1610 \end{array}$	(c) $\begin{array}{r} 90 \\ \times 36 \\ \hline 3240 \end{array}$		
(d) $\begin{array}{r} 30 \\ \times 48 \\ \hline 1440 \end{array}$	(e) $\begin{array}{r} 80 \\ \times 61 \\ \hline 4880 \end{array}$			
6. (a) $\begin{array}{r} 63 \times 58 \\ 3654 \end{array}$	(b) $\begin{array}{r} 84 \times 78 \\ 6552 \end{array}$	(c) $\begin{array}{r} 53 \times 49 \\ 2597 \end{array}$		
(d) $\begin{array}{r} 45 \times 61 \\ 2745 \end{array}$	(e) $\begin{array}{r} 67 \times 37 \\ 2479 \end{array}$			

BRAINTICKLER

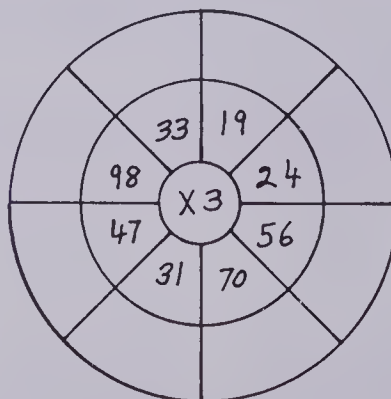
Grandfather Maloney said:
My age this year is a multiple of 7.
My age last year was a multiple of 6.
I have lived for more than half a century.
How old am I? **91**

64 Multiplying 2 digits by 2 digits

Complete Exercises 1 and 2 orally, discussing the value of each ■. You may also wish to demonstrate Exercise 5(a), showing what to do with a zero in the multiplicand.

ACTIVITIES

1. Have the children help prepare multiplication wheels for distribution.



2. Have the students complete a multiplication matrix.

×	20	30	40	50	60	70	80	90
20								
30								
40								
50								
60								
70								
80								
90								

3. Have the students prepare their own matrices for exchange with other groups or classmates.

Example

×	2	4	20	40	10	1000
4						
8						
40						
80						
10						

(Continued on page 70)

Basketball Practice

Gina practised 105 free throws a day for 15 d.
How many free throws did she practise altogether?

$$\begin{array}{r} \text{Multiply} \quad 105 \\ \times 15 \\ \hline 525 \quad (5 \times 105) \\ 1050 \quad (10 \times 105) \\ \hline 1575 \end{array}$$

She practised 1575 free throws altogether.



Exercises
Copy and complete.

- | | | | |
|--|---|---|---|
| $\begin{array}{r} 308 \\ \times 36 \\ \hline 1848 \\ 9240 \\ \hline 11088 \end{array}$ <p>(6 × 308)
(30 × 308)</p> | $\begin{array}{r} 493 \\ \times 48 \\ \hline 3944 \\ 19720 \\ \hline 23664 \end{array}$ <p>(8 × 493)
(40 × 493)</p> | 3. (a) $\begin{array}{r} 643 \\ \times 20 \\ \hline 12860 \end{array}$ | (b) $\begin{array}{r} 807 \\ \times 32 \\ \hline 25824 \end{array}$ |
| 4. (a) $\begin{array}{r} 470 \\ \times 85 \\ \hline 39550 \end{array}$ | (b) $\begin{array}{r} 607 \\ \times 76 \\ \hline 46132 \end{array}$ | (c) $\begin{array}{r} 584 \\ \times 31 \\ \hline 18104 \end{array}$ | (d) $\begin{array}{r} 982 \\ \times 65 \\ \hline 63830 \end{array}$ |
| 5. (a) $\begin{array}{r} 648 \\ \times 50 \\ \hline 32400 \end{array}$ | (b) $\begin{array}{r} 881 \\ \times 26 \\ \hline 22906 \end{array}$ | (c) $\begin{array}{r} 300 \\ \times 85 \\ \hline 25500 \end{array}$ | (d) $\begin{array}{r} 294 \\ \times 36 \\ \hline 10584 \end{array}$ |
| 6. $\begin{array}{r} \$7.11 \\ \times 38 \\ \hline \$270.18 \end{array}$
<small>Dollars and cents</small> | $\begin{array}{r} \$3.78 \\ \times 21 \\ \hline \$79.38 \end{array}$ | (c) $\begin{array}{r} \$9.00 \\ \times 87 \\ \hline \$783.00 \end{array}$ | (d) $\begin{array}{r} \$6.58 \\ \times 16 \\ \hline \$105.28 \end{array}$ |
| 7. (a) $\begin{array}{r} \$5.99 \\ \times 13 \\ \hline \$77.87 \end{array}$ | (b) $\begin{array}{r} \$9.75 \\ \times 36 \\ \hline \$351.00 \end{array}$ | (c) $\begin{array}{r} \$8.03 \\ \times 84 \\ \hline \$674.52 \end{array}$ | (d) $\begin{array}{r} \$3.00 \\ \times 27 \\ \hline \$81.00 \end{array}$ |
| 8. Maurice practised 150 free throws every day for 28 d.
How many free throws altogether? 4200 | | | |

Multiplying 3 digits by 2 digits 65

OBJECTIVE

To multiply 3-digit numbers by 2-digit numbers

PACING

Level A 1-3, 5, 6
Level B 3-5, 7, 8
Level C 4-8

RELATED AIDS

BFA COMP LAB II — 40, 41.
BFA PROB. SOLVING LAB II — 85, 89.
CALC. ACTIVITY MASTERS — 14-16, 48.

SUGGESTIONS

Initial Activity Have students participate in a mental drill involving multiplication and addition, e.g., $9 \times 4 + 7$, $4 \times 8 + 3$, etc.

Review the procedure for multiplying a 2-digit number by a 2-digit number. Use the display at the top of the page to progress to multiplying a 3-digit number by a 2-digit number. Emphasize the similarity of the process to what was done in the previous lesson.

USING THE BOOK

Complete Exercises 1, 2, 6(a), and 6(b) as a group before assigning the exercises. You may wish to remind the students that though all answers for Exercises 6 and 7 should be expressed in dollars and cents, the computation procedure is the same.

Example

$$\begin{array}{r} 74596 \\ \times 12 \\ \hline \end{array}$$

$$\begin{array}{r} 287 \\ \times 51 \\ \hline \end{array}$$

Score 2 points.

$$\begin{array}{r} 443 \\ \times 77 \\ \hline \end{array}$$

$$\begin{array}{r} 746 \\ \times 33 \\ \hline \end{array}$$

$$\begin{array}{r} 596 \\ \times 12 \\ \hline \end{array}$$

$$\begin{array}{r} 287 \\ \times 51 \\ \hline \end{array}$$

Score 4 points.

$$\begin{array}{r} 746 \\ \times 33 \\ \hline \end{array}$$

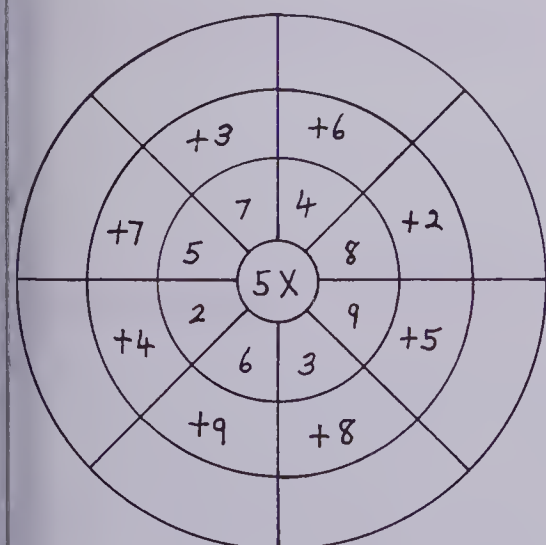
$$\begin{array}{r} 443 \\ \times 77 \\ \hline \end{array}$$

EXTRA PRACTICE

- | | | |
|--|--|--|
| 1. $\begin{array}{r} 635 \\ \times 69 \\ \hline [43815] \end{array}$ | 2. $\begin{array}{r} 527 \\ \times 34 \\ \hline [17918] \end{array}$ | 3. $\begin{array}{r} 886 \\ \times 32 \\ \hline [28352] \end{array}$ |
| 4. $\begin{array}{r} 476 \\ \times 90 \\ \hline [42840] \end{array}$ | 5. $\begin{array}{r} \$5.27 \\ \times 41 \\ \hline [\$216.07] \end{array}$ | |

ACTIVITIES

To provide practice with the type of mental process required while using the multiplication algorithm, have the children prepare drill wheels as shown. Note that the first two numbers are multiplied together and that the third is added to the product.



2. Have students complete multiplication puzzles as shown. You may wish to provide calculators to

help and/or provide assistance while students make up their own puzzles for exchange with classmates.

$\begin{array}{r} 476 \\ \times 25 \\ \hline 2300 \\ 9200 \\ \hline 01000 \end{array}$	$\begin{array}{r} 764 \\ \times 97 \\ \hline 0048 \\ 60700 \\ \hline 04000 \end{array}$	$\begin{array}{r} \$8.53 \\ \times 38 \\ \hline 0800 \\ 05090 \\ \hline \$000.04 \end{array}$
--	---	---

3. To review estimation by rounding, approximations, and "number reasonableness" make up a deck of 40 to 50 cards each showing a 3-digit by 2-digit question. Players (from 2 to 4) (a) shuffle deck, (b) deal 4 cards per player, (c) use mental powers of rounding and estimating to place their cards in order (by product) from smallest to largest, (d) use calculators to check products and ordering, (e) score 1 point for each correctly-placed card.

The winner is the player with the most points after a predetermined number of rounds.

OBJECTIVE

To multiply 3-digit numbers by 3-digit numbers

PACING

Level A 1-5
Level B All
Level C 3-7

RELATED AIDS

BFA COMP LAB II — 43.

BFA PROB. SOLVING LAB II — 93.

SUGGESTIONS

Initial Activity Continue providing practice in using multiplication facts by means of drills, games, and flash cards.

Review the procedure for multiplying 3 digits by 2 digits. Remind the students about the use of the zero as a placeholder when multiplying by tens.

Go over the example in the display at the top of the pupil page. Draw particular attention to the multiplication by hundreds and the use of two zeros as placeholders. Use coloured chalk or cut out coloured construction paper circles to emphasize this. Do several more examples of multiplication by 3 digits.

USING THE BOOK

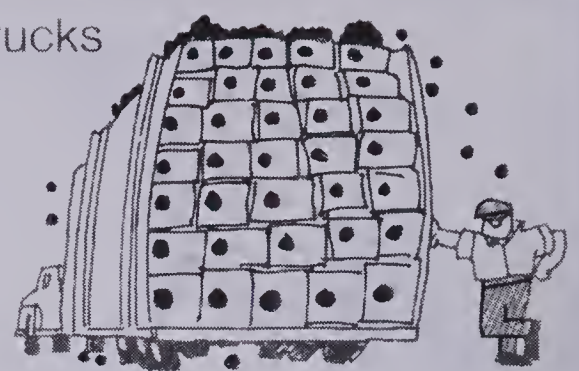
Complete Exercises 1 and 2 orally, consolidating the ideas and procedures that have been reviewed and/or introduced over the past several pages. Exercise 4(a) has already been completed and the answer for 4(b) is also in the back of the text for those who need some guidance as to what to do when there are zeros in the one's place of the multiplier. The students may need to be reminded that the answers for Exercise 6 should be expressed as dollars and cents amounts.

ACTIVITIES

1. Using advertisements from newspapers, have students calculate the cost of buying large quantities of the same item, e.g., 150 jugs of milk, 367 record albums, etc.

2. If you have not already done so, see the multiplication ideas listed as Activities 2 and 3 on page 65. Adjust them to reflect the multiplication of 3-digit numbers by 3-digit numbers.

Trucks



A truck holds 146 crates of oranges.
How many crates will 126 trucks hold?

Multiply

$$\begin{array}{r} 146 \\ \times 126 \\ \hline 876 \\ 2920 \\ 18000 \\ \hline 18498 \end{array}$$

126 trucks will hold 18 648 crates of oranges.

Exercises
Copy and complete.

<p>1. $\begin{array}{r} 146 \\ \times 126 \\ \hline 876 \\ 2920 \\ 18000 \\ \hline 18498 \end{array}$</p>	<p>2. $\begin{array}{r} 136 \\ \times 142 \\ \hline 1472 \\ 29120 \\ 18080 \\ \hline 19312 \end{array}$</p>	<p>3. $\begin{array}{r} 146 \\ \times 126 \\ \hline 876 \\ 2920 \\ 18000 \\ \hline 18498 \end{array}$</p>
<p>4. (a) $\begin{array}{r} 127 \\ \times 340 \\ \hline 5080 \\ 38100 \\ \hline 43180 \end{array}$</p>	<p>(b) $\begin{array}{r} 659 \\ \times 860 \\ \hline 566740 \\ 57080 \\ \hline 566740 \end{array}$</p>	<p>(c) $\begin{array}{r} 387 \\ \times 490 \\ \hline 165130 \\ 38700 \\ \hline 190620 \end{array}$</p>
<p>5. (a) $\begin{array}{r} 556 \\ \times 364 \\ \hline 202384 \\ 33360 \\ \hline 202384 \end{array}$</p>	<p>(b) $\begin{array}{r} 371 \\ \times 419 \\ \hline 155299 \\ 148570 \\ \hline 155299 \end{array}$</p>	<p>(c) $\begin{array}{r} 646 \\ \times 441 \\ \hline 284886 \\ 258080 \\ \hline 284886 \end{array}$</p>
<p>6. (a) $\\$9.47 \times 324$</p>	<p>(b) $\\$8.76 \times 865$</p>	<p>7. Each truck carries 244 crates of grapefruit. How many crates will 319 trucks carry? 77 836</p>
<p>(c) $\\$7.73 \times 628$</p>	<p>(d) $\\$4.38 \times 480$</p>	
<p>(e) $\\$6.61 \times 193$</p>		

66 Multiplying 3 digits by 2 digits

ANSWERS:

6. (a) \$3068.28 (b) \$7577.40 (c) \$4854.44 (d) \$2102.40
(e) \$1275.73

EXTRA PRACTICE

1. $\begin{array}{r} 768 \\ \times 357 \\ \hline [274\ 176] \end{array}$ 2. $\begin{array}{r} 442 \\ \times 807 \\ \hline [356\ 694] \end{array}$ 3. $\begin{array}{r} 601 \\ \times 346 \\ \hline [207\ 946] \end{array}$

4. $\begin{array}{r} 985 \\ \times 650 \\ \hline [640\ 250] \end{array}$ 5. $\begin{array}{r} \$3.69 \\ \times 791 \\ \hline [\$2918.79] \end{array}$

Zeros in the Multiplier

Multiply

Long way	Short way	Long way	Short way
$\begin{array}{r} 748 \\ \times 403 \\ \hline 2244 \\ 0000 \\ 299200 \\ \hline 301444 \end{array}$	$\begin{array}{r} 748 \\ \times 403 \\ \hline 2244 \\ 299200 \\ \hline 301444 \end{array}$	$\begin{array}{r} 748 \\ \times 4003 \\ \hline 2244 \\ 0000 \\ 2992000 \\ \hline 2994244 \end{array}$	$\begin{array}{r} 748 \\ \times 4003 \\ \hline 2244 \\ 2992000 \\ \hline 2994244 \end{array}$
Check by rounding $700 \times 400 = 280\,000$		Check by rounding $700 \times 4000 = 2\,800\,000$	

Exercises

Calculate. Use the short way. Check by rounding to see if your answer is reasonable.

1.
$$\begin{array}{r} 385 \\ \times 209 \\ \hline 3465 \\ 77000 \\ 80000 \\ \hline 80665 \end{array}$$
 Check $400 \times 200 = 80\,000$
2.
$$\begin{array}{r} 468 \\ \times 603 \\ \hline 1404 \\ 280800 \\ 282204 \end{array}$$
 Check $500 \times 600 = 300\,000$
3. (a)
$$\begin{array}{r} \$9.37 \\ \times 805 \\ \hline \$7542.85 \end{array}$$
 (b)
$$\begin{array}{r} \$8.20 \\ \times 602 \\ \hline \$4936.40 \end{array}$$
- (c)
$$\begin{array}{r} \$7.05 \\ \times 304 \\ \hline \$2143.20 \end{array}$$
 (d)
$$\begin{array}{r} \$1.57 \\ \times 601 \\ \hline \$943.57 \end{array}$$
4. (a)
$$\begin{array}{r} 46 \\ \times 5003 \\ \hline 230138 \end{array}$$
 (b)
$$\begin{array}{r} 38 \\ \times 7006 \\ \hline 266228 \end{array}$$
 (c)
$$\begin{array}{r} 97 \\ \times 3004 \\ \hline 291388 \end{array}$$
 (d)
$$\begin{array}{r} 85 \\ \times 8002 \\ \hline 680170 \end{array}$$
 (e)
$$\begin{array}{r} 49 \\ \times 2007 \\ \hline 98343 \end{array}$$
5. (a)
$$\begin{array}{r} 483 \\ \times 5007 \\ \hline 2418381 \end{array}$$
 (b)
$$\begin{array}{r} 984 \\ \times 6002 \\ \hline 5905968 \end{array}$$
 (c)
$$\begin{array}{r} 701 \\ \times 4007 \\ \hline 2808907 \end{array}$$
 (d)
$$\begin{array}{r} 947 \\ \times 8005 \\ \hline 7580735 \end{array}$$
 (e)
$$\begin{array}{r} 389 \\ \times 9005 \\ \hline 3502945 \end{array}$$
6. (a) 725×4017 (b) 598×3049 (c) 805×9048 (d) 559×3074
7. (a) 492×4105 (b) 874×9407 (c) 184×3705 (d) 384×5604

Zeros in the multiplier 67

OBJECTIVE

To multiply when there is one or more zeros in the multiplier

PACING

Level A 1-3, 4(a)-4(c), 5(a)-5(c)
Level B 1-6
Level C 3-7

RELATED AIDS

HMS—DM17.
BFA COMP LAB II—42.

SUGGESTIONS

Initial Activity Briefly review the multiplication algorithm as presented so far. Emphasize the meaning of each row of computation:

$$\begin{array}{r} 513 \\ \times 78 \\ \hline \end{array}$$

← 8×513
← 70×513

and especially the use of zero as a placeholder.

Present an example such as 312×503 . Guide the children through the algorithm, again stressing the origin of each row of numbers. Repeat as necessary.

USING THE BOOK

Read through the two examples at the top of the pupil page. You may wish to specify for certain individuals or groups that they use the "long way" of computing for now. With the computations now starting to yield 6 and more digit answers, you may wish to have answers checked by rounding to ensure that they are at least reasonable.

Complete Exercises 1 and 2 orally using the "short way" (or the "long way" for those who need it). The answers for the first questions in Exercises 3, 4, and 5 are noted in the back of the pupil text for those who wish to check their progress.

ANSWERS:

6. (a) 2 912 325 (b) 1 823 302 (c) 7 283 640 (d) 1 718 366
7. (a) 2 019 660 (b) 8 221 718 (c) 6 81 720 (d) 2 151 936

ACTIVITIES

- See the "500 Grand" idea as described in the Activity Reservoir.
- To provide practice in the area of multiplication facts, see the "Bingo" idea as described in the Activity Reservoir.
- Some of the children may have access to the necessary information to record data on "Spec Sheets" such as:

- I have lived _____ days.
- I have lived _____ hours.
- I have been at school _____ days this year so far.
- My heart beats _____ times per minute (resting).
It would beat _____ times in one day.
It would beat _____ times in one week.
etc.

OBJECTIVE

To multiply tenths by single-digit whole numbers

PACING

Level A 1-7
Level B All
Level C 3-10

RELATED AIDS

BFA COMP LAB II — 101, 102.

SUGGESTIONS

Initial Activity Review, if necessary, (a) the multiplication algorithm for up to 4 digits by 1 digit and (b) place value to hundredths.

Ask: "What is 8×9 ? Will 8.3×9 be 7.47, 74.7, or 747?"

Develop the reason for the placement of the decimal by estimation. Point out that 9.2×6 is a little more than $9 \times 6 = 54$. Therefore the answer is 55.2 not 5.52 nor 552.

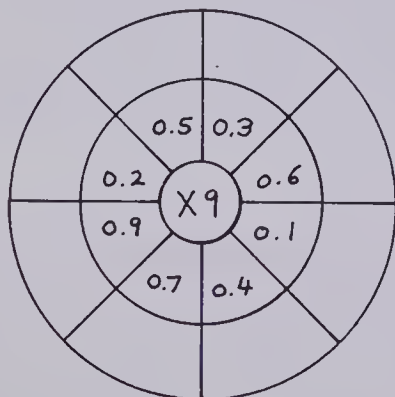
USING THE BOOK

Read through the display situation at the top of the pupil page. You may even wish to perform a similar demonstration with a suitably-sized set of books (or other items). Point out that (a) the multiplication process is virtually the same and (b) the sum of the number of decimal places in the factors is equal to the number of decimal places in the product.

Before assigning the exercises, complete Exercises 1(a), 3(a), and 5(a) at the chalkboard.

ACTIVITIES

1. Have the children help in the preparation of multiplication wheels.

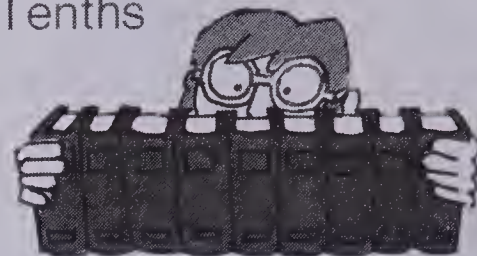


2. Provide 2 decks of cards. One deck is made up of whole numbers from 1 to 9. The other deck is made up of a selection of any numbers containing one decimal place. Students play in small groups. Each

Multiplying Tenths

A library receives a reference set that has 9 books the same size. Each book is 8.3 cm thick. How much shelf space will be needed?

$$\begin{array}{r} \text{Multiply.} \quad 8.3 \quad \leftarrow 1 \text{ decimal place} \\ \times \quad 9 \quad \leftarrow 0 \text{ decimal places} \\ \hline 74.7 \quad \leftarrow 1 \text{ decimal place} \end{array}$$



74.7 cm of shelf space is needed

The sum of the number of decimal places in the factors is equal to the number of decimal places in the product.

Exercises

1. $\begin{array}{r} 0.4 \\ \times 6 \\ \hline 2.4 \end{array}$ (c) $\begin{array}{r} 0.7 \\ \times 3 \\ \hline 2.1 \end{array}$ (d) $\begin{array}{r} 0.5 \\ \times 4 \\ \hline 2.0 \end{array}$ (e) $\begin{array}{r} 0.7 \\ \times 2 \\ \hline 1.4 \end{array}$ (f) $\begin{array}{r} 0.9 \\ \times 5 \\ \hline 4.5 \end{array}$
2. (a) $7 \times 0.2 = 1.4$ (b) $3 \times 0.8 = 2.4$ (c) $6 \times 0.1 = 0.6$ (d) $8 \times 0.6 = 4.8$
3. $\begin{array}{r} 6.3 \\ \times 4 \\ \hline 25.2 \end{array}$ (c) $\begin{array}{r} 9.7 \\ \times 6 \\ \hline 58.2 \end{array}$ (d) $\begin{array}{r} 4.8 \\ \times 2 \\ \hline 9.6 \end{array}$ (e) $\begin{array}{r} 8.7 \\ \times 5 \\ \hline 43.5 \end{array}$ (f) $\begin{array}{r} 1.3 \\ \times 9 \\ \hline 11.7 \end{array}$
4. (a) $4 \times 5.1 = 20.4$ (b) $8 \times 6.3 = 50.4$ (c) $3 \times 2.7 = 8.1$ (d) $1 \times 5.8 = 5.8$ (e) $7 \times 3.8 = 26.6$
5. $\begin{array}{r} 26.5 \\ \times 3 \\ \hline 79.5 \end{array}$ (b) $\begin{array}{r} 88.7 \\ \times 4 \\ \hline 354.8 \end{array}$ (c) $\begin{array}{r} 37.1 \\ \times 6 \\ \hline 222.6 \end{array}$ (d) $\begin{array}{r} 50.8 \\ \times 7 \\ \hline 355.6 \end{array}$ (e) $\begin{array}{r} 93.9 \\ \times 5 \\ \hline 469.5 \end{array}$
6. (a) $7 \times 64.9 = 454.3$ (b) $4 \times 83.7 = 334.8$ (c) $8 \times 30.2 = 241.6$ (d) $1 \times 98.7 = 98.7$
7. (a) $237.4 \times 6 = 1424.4$ (b) $489.6 \times 2 = 979.2$ (c) $207.4 \times 8 = 1659.2$ (d) $321.8 \times 4 = 1287.2$
8. (a) $4 \times 312.7 = 1250.8$ (b) $9 \times 400.5 = 3604.5$ (c) $1 \times 857.6 = 857.6$ (d) $3 \times 805.6 = 2416.8$
9. One book is 3.8 cm thick. How thick are 8 books? **30.4 cm**
10. A shelf is 32.4 cm high. How high are 5 shelves? **162.0 cm**

68 Multiplying tenths by 1-digit whole numbers

ANSWERS:

7. (a) 1424.4 (b) 979.2 (c) 1659.2 (d) 1287.2
8. (a) 1250.8 (b) 3604.5 (c) 857.6 (d) 2416.8

student is dealt a whole-number card. Each in turn picks a decimal number from the pile which is placed face down in the centre. The decimal number is multiplied by the whole number. If this is done correctly, the student takes a score equal to the product. The winner is the player with the greatest point total after a predetermined number of turns.

3. Use the 2 decks of cards prepared for Activity 2. Form teams of students. First member of the team runs up and selects a card from each

deck. The student compiles the answer on the board. If it is correct, the next person on the team runs up and does the same. The first team finished wins.

EXTRA PRACTICE

1. $\begin{array}{r} 7.98 \\ \times 3 \\ \hline [23.94] \end{array}$
2. $\begin{array}{r} 235.9 \\ \times 7 \\ \hline [1651.3] \end{array}$
3. $\begin{array}{r} 482.1 \\ \times 4 \\ \hline [1928.4] \end{array}$
4. $\begin{array}{r} 39.7 \\ \times 5 \\ \hline [198.5] \end{array}$
5. $\begin{array}{r} 661.3 \\ \times 8 \\ \hline [5290.4] \end{array}$

Multiplying Hundredths

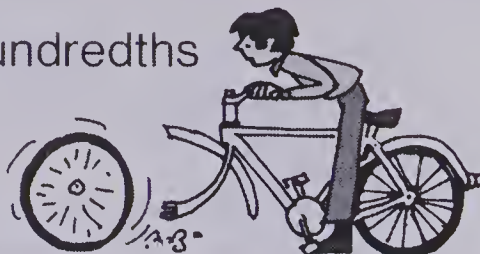
A bicycle wheel travels 2.32 m in one turn.

How far will it travel in 9 turns?

Multiply:

$$\begin{array}{r} 2.32 \\ \times 9 \\ \hline 20.88 \end{array}$$

2.32 ← 2 decimal places
 $\times 9$ ← 0 decimal places
 20.88 ← 2 decimal places



It travels 20.88 m in 9 turns.

The sum of the number of decimal places in the factors is equal to the number of decimal places in the product.

Exercises

- $$\begin{array}{r} \$0.75 \\ \times 3 \\ \hline \$2.25 \end{array}$$

$$\begin{array}{r} \$0.84 \\ \times 7 \\ \hline \$5.88 \end{array}$$

$$\begin{array}{r} \$0.68 \\ \times 7 \\ \hline \$4.76 \end{array}$$

$$\begin{array}{r} \$0.37 \\ \times 9 \\ \hline \$3.33 \end{array}$$

$$\begin{array}{r} \$0.48 \\ \times 6 \\ \hline \$2.88 \end{array}$$
- $$\begin{array}{r} 0.68 \\ \times 3 \\ \hline 2.04 \end{array}$$

$$\begin{array}{r} 0.63 \\ \times 7 \\ \hline 4.41 \end{array}$$

$$\begin{array}{r} 0.79 \\ \times 5 \\ \hline 3.95 \end{array}$$

$$\begin{array}{r} 0.84 \\ \times 6 \\ \hline 5.04 \end{array}$$

$$\begin{array}{r} 0.12 \\ \times 9 \\ \hline 1.08 \end{array}$$
- $$\begin{array}{r} \$4.68 \\ \times 4 \\ \hline \$18.72 \end{array}$$

$$\begin{array}{r} \$3.05 \\ \times 8 \\ \hline \$24.40 \end{array}$$

$$\begin{array}{r} \$9.61 \\ \times 7 \\ \hline \$67.27 \end{array}$$

$$\begin{array}{r} \$8.35 \\ \times 4 \\ \hline \$33.40 \end{array}$$

$$\begin{array}{r} \$6.00 \\ \times 3 \\ \hline \$18.00 \end{array}$$
- $$\begin{array}{r} 6.39 \\ \times 6 \\ \hline 38.34 \end{array}$$

$$\begin{array}{r} 8.02 \\ \times 5 \\ \hline 40.10 \end{array}$$

$$\begin{array}{r} 9.8 \\ \times 3 \\ \hline 29.4 \end{array}$$

$$\begin{array}{r} 8.17 \\ \times 5 \\ \hline 40.85 \end{array}$$

$$\begin{array}{r} 4.63 \\ \times 8 \\ \hline 37.04 \end{array}$$
- $$\begin{array}{r} 46.31 \\ \times 7 \\ \hline 324.17 \end{array}$$

$$\begin{array}{r} 39.06 \\ \times 3 \\ \hline 117.18 \end{array}$$

$$\begin{array}{r} 98.17 \\ \times 7 \\ \hline 687.19 \end{array}$$

$$\begin{array}{r} 30.95 \\ \times 6 \\ \hline 185.70 \end{array}$$

$$\begin{array}{r} 88.96 \\ \times 5 \\ \hline 444.80 \end{array}$$
- $$(a) 3 \times 27.38$$

$$(b) 0.9 \times 35.6$$

$$(c) 4 \times 31.05$$

$$(d) 2 \times 48.2$$
- $$(a) 6 \times \$317.62$$

$$(b) 8 \times \$419.05$$

$$(c) 5 \times \$371.00$$
- $$\begin{array}{l} \text{One small tree costs } \$19.95. \\ \text{How much would 7 trees cost? } \end{array}$$

$$\begin{array}{l} \text{A bicycle wheel travels } 2.68 \text{ m in one turn.} \\ \text{How far will it travel in 8 turns? } \end{array}$$

Multiplying hundredths by 1-digit whole numbers 69

OBJECTIVE

To multiply hundredths by single-digit whole numbers

PACING

Level A 1-6, parts (a)-(c); 8; 9
 Level B 1-6, parts (a)-(c); 7-9
 Level C 1-5, parts (c)-(e); 7-9

RELATED AIDS

HMS — DM18.

BFA PROB. SOLVING LAB II — 113.

SUGGESTIONS

Initial Activity Present the problem in the display at the top of the page. Go over the solution step by step. Do several more examples of the same kind. Lead students to the discovery that the sum of the number of decimal places in the two factors is equal to the number of decimal places in the product.

Have the students estimate the product by dealing with the whole numbers only as described in the Initial Activity on page 68.

Emphasize the estimating system before relying on the rule.

USING THE BOOK

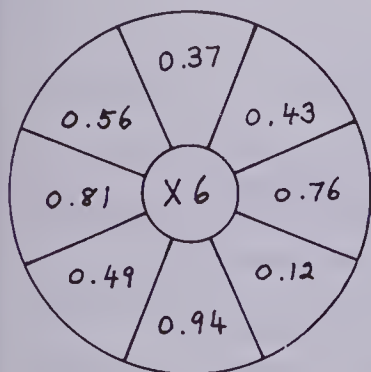
Complete Exercises 1(a) and 1(b) at the chalkboard before assigning the exercises. You may wish to do likewise with some of the other exercises for some groups. Remind the children to watch for dollar amounts and to record their answers accordingly.

ANSWERS:

6. (a) 82.14 (b) 32.04 (c) 124.20 (d) 96.4
 7. (a) \$1905.72 (b) \$3352.40 (c) \$1855.00

ACTIVITIES

Play "Clock Multiplication".



First one to write all the answers correctly wins.

2. Make a bulletin-board display of numbers with 2 decimal places. In

the centre, place a single-digit whole number. Students choose any 5 of the decimal numbers displayed and multiply them by the whole number.

3. Have students make up additional problems similar to the ones in Exercises 8 and 9 at the bottom of the page. Use the newspaper as a possible source of correct prices. Have the children trade problems for solutions.

EXTRA PRACTICE

- $$\begin{array}{r} \$8.97 \\ \times 5 \\ \hline [\$44.85] \end{array}$$
- $$\begin{array}{r} 4.83 \\ \times 9 \\ \hline [43.47] \end{array}$$
- $$\begin{array}{r} 29.34 \\ \times 6 \\ \hline [176.04] \end{array}$$
- $$\begin{array}{r} 734.01 \\ \times 4 \\ \hline [2936.04] \end{array}$$
- $$\begin{array}{r} \$827.36 \\ \times 8 \\ \hline [\$6618.88] \end{array}$$

OBJECTIVE

To multiply a whole number by 0.1

PACING

Level A All

Level B All

Level C 1-6, parts (a), (b), (d), and (e);
7

SUGGESTIONS

Initial Activity Continue to provide practice with multiplication facts by the use of drills, flash cards, games, etc. Particularly focus on multiplication by 1 for this objective. Quick recall of facts is essential if students are to complete assignments successfully.

USING THE BOOK

Present the problem in the display at the top of the page. Review with the students the commutative property of multiplication, i.e., $6 \times 3 = 3 \times 6$. Indicate that the use of this property will make the solving of the problem easier. Take the students through the steps displayed. Do several more examples on the board for consolidation.

You may wish to do all the questions orally with the students. Alternatively, have them write the products only.

ACTIVITIES

1. Write a series of numbers on the board. Make a cutout of a small coloured circle to represent a decimal point. Each number is to be multiplied by 0.1. Students in turn place the decimal point in the appropriate place in each number.

2. "Flash Card Drill". Make up a series of flash cards with whole numbers. Hold each one up in turn and students multiply it by 0.1.

3. See the "Input-Output" idea as described in the Activity Reservoir.

EXTRA PRACTICE

- | | | |
|--|--|--|
| 1. $\begin{array}{r} 63 \\ \times 0.1 \\ \hline [6.3] \end{array}$ | 2. $\begin{array}{r} 854 \\ \times 0.1 \\ \hline [85.4] \end{array}$ | 3. $\begin{array}{r} 528 \\ \times 0.1 \\ \hline [52.8] \end{array}$ |
| 4. $\begin{array}{r} 7285 \\ \times 0.1 \\ \hline [728.5] \end{array}$ | 5. $\begin{array}{r} 9824 \\ \times 0.1 \\ \hline [982.4] \end{array}$ | 6. $\begin{array}{r} 6015 \\ \times 0.1 \\ \hline [601.5] \end{array}$ |

Model Railway

One piece of track on Doug's model railway measured 0.1 m. He laid 15 pieces of track end to end. How long was the track?

There are two ways to multiply:

$\begin{array}{r} 15 \times 0.1 \\ \hline 0.1 \\ \times 15 \\ \hline 05 \\ 10 \\ \hline 1.5 \end{array}$	$\begin{array}{r} 0.1 \times 15 \\ \hline 15 \\ \times 0.1 \\ \hline 1.5 \end{array}$
<p>1 decimal place 1 decimal place</p>	



His track was 1.5 m long.
Which way was easier?

Exercises

- | | | | | |
|---|---|---|---|---|
| 1. $\begin{array}{r} 4 \\ \times 0.1 \\ \hline 0.4 \end{array}$ | 2. $\begin{array}{r} 9 \\ \times 0.1 \\ \hline 0.9 \end{array}$ | (c) $\begin{array}{r} 7 \\ \times 0.1 \\ \hline 0.7 \end{array}$ | (d) $\begin{array}{r} 3 \\ \times 0.1 \\ \hline 0.3 \end{array}$ | (e) $\begin{array}{r} 6 \\ \times 0.1 \\ \hline 0.6 \end{array}$ |
| 3. $\begin{array}{r} 17 \\ \times 0.1 \\ \hline 1.7 \end{array}$ | (b) $\begin{array}{r} 28 \\ \times 0.1 \\ \hline 2.8 \end{array}$ | (c) $\begin{array}{r} 95 \\ \times 0.1 \\ \hline 9.5 \end{array}$ | (d) $\begin{array}{r} 71 \\ \times 0.1 \\ \hline 7.1 \end{array}$ | (e) $\begin{array}{r} 34 \\ \times 0.1 \\ \hline 3.4 \end{array}$ |
| 4. $\begin{array}{r} 237 \\ \times 0.1 \\ \hline 23.7 \end{array}$ | (b) $\begin{array}{r} 448 \\ \times 0.1 \\ \hline 44.8 \end{array}$ | (c) $\begin{array}{r} 205 \\ \times 0.1 \\ \hline 20.5 \end{array}$ | (d) $\begin{array}{r} 446 \\ \times 0.1 \\ \hline 44.6 \end{array}$ | (e) $\begin{array}{r} 517 \\ \times 0.1 \\ \hline 51.7 \end{array}$ |
| 5. (a) 0.1×215 21.5 | (b) 0.1×364 36.4 | (c) 0.1×912 91.2 | (d) 0.1×603 60.3 | |
| 6. (a) $\begin{array}{r} 4832 \\ \times 0.1 \\ \hline 483.2 \end{array}$ | (b) $\begin{array}{r} 6258 \\ \times 0.1 \\ \hline 625.8 \end{array}$ | (c) $\begin{array}{r} 7682 \\ \times 0.1 \\ \hline 768.2 \end{array}$ | (d) $\begin{array}{r} 4235 \\ \times 0.1 \\ \hline 423.5 \end{array}$ | |
| 7. One length of track is 0.1 m long.
How long are (a) 35 lengths of track? 3.5 m (b) 412 lengths of track? 41.2 m | | | | |

70 Multiplying a whole number by 0.1

(Continued from page 64)

EXTRA PRACTICE

- | | | |
|--|---|---|
| 1. (a) $\begin{array}{r} 34 \\ \times 28 \\ \hline [952] \end{array}$ | (b) $\begin{array}{r} 62 \\ \times 86 \\ \hline [5332] \end{array}$ | (c) $\begin{array}{r} 49 \\ \times 80 \\ \hline [3920] \end{array}$ |
| (d) $\begin{array}{r} 32 \\ \times 25 \\ \hline [800] \end{array}$ | (e) $\begin{array}{r} 46 \\ \times 13 \\ \hline [598] \end{array}$ | (f) $\begin{array}{r} 80 \\ \times 19 \\ \hline [1520] \end{array}$ |
| 2. (a) 46×12 [552] (b) 37×18 [666]
(c) 23×54 [1242] (d) 87×11 [957]
(e) 62×41 [2542] (f) 93×10 [930] | | |

3. Andrea delivered 67 newspapers each day.
How many newspapers did she deliver in 28 d? [1876]
4. Mr. Hanna drives 56 km each day going to and from work.
How many kilometres will he drive in 33 d? [2128 km]
5. Ms Lee's car uses 55 L of gasoline each week.
How many litres would she use in 26 weeks? [1430 L]

Bicycle Rides

Kevin rode his bicycle 175 km during the summer.
Joe rode his bicycle seven tenths of Kevin's distance.
How far did Joe ride his bicycle?

Joe rode 0.7 of 175 km
 $= 0.7 \times 175 \text{ km}$
 $= 122.5 \text{ km}$

175 \leftarrow 0 decimal places
 $\times 0.7 \leftarrow$ 1 decimal place
 122.5 \leftarrow 1 decimal place



Exercises

- (a) $6 \times 0.4 = 2.4$

(b) $5 \times 0.7 = 3.5$

(c) $8 \times 0.6 = 4.8$

(d) $9 \times 0.3 = 2.7$

(e) $7 \times 0.5 = 3.5$
- (a) $27 \times 0.6 = 16.2$

(b) $93 \times 0.4 = 37.2$

(c) $85 \times 0.7 = 59.5$

(d) $69 \times 0.9 = 62.1$

(e) $52 \times 0.1 = 5.2$
- (a) $0.3 \times 47 = 14.1$

(b) $0.6 \times 85 = 51.0$

(c) $0.7 \times 49 = 34.3$

(d) $0.5 \times 61 = 30.5$
- (a) $467 \times 0.4 = 186.8$

(b) $987 \times 0.4 = 394.8$

(c) $405 \times 0.7 = 283.5$

(d) $982 \times 0.8 = 785.6$

(e) $853 \times 0.5 = 426.5$
- (a) $4832 \times 0.4 = 1932.8$

(b) $5937 \times 0.8 = 4749.6$

(c) $6502 \times 0.9 = 5851.8$

(d) $3517 \times 0.2 = 703.4$

(e) $9881 \times 0.6 = 5928.6$
- (a) $87\,432 \times 0.4 = 34\,972.8$

(b) $92\,411 \times 0.3 = 27\,723.3$

(c) $49\,027 \times 0.5 = 24\,513.5$

(d) $38\,419 \times 0.9 = 34\,577.1$

(e) $13\,950 \times 0.6 = 8370.0$
- Chris rode his bicycle 120 km.
Yvonne rode hers eight tenths of Chris' distance.
How far did she ride? **96.0 km**
- Gloria rode 86 km on her ten-speed bike.
Marcos went only four tenths of her distance.
How far did he ride? **34.4 km**

Multiplying whole numbers by tenths 71

USING THE BOOK

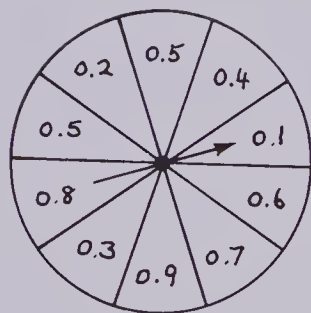
Read through the example at the top of the page, consolidating the ideas presented in the Initial Activity. Complete Exercises 1, 2(a), and 4(a) orally before assigning the rest.

ACTIVITIES

1. Play "Concentration" as described in the Activity Reservoir. Use cards such as:

0.8×7	5.6
8×7	56
0.3×9	2.7

2. Use an HMS textbook and a spinner (or appropriately labelled deck of cards) to play "Flip 500".



Each player starts with a personal total of 500. Players take turns: (a) flipping textbook open to any page — randomly; (b) noting the page number on the left; (c) twirling the spinner (or taking a card); (d) multiplying the two numbers together (i.e., page number and spinner number; and (e) if correct (use a calculator to check), subtracting the product from their personal totals. Winner is the player closest to zero after 5 turns each.

3. Have the students make up problems similar to those shown in Exercises 7 and 8 on this pupil page. Encourage them to use the names of classmates. Display them on a bulletin board. Encourage others to solve these extra problems if and/or when they have time.

OBJECTIVE

To multiply whole numbers by tenths

PACING

Level A 1, 2, 4, 5, 7, 8
 Level B 2-8
 Level C 2-8

MATERIALS

flash cards, drill sheets, graph paper for matrices

SUGGESTIONS

Initial Activity Review basic multiplication facts by providing oral drill or flash cards, drill wheels (see page 69, Activity 1), and/or multiplication matrices (see page 64, Activity 2).

Provide an example problem such as:

David has 27 coins in his collection. His mother has 5 times this amount in hers.

How many coins are in David's mother's collection?

Emphasize: The basic idea [coins]; the facts [27 coins, 5 times that amount]; what is being asked; the correct operation [multiplication]. Before completing the calculation, ask "Do you expect David's mother has more or less coins in her collection than David?" Point out that, because the calculation contains 27 (i.e., whole number), 5 (whole number greater than 1), and " \times ", the expected answer will be greater (i.e., 5 times greater) and that therefore David's mother will be seen to have more coins in her collection.

Repeat using similar examples (i.e., whole number (up to 3 digits) greater than one multiplied by a single-digit whole number greater than one) to establish the pattern and process. Then, present a problem such as:

Mr. Hoffman uses 55 L of gasoline each week.

Ms Thomas uses 0.7 of that amount. How much gasoline does she use each week?

Solve the problem, ascertaining the operation is still multiplication (key word is "of") but that the final answer will be *less* than 55 L because 0.7 is less than 1. Repeat with similar examples.

OBJECTIVE

To multiply whole numbers by tenths

PACING

Level A 1-3, 5, 7

Level B 2-8

Level C 2-8

SUGGESTIONS

Initial Activity Continue to provide practice with multiplication facts by the use of drills, flash cards, games, etc.

Go over the problem that is presented in the display at the top of the pupil page. Do several more examples to consolidate the concept.

USING THE BOOK

You may wish to do Exercises 1(a), 3(a), and 5(a) with less able students before assigning the rest of the page. Stop the class periodically and ask them to estimate the answers using the procedures described in the Initial Activity on page 68.

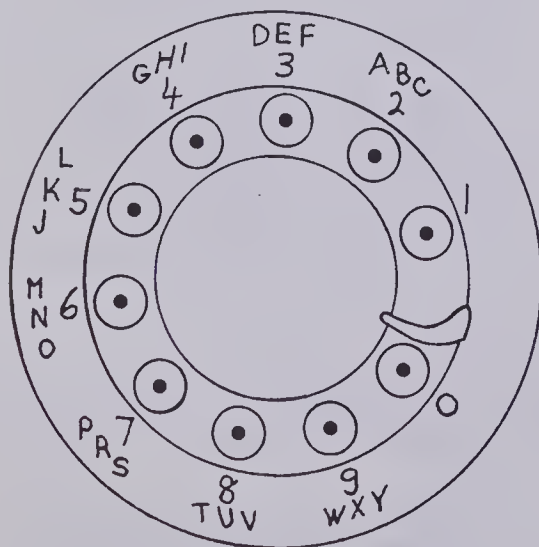
ACTIVITIES

1. If you have not already done so, see Activities 1 and 2 on page 68. Adjust the numbers on the cards accordingly:

312	4.3	112	12.9
-----	-----	-----	------

2. See "500 Grand" in the Activity Reservoir.

3. Make a large telephone dial.



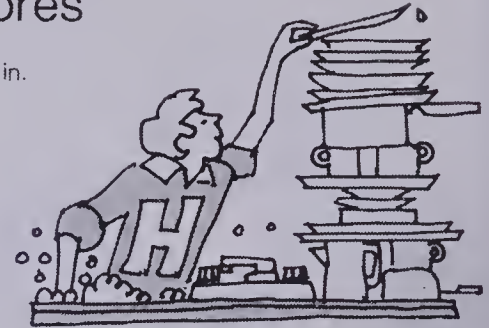
Make out problems such as:

- Dial a 7-letter word. Find out its sum. Multiply the sum by 7.8.
- Dial your first name. Find the sum of your name. Multiply the sum by 24.6.

Daily Chores

On Saturday, Tim washed the supper dishes in 25 min.
Tina took 1.5 times as long to do them on Sunday.
How many minutes did she take?

$$\begin{array}{r} \text{Multiply.} \quad 25 \leftarrow 0 \text{ decimal places} \\ \times 1.5 \leftarrow 1 \text{ decimal place} \\ \hline 125 \\ 250 \\ \hline 37.5 \leftarrow 1 \text{ decimal place} \end{array}$$



She took 37.5 min.

Exercises

Calculate.

- $$\begin{array}{r} 68 \\ \times 1.7 \\ \hline 476 \\ 680 \\ \hline 1156 \end{array}$$
- $$\begin{array}{r} 26 \\ \times 3.4 \\ \hline 884 \\ 60 \\ \hline 4320 \end{array}$$
- $$\begin{array}{r} 85 \\ \times 5.8 \\ \hline 4930 \\ 27 \\ \hline 702 \end{array}$$
- $$\begin{array}{r} 811 \\ \times 3.9 \\ \hline 31629 \end{array}$$
- $$\begin{array}{r} 140 \\ \times 3.5 \\ \hline 4900 \end{array}$$
- $$\begin{array}{r} 485 \\ \times 8.1 \\ \hline 39285 \end{array}$$
- $$\begin{array}{r} 602 \\ \times 5.8 \\ \hline 34916 \end{array}$$
- $$\begin{array}{r} 596 \\ \times 4.7 \\ \hline 28012 \end{array}$$
- $$\begin{array}{r} 811 \\ \times 3.9 \\ \hline 31629 \end{array}$$
- $$\begin{array}{r} 140 \\ \times 3.5 \\ \hline 4900 \end{array}$$
- $$\begin{array}{r} 6.8 \times 4276 \\ 29076.8 \end{array}$$
- $$\begin{array}{r} 9.1 \times 6035 \\ 54918.5 \end{array}$$
- $$\begin{array}{r} 3.7 \times 4270 \\ 15799.0 \end{array}$$
- $$\begin{array}{r} 2.3 \times 6915 \\ 15904.5 \end{array}$$
- $$\begin{array}{r} 583 \\ \times 27.4 \\ \hline \end{array}$$
- $$\begin{array}{r} 462 \\ \times 33.1 \\ \hline \end{array}$$
- $$\begin{array}{r} 329 \\ \times 40.6 \\ \hline \end{array}$$
- $$\begin{array}{r} 517 \\ \times 31.4 \\ \hline \end{array}$$
- $$\begin{array}{r} 730 \\ \times 65.1 \\ \hline \end{array}$$
- $$\begin{array}{r} 2407 \\ \times 3.4 \\ \hline \end{array}$$
- $$\begin{array}{r} 6829 \\ \times 2.6 \\ \hline \end{array}$$
- $$\begin{array}{r} 8160 \\ \times 3.4 \\ \hline \end{array}$$
- $$\begin{array}{r} 3842 \\ \times 2.8 \\ \hline \end{array}$$
- $$\begin{array}{r} 8563 \\ \times 4.5 \\ \hline \end{array}$$
- Karen worked 12 h at the gas station.
Gail worked 3.6 times as long.
How many hours did Gail work? **43.2 h**
- Lois cleaned her room in 44 min.
Gary took 2.4 times as long.
How long did Gary take? **105.6 min**

72 Multiplication of whole numbers by decimals (tenths)

ANSWERS:

5. (a) 15 974.2 (b) 15 292.2 (c) 13 357.4 (d) 16 233.8 (e) 47 523.0
6. (a) 8183.8 (b) 17 755.4 (c) 27 744.0 (d) 10 757.6 (e) 38 533.0

EXTRA PRACTICE

- $$\begin{array}{r} 95 \\ \times 3.4 \\ \hline 323.0 \end{array}$$
- $$\begin{array}{r} 682 \\ \times 9.5 \\ \hline 6479.0 \end{array}$$
- $$\begin{array}{r} 4075 \\ \times 7.3 \\ \hline 29747.5 \end{array}$$
- $$\begin{array}{r} 4824 \\ \times 2.9 \\ \hline 13989.6 \end{array}$$
- $$\begin{array}{r} 276 \\ \times 2.5 \\ \hline 690.0 \end{array}$$
- $$\begin{array}{r} 724 \\ \times 4.4 \\ \hline 3185.6 \end{array}$$
- $$\begin{array}{r} 2217 \\ \times 3.6 \\ \hline 7981.2 \end{array}$$
- $$\begin{array}{r} 380 \\ \times 1.9 \\ \hline 722.0 \end{array}$$
- $$\begin{array}{r} 163 \\ \times 8.2 \\ \hline 1336.6 \end{array}$$
- $$\begin{array}{r} 3482 \\ \times 5.6 \\ \hline 19499.2 \end{array}$$

Hundredths and Thousandths

Multiply

$$\begin{array}{r} 95 \\ \times 0.1 \\ \hline 9.5 \end{array}$$

0 decimal places
1 decimal place
1 decimal place

$$\begin{array}{r} 95 \\ \times 0.01 \\ \hline 0.95 \end{array}$$

0 decimal places
2 decimal places
2 decimal places

$$\begin{array}{r} 395 \\ \times 0.001 \\ \hline 0.395 \end{array}$$

0 decimal places
3 decimal places
3 decimal places

The sum of the number of decimal places in the factors is equal to the number of decimal places in the product.

Exercises

1. (a) $\begin{array}{r} 57 \\ \times 0.01 \\ \hline 0.57 \end{array}$

0 decimal places
2 decimal places
2 decimal places

(c) $\begin{array}{r} 63 \\ \times 0.01 \\ \hline 0.63 \end{array}$

(d) $\begin{array}{r} 83 \\ \times 0.01 \\ \hline 0.83 \end{array}$

2. (a) $0.01 \times 49 = 0.49$ (b) $0.01 \times 97 = 0.97$ (c) $0.01 \times 56 = 0.56$ (d) $0.01 \times 39 = 0.39$

3. $0.01 \times 483 = 4.83$ (b) $0.01 \times 857 = 8.57$ (c) $0.01 \times 924 = 9.24$ (d) $0.01 \times 684 = 6.84$

4. (a) $\begin{array}{r} 83 \\ \times 0.001 \\ \hline 0.083 \end{array}$

0 decimal places
3 decimal places
3 decimal places

(b) $\begin{array}{r} 65 \\ \times 0.001 \\ \hline 0.065 \end{array}$

(c) $\begin{array}{r} 96 \\ \times 0.001 \\ \hline 0.096 \end{array}$

(d) $\begin{array}{r} 55 \\ \times 0.001 \\ \hline 0.055 \end{array}$

5. $\begin{array}{r} 834 \\ \times 0.001 \\ \hline 0.834 \end{array}$ (b) $\begin{array}{r} 208 \\ \times 0.001 \\ \hline 0.208 \end{array}$ (c) $\begin{array}{r} 769 \\ \times 0.001 \\ \hline 0.769 \end{array}$ (d) $\begin{array}{r} 153 \\ \times 0.001 \\ \hline 0.153 \end{array}$ (e) $\begin{array}{r} 429 \\ \times 0.001 \\ \hline 0.429 \end{array}$

6. $0.001 \times 7491 = 7.491$ (b) $0.001 \times 4685 = 4.685$ (c) $0.001 \times 4876 = 4.876$
(d) $0.001 \times 5876 = 5.876$ (e) $0.001 \times 3471 = 3.471$ (f) $0.001 \times 2008 = 2.008$

OBJECTIVE

To multiply by 0.01 and 0.001

PACING

Level A All

Level B All

Level C All

SUGGESTIONS

Initial Activity Review previous work done on multiplying by 0.1 (page 70). Have the children study the display at the top of the page. From the examples students should be asked to identify (a) the relationship between the number of decimal places in the factors and the number of decimal places in the product and (b) the relationship between the numerals in the first factor and the numerals in the product (they are the same numerals with the addition of the 0 as a placeholder in the product).

USING THE BOOK

All or more exercises can be done orally. Alternatively have students write the products only.

Multiplying by 0.01 and 0.001 73

ACTIVITIES

1. Play "Beat the Clock" by preparing examples similar to the ones in the text. Students fill in answers only. They are given a time limit and try to "Beat the Clock".

2. Write examples which show the multiplicand and the product only. Students have to determine the multiplier.

Example
 $27 \times \underline{\quad} = 2.7$
 $31 \times \underline{\quad} = 0.31$ etc.

3. Make up 2 stacks of cards. One stack has cards which have 0.1 or 0.01 or 0.001 written on them. The

other stack has whole numbers. Both stacks are shuffled and placed face down. Each player in turn takes the top card off each stack and multiplies the numbers. If he does it successfully he wins a point. The one with the most points at the end of the time limit wins.

EXTRA PRACTICE

1. $\begin{array}{r} 52 \\ \times 0.01 \\ \hline [0.52] \end{array}$ 2. $\begin{array}{r} 376 \\ \times 0.01 \\ \hline [3.76] \end{array}$ 3. $\begin{array}{r} 483 \\ \times 0.001 \\ \hline [0.483] \end{array}$

4. $\begin{array}{r} 19 \\ \times 0.1 \\ \hline [1.9] \end{array}$ 5. $\begin{array}{r} 283 \\ \times 0.001 \\ \hline [0.283] \end{array}$ 6. $\begin{array}{r} 4890 \\ \times 0.001 \\ \hline [4.890] \end{array}$

OBJECTIVE

To multiply whole numbers by 2-digit decimals (hundredths)

PACING

Level A 1-7
Level B 3-8
Level C 3-8

RELATED AIDS

HMS — DM19.
BFA COMP LAB II — 104.
BFA PROB. SOLVING LAB II — 138.

SUGGESTIONS

Initial Activity Remind students that the number of decimal places in the factors is equal to the number of decimal places in the product. Have the children study the display at the top of the page to ascertain how this applies to the examples given.

USING THE BOOK

Have your more able students do as many questions as they can writing only the product. They will not need to do as many questions as your less able students and should be assigned a puzzle or interesting problem to do when they have completed their assigned exercises. Your less able students may require two days to complete their work.

ACTIVITIES

1. Make a collection of items such as a candy bar, toothpaste, packet of spices, etc. Do not show prices. Each student is given an imaginary twenty dollars to spend. They are allowed to buy what they want from the display. They may buy more than one of any item. The student who comes closest to spending his twenty dollars without going over wins.

2. Have the children prepare for an imaginary trip to a camp for a weekend. Using newspaper advertisements, plan the kind of food, the quantity of each item, and the cost of the food for the trip.

3. Have the students design a poster advertising something for sale. Underneath they write one or more word problems pertaining to their advertisement. The posters can be displayed and the word problems solved by other class members.

Multiplying by Hundredths

Multiply

$$\begin{array}{r} 18 \\ \times 0.48 \\ \hline 144 \\ 720 \\ \hline 864 \end{array}$$

18 ← 2 decimal places
× 0.48 ← 2 decimal places
864 ← 2 decimal places

$$\begin{array}{r} 63 \\ \times 2.47 \\ \hline 441 \\ 2520 \\ 12600 \\ \hline 15561 \end{array}$$

63 ← 2 decimal places
× 2.47 ← 2 decimal places
15561 ← 2 decimal places

Exercises

Copy and complete.

- 0.25×19 There are 2 decimal places in the product.
4.75
- 0.57×536 There are 2 decimal places in the product.
305.52
- | | | | |
|---|--|--|---|
| $\begin{array}{r} 19 \\ \times 0.12 \\ \hline 38 \\ 190 \\ \hline 2.28 \end{array}$ | $\begin{array}{r} 27 \\ \times 0.37 \\ \hline 189 \\ 810 \\ \hline 9.99 \end{array}$ | $\begin{array}{r} 43 \\ \times 0.48 \\ \hline 344 \\ 1740 \\ \hline 20.64 \end{array}$ | $\begin{array}{r} 2.42 \times 25 \\ \hline 60.50 \end{array}$ |
| | $\begin{array}{r} 14 \\ \times 0.56 \\ \hline 84 \\ 700 \\ \hline 7.84 \end{array}$ | $\begin{array}{r} 13 \\ \times 0.52 \\ \hline 104 \\ 650 \\ \hline 6.76 \end{array}$ | $\begin{array}{r} 0.49 \times 13 \\ \hline 6.37 \end{array}$ |
- | | | | |
|--|---|---|---|
| $\begin{array}{r} 27 \\ \times 3.42 \\ \hline 198 \\ 810 \\ 5400 \\ \hline 92.34 \end{array}$ | $\begin{array}{r} 38 \\ \times 1.36 \\ \hline 228 \\ 496 \\ 5072 \\ \hline 51.68 \end{array}$ | $\begin{array}{r} 45 \\ \times 6.27 \\ \hline 270 \\ 900 \\ 27000 \\ \hline 282.15 \end{array}$ | $\begin{array}{r} 84 \\ \times 8.41 \\ \hline 336 \\ 672 \\ 67200 \\ \hline 706.44 \end{array}$ |
| $\begin{array}{r} 51 \\ \times 2.59 \\ \hline 255 \\ 1020 \\ 13050 \\ \hline 132.09 \end{array}$ | | | |
- | | | | |
|--|--|---|--|
| $\begin{array}{r} 452 \times 85 \\ \hline 38420 \end{array}$ | $\begin{array}{r} 676 \times 41 \\ \hline 27716 \end{array}$ | $\begin{array}{r} 0.95 \times 73 \\ \hline 69.35 \end{array}$ | $\begin{array}{r} 8.64 \times 49 \\ \hline 423.36 \end{array}$ |
|--|--|---|--|
- | | | | |
|---|--|--|--|
| $\begin{array}{r} 835 \\ \times 2.81 \\ \hline 6680 \\ 18700 \\ 234635 \\ \hline 2346.35 \end{array}$ | $\begin{array}{r} 902 \\ \times 0.59 \\ \hline 51118 \\ \hline 532.18 \end{array}$ | $\begin{array}{r} 685 \\ \times 5.37 \\ \hline 36245 \\ 20100 \\ 367845 \\ \hline 3678.45 \end{array}$ | $\begin{array}{r} 418 \\ \times 1.74 \\ \hline 2924 \\ 7104 \\ 72732 \\ \hline 727.32 \end{array}$ |
| $\begin{array}{r} 853 \\ \times 0.85 \\ \hline 6824 \\ 68240 \\ 72505 \\ \hline 725.05 \end{array}$ | | | |
- | | | | |
|---|--|---|--|
| $\begin{array}{r} 4873 \\ \times 0.45 \\ \hline 24365 \\ 194910 \\ 219285 \end{array}$ | $\begin{array}{r} 3321 \\ \times 0.68 \\ \hline 21348 \\ 209808 \\ 565828 \end{array}$ | $\begin{array}{r} 5382 \\ \times 0.76 \\ \hline 323352 \\ 375024 \\ 409032 \end{array}$ | $\begin{array}{r} 6350 \\ \times 0.78 \\ \hline 50800 \\ 508000 \\ 495300 \end{array}$ |
| $\begin{array}{r} 9476 \\ \times 4.53 \\ \hline 47888 \\ 428160 \\ 4292628 \end{array}$ | | | |

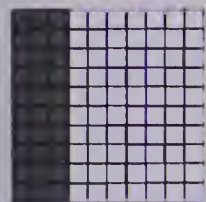
74 Multiplying whole numbers by two decimals

EXTRA PRACTICE

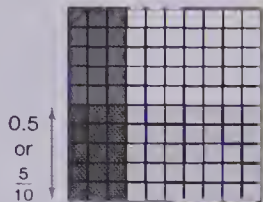
- | | | |
|--|---|---|
| 1. $\begin{array}{r} 14 \\ \times 0.28 \\ \hline [3.92] \end{array}$ | 2. $\begin{array}{r} 36 \\ \times 3.43 \\ \hline [123.48] \end{array}$ | 3. $\begin{array}{r} 493 \\ \times 7.51 \\ \hline [3702.43] \end{array}$ |
| 4. $\begin{array}{r} 982 \\ \times 4.36 \\ \hline [4281.52] \end{array}$ | 5. $\begin{array}{r} 4327 \\ \times 0.72 \\ \hline [3115.44] \end{array}$ | 6. $\begin{array}{r} 9916 \\ \times 0.85 \\ \hline [8428.60] \end{array}$ |

Multiplying Tenths

Multiply 0.3×0.5 .



0.3 or $\frac{3}{10}$



0.5
or
 $\frac{5}{10}$

0.3

0.5 ← 1 decimal place
 $\times 0.3$ ← 1 decimal place

0.15 ← 2 decimal places

$$0.3 \times 0.5 = 0.15$$

three tenths \times five tenths = fifteen hundredths

The sum of the number of decimal places in the factors is equal to the number of decimal places in the product.

Exercises
Multiply.

1. (a) 0.7
 $\times 0.4$

0.28

(b) 0.6
 $\times 0.3$

0.18

(c) 0.5
 $\times 0.2$

0.10

(d) 0.8
 $\times 0.7$

0.56

(e) 0.9
 $\times 0.2$

0.18

2. (a) 0.1×0.4
0.04

(b) 0.2×0.3
0.06

(c) 0.8×0.1
0.08

(d) 0.3×0.3
0.09

(e) 0.2×0.4
0.08

3. (a) 17.1
 $\times 0.5$

8.55

(b) 6.7
 $\times 0.3$

2.01

(c) 9.5
 $\times 0.6$

5.70

(d) 8.2
 $\times 0.1$

0.82

(e) 5.1
 $\times 0.9$

4.59

4. (a) 36.5
 $\times 0.3$

10.95

(b) 91.7
 $\times 0.8$

73.36

(c) 82.4
 $\times 0.8$

65.92

(d) 60.3
 $\times 0.3$

18.09

(e) 95.9
 $\times 0.9$

86.31

BRAINTICKLER

Find the rule:

When Jim said 10, Tom said 1.

When Jim said 5, Tom said one half.

When Jim said 1, Tom said one tenth.

Multiplying tenths by tenths 75

ANSWERS:

Braintickler: Multiply by 0.1.

USING THE BOOK

Discuss the display at the top of the page, noting its similarity to what took place in the Initial Activity demonstration.

Do Exercises 1 and 2 orally with the students. Do Exercise 3(a) on the board, drawing attention to the necessity to insert a zero as a placeholder. Assign the rest of the page.

ACTIVITIES

1. Have the students complete a matrix such as:

\times	0.1	0.2	0.3	etc.
0.1				
0.2				
0.3				
etc.				

2. Have the children, in threes, continue drawing overlaps as outlined in the Initial Activity.

3. To consolidate the multiplication skills presented thus far, see the "Coded Riddles" idea listed in the Activity Reservoir.

OBJECTIVE

To multiply tenths by tenths

PACING

Level A All

Level B All

Level C All

MATERIALS

2 overhead (10 cm \times 10 cm) transparencies showing



100 pennies, graph paper (see DM69)

RELATED AIDS

BFA COMP LAB II — 103.

SUGGESTIONS

Initial Activity Pose a riddle such as: "I am a small coin. I am one tenth of one tenth. What am I?" After you receive the answer, review the steps taken to arrive at the solution.

What is the whole unit of money? [one dollar]

What is one tenth of a dollar? [a dime]

What is one tenth of a dime? [a cent]

What fraction of a dollar is a cent? [one hundredth]

Remind students that "of" and " \times " mean the same in multiplication. Thus one tenth \times one tenth is one hundredth. You may wish to demonstrate using actual pennies or representatives of 100 pennies on the chalkboard.

On the overhead projector, show one of the transparencies which have a 10 cm square divided into tenths. Use a watercolour marker to colour in one tenth. Do the same thing to the second transparency but use a different colour marker. Overlap the two transparencies so that they show hundredths. Point out that a numerical way of showing this is:

$$\begin{array}{r} 0.1 \leftarrow 1 \text{ decimal place} \\ \times 0.1 \leftarrow 1 \text{ decimal place} \\ \hline 01 \\ 000 \\ \hline 0.01 \leftarrow 2 \text{ decimal places} \end{array}$$

Repeat this procedure several times, each time showing the graphic and numerical forms of the demonstration together.

(Continued on page 77)

OBJECTIVES

To provide practice in multiplying decimal numbers by decimal numbers involving tenths
To introduce the distributive property of multiplication

PACING

Level A All
Level B All
Level C All

RELATED AIDS

BFA COMP LAB II — 103.
CALC. ACTIVITY MASTERS — 68.

SUGGESTIONS

Initial Activity Go over examples such as those shown in the display at the top of the page. Reinforce the rule that the sum of the number of decimal places in the factors is equal to the sum of the number of places in the product.

Present a problem such as:
Gina and Paul use the same bike for delivering papers.

They each have their own, separate paper route.

She travels 3.4 km each day; he travels 2.8 km each day.

How far would that bike travel in six days?

Point out that there are *two* ways of finding the answer:

<i>Gina</i>		<i>Paul</i>
$6 \times (3.4 + 2.8)$	$6 \times 3.4 +$	6×2.8
↓	or	↓
$6 \times (6.2)$		$20.4 +$
↓		↓
37.2		16.8
		↓
		37.2

Repeat using different numbers and situations (i.e., training for a cross-country run, swimming lengths, length of ski runs, etc.).

USING THE BOOK

Allow students to solve any 2 questions in Exercises 1 to 6. The choice is theirs. Also, be sure to remind the students about the order of operations rules (i.e., Brackets first; then Multiplication, Division, Addition, Subtraction in the order that they appear). They will need this information before attempting Exercises 7 to 10. These exercises should be completed by all. Be certain, when the work has been corrected to point out the relationship

More Decimal Multiplication

Multiply.

$$\begin{array}{r} 3.4 \leftarrow 1 \text{ decimal place} \\ \times 1.4 \leftarrow 1 \text{ decimal place} \\ \hline 136 \\ 340 \\ \hline 4.76 \leftarrow 2 \text{ decimal places} \end{array}$$

$$\begin{array}{r} 13.2 \leftarrow 1 \text{ decimal place} \\ \times 3.5 \leftarrow 1 \text{ decimal place} \\ \hline 660 \\ 3960 \\ \hline 46.20 \leftarrow 2 \text{ decimal places} \end{array}$$

$$\begin{array}{r} 476.7 \leftarrow 1 \text{ decimal place} \\ \times 3.1 \leftarrow 1 \text{ decimal place} \\ \hline 4767 \\ 143010 \\ \hline 1477.77 \leftarrow 2 \text{ decimal places} \end{array}$$

Exercises

<p>1. \bullet $\begin{array}{r} 4.6 \\ \times 3.2 \\ \hline 92 \\ 1380 \\ \hline 14.72 \end{array}$</p> <p>3. \bullet $\begin{array}{r} 12.6 \\ \times 5.8 \\ \hline 73.08 \end{array}$</p> <p>4. (a) $\begin{array}{r} 3.3 \times 48.6 \\ \hline 160.38 \end{array}$</p> <p>5. \bullet $\begin{array}{r} 397.5 \\ \times 6.2 \\ \hline 2464.50 \end{array}$</p> <p>6. (a) 4.7×760.5 $\hline 3574.35$</p>	<p>\bullet $\begin{array}{r} 9.5 \\ \times 4.1 \\ \hline 38.95 \end{array}$</p> <p>(d) $\begin{array}{r} 9.5 \\ \times 8.1 \\ \hline 76.95 \end{array}$</p> <p>(b) $\begin{array}{r} 43.9 \\ \times 2.7 \\ \hline 118.53 \end{array}$</p> <p>(b) $\begin{array}{r} 5.4 \times 53.9 \\ \hline 291.06 \end{array}$</p> <p>(b) $\begin{array}{r} 402.8 \\ \times 3.1 \\ \hline 1248.68 \end{array}$</p> <p>(b) 9.3×581.4 $\hline 5407.02$</p>	<p>(c) $\begin{array}{r} 6.6 \\ \times 5.8 \\ \hline 38.28 \end{array}$</p> <p>(e) $\begin{array}{r} 4.9 \\ \times 5.9 \\ \hline 28.91 \end{array}$</p> <p>(c) $\begin{array}{r} 65.1 \\ \times 4.3 \\ \hline 279.93 \end{array}$</p> <p>(c) $\begin{array}{r} 981.3 \\ \times 9.5 \\ \hline 9322.35 \end{array}$</p> <p>(c) 6.5×800.4 $\hline 5202.60$</p>
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Calculate. Compare the answers in each pair.

7. (a) $2 \times (3.1 + 1.4)$ 9	8. (a) $4 \times (5.2 + 6.7)$ 47.6	<p>Remember:</p> <p>1st. Do operations in brackets.</p> <p>2nd. Do multiplication then addition.</p>
(b) $2 \times 3.1 + 2 \times 1.4$ 9	(b) $4 \times 5.2 + 4 \times 6.7$ 47.6	
9. (a) $1.3 \times (2.1 + 3.2)$ 6.89	10. (a) $3.2 \times (1.5 + 4.6)$ 19.52	
(b) $1.3 \times 2.1 + 1.3 \times 3.2$ 6.89	(b) $3.2 \times 1.5 + 3.2 \times 4.6$ 19.52	

76 Multiplying decimals, distributive property

between the answers for the (a) and (b) parts of Exercises 7 to 10.

ACTIVITIES

1. Using a metronome, set an appropriate speed to call out multiplication facts in time to the metronome. Students write answers at the speed dictated.

2. Play "Teacher Torture". Students make up questions for you, the teacher, to answer. If you answer correctly, you get a point; if you are wrong, the students must supply the

correct answer before claiming a point.

3. See "Input-Output" as described in the Activity Reservoir.

EXTRA PRACTICE

1. (a) 9.6×32.5 [312.00]
(b) 48.7×3.6 [175.32]
(c) 150.9×4.8 [724.32]
(d) 374.7×5.3 [1985.91]
(e) 333.3×3.3 [1099.89]
2. (a) $6 \times (4.1 + 2.8)$ [41.40]
(b) $6 \times 4.1 + 6 \times 2.8$ [41.40]
3. (a) $3.7 \times (3.1 + 5.6)$ [32.19]
(b) $3.7 \times 3.1 + 3.7 \times 5.6$ [32.19]

Multiplying Two Decimal Numbers

Multiply

$$\begin{array}{r} 27.3 \leftarrow 1 \text{ decimal place} \\ \times 2.27 \leftarrow 2 \text{ decimal places} \\ \hline \end{array}$$

$$\begin{array}{r} 1911 \\ 5460 \\ 54600 \\ \hline 61.971 \leftarrow 3 \text{ decimal places} \end{array}$$

Check by rounding $2 \times 30 = 60$.

$$\begin{array}{r} 9.32 \leftarrow 2 \text{ decimal places} \\ \times 1.16 \leftarrow 2 \text{ decimal places} \\ \hline \end{array}$$

$$\begin{array}{r} 5592 \\ 9320 \\ 93200 \\ \hline 10.8112 \leftarrow 4 \text{ decimal places} \end{array}$$

Check by rounding $1 \times 9 = 9$.

Exercises

1. How many decimal places in each?

● 32.3 1 ● 4.175 3 (c) 234.58 2 (d) 0.001 3 (e) 5.7684 4

2. Calculate. Check by rounding.

● 38.4 Check 40 ● 74.7 Check 70 ● 85.3 90 (d) 93.6 90
 $\times 2.76$ $\times 3$ $\times 4.35$ $\times 4$ $\times 6.28$ $\times 6$ $\times 2.17$ $\times 2$
 105.984 120 324.945 280 535.684 540 203.112 180

3. ● 45.73 ● 64.85 (c) 14.63 (d) 76.58 (e) 43.37
 $\times 2.36$ $\times 0.14$ $\times 5.61$ $\times 4.45$ $\times 6.06$
 107.9228 9.0790 82.0743 340.7810 262.8222

(f) 5.92×60.49 (g) 3.08×94.17 (h) 0.63×27.14
 358.1008 290.0436 17.0982

4. (a) 6.35×4.58 (b) 76.3×1.86 (c) 5.09×175.4
 29.0830 141.918 892.786
(d) 6.42×5.37 (e) 3.01×9.75 (f) 8.17×2.01
 34.4754 29.3475 16.4217

5. ● 1.3 ● 4.27 (c) 6.83 (d) 5.08 (e) 63.4
 $\times 15.37$ $\times 10.75$ $\times 27.62$ $\times 72.97$ $\times 40.91$
 19.981 45.9025 188.6446 370.6876 2593.694

Multiplying two decimals 77

OBJECTIVE

To multiply hundredths by hundredths

PACING

Level A 1-4

Level B 1-4

Level C 2-5

RELATED AIDS

HMS — DM20.

BFA COMP LAB II — 105-107.

CALC. ACTIVITY MASTERS — 79.

SUGGESTIONS

Initial Activity Continue to do drills and game activities which involve quick recall of multiplication facts.

Have the children study the display at the top of the page. Draw their attention to the relationship between the number of decimal places in the question and number of decimal places in the answer.

USING THE BOOK

You may prefer to do Exercise 1 orally with the students. With less able students you may wish to do Exercises 2(a) and 3(a) on the board before assigning the rest of the row.

ACTIVITIES

1. See "500 Grand" in the Activity Reservoir.

2. Provide partially blank multiplication questions to play "Tic Tac Times".

$$\begin{array}{r} 47.6 \\ \times 2.38 \\ \hline \square \square \square 8 \\ 1 \square \square \square 0 \\ 9 \square \square \square 0 \\ \hline 113.288 \end{array}$$

Players fill in appropriate correct numbers. "Tic Tac Toe" rules apply. The first person to write three numerals horizontally, vertically, or diagonally gets the product. The player with the highest product total after 5 games wins.

3. Have the children (using calculators to check) help make up partially blank "Tic Tac Times" sheets for Activity 2 above. Have them vary the multiplicands and multipliers (i.e., number of decimal places, etc.).

(Continued from page 75)

Provide graph paper (or use DM69) and divide the class into groups of 3. Have the first student colour some portion (less than one but in tenths) of a 10×10 grid vertically. The second student does likewise but horizontally so that this second colouring job overlaps the first.

The third student writes the appropriate computation and states in hundredths what the product or overlap is. Have the children change roles so that each has a turn performing each of the three steps.

OBJECTIVE

To multiply using thousandths and ten thousandths

PACING

Level A 1-3
Level B 2-4
Level C 2-5

RELATED AIDS

BFA COMP LAB II — 108, 109.

SUGGESTIONS

Initial Activity Go over the examples in the display at the top of the page drawing attention to the number of decimal places in the question and the corresponding number of decimal places in the answer. With less able students you may also wish to draw particular attention to the second example which shows multiplication by 4 digits.

USING THE BOOK

Assign the exercises. You may wish to complete several examples orally with some of the students. Less able students might also benefit from completing their work on graph paper, to help keep columns of numbers properly aligned.

ACTIVITIES

1. Divide the class into 2 teams. Each member of a team makes up a question similar to the ones in the exercise. At a given signal each member passes his question to a member of the opposing team who solves it. The team with the largest number of correct solutions wins.

2. If you have not already done so, see the "Tic Tac Times" idea described in Activity 2 on page 77.

3. See the "Square It" idea described in the Activity Reservoir.

EXTRA PRACTICE

1. (a) 59×0.146 [8.614] (b) 382×0.147 [56.154] (c) 29×8.176 [237.104]
(d) 449×3.156 [1417.044] (e) 948×2.685 [2545.380]

More Decimal Places

Multiply.

$$\begin{array}{r} 382 \leftarrow 0 \text{ decimal places} \\ \times 0.476 \leftarrow 3 \text{ decimal places} \\ \hline 2292 \\ 26740 \\ 152800 \\ \hline 181.832 \leftarrow 3 \text{ decimal places} \end{array}$$

$$\begin{array}{r} 94.3 \leftarrow 1 \text{ decimal place} \\ \times 0.1763 \leftarrow 4 \text{ decimal places} \\ \hline 2829 \\ 56580 \\ 660100 \\ 943000 \\ \hline 16.62509 \leftarrow 5 \text{ decimal places} \end{array}$$

Exercises

- | | | | | |
|--|--|--|---|---|
| 1. 82×0.001
0.082 | 2. 57×0.014
0.798 | 3. 385×0.407
156.695 | (d) 902×0.239
215.578 | (e) 4483×0.746
3344.318 |
| 4. (a) 37×0.0761
2.8157 | (b) 85×0.1358
11.5430 | (c) 368×0.2074
76.3232 | (d) 485×0.0003
0.1455 | (e) 8563×0.0261
223.4943 |
| 5. 48×4.176
200.448 | (b) 25×6.284
157.100 | (c) 603×8.481
5114.043 | (d) 764×2.702
2064.328 | (e) 946×5.817
5502.882 |
| 6. 13.74×0.762
10.46988 | (b) 26.51×8.108
214.94308 | (c) 81.04×9.121
739.16584 | (d) 60.43×0.009
0.54387 | |
| 7. (a) 5.3×0.4009
2.12477 | (b) 7.5×0.0174
0.13050 | (c) 46.5×2.1605
100.46325 | (d) 13.7×4.1267
56.53579 | |

BRAINTICKLER

Multiply 37 by each of these numbers: 3, 6, 9, 12.
Your products will be interesting. Three more numbers larger than 12 continue the pattern. What are they?



78 Multiply decimals by 3 and 4 digit decimal.

ANSWERS:

Braintickler: 111, 222, 333, 444
15, 18, 21

Tune Up

Calculate.

1. (a) 9×7 **63** (b) 3×8 **24** (c) 6×0 **0** (d) 7×5 **35** (e) 4×9 **36**
2. (a) 3×14 **42** (b) 9×28 **252** (c) 7×65 **455** (d) 1×84 **84** (e) 6×93 **558**
3. (a) 23×47 **1081** (b) 25×87 **2175** (c) 68×41 **2788** (d) 51×32 **1632** (e) 35×13 **455**
4. (a) 78×448 **34 944** (b) 87×379 **32 973** (c) 35×107 **3745** (d) $29 \times \$2.83$ **\$82.07** (e) $84 \times \$9.13$ **\$766.92**
5. (a) 205×876 **179 580** (b) 632×449 **283 768** (c) 902×817 **736 934** (d) 430×259 **111 370** (e) 106×365 **38 690**
6. (a) 10×64 **640** (b) 100×83 **8300** (c) $1 \times \$8.37$ **\$8.37** (d) $1000 \times \$6.82$ **\$6820.00**

Calculate. Compare the answers in each pair.

7. (a) $7 \times 6 = 42$ (b) 6×7 **42** 8. (a) 15×3 **45** (b) 3×15 **45**
9. (a) 100×56 **5600** (b) 56×100 **5600** 10. (a) $6 \times (2.3 + 1.9)$ **25.2** (b) $6 \times 2.3 + 6 \times 1.9$ **25.2**
11. (a) $4 \times (3.7 + 2.6)$ **25.2** 12. (a) $1.8 \times (0.5 + 3.7)$ **7.56** 13. (a) $7.1 \times (3.3 + 1.0)$ **30.53**
(b) $4 \times 3.7 + 4 \times 2.6$ **25.2** (b) $1.8 \times 0.5 + 1.8 \times 3.7$ **7.56** (b) $7.1 \times 3.3 + 7.1 \times 1.0$ **30.53**

Calculate.

14. (a) $2 \times \$7.34$ **\$14.68** (b) $6 \times \$3.75$ **\$22.50** (c) $3 \times \$27.48$ **\$82.44** (d) $5 \times \$352.15$ **\$1760.75**
15. (a) 0.1×25 **2.5** (b) 0.4×583 **233.2** (c) 0.7×351 **245.7** (d) 0.6×7685 **4611.0**
16. (a) 4.8×97 **465.6** (b) 6.7×387 **2592.9** (c) 4.5×596 **2682.0** (d) 3.8×3852 **14 637.6**
17. (a) 0.01×3 **0.03** (b) 0.001×481 **0.481** (c) 0.01×582 **5.82** (d) 0.001×693 **0.693**
18. (a) 4.76×7 **33.32** (b) 6.85×93 **637.05** (c) 0.76×4735 **3598.60** (d) 3.03×517 **1566.51**
19. (a) 0.9×0.5 **0.45** (b) 0.5×4.6 **2.30** (c) 0.4×60.5 **24.20** (d) 0.7×99.6 **69.72**
20. (a) 5.7×6.8 **38.76** (b) 7.3×30.7 **224.11** (c) 8.1×573.7 **4646.97** (d) 6.2×600.8 **3724.96**
21. (a) 3.56×16.4 **58.384** (b) 1.67×13.7 **22.879** (c) 5.01×26.43 **132.4143** (d) 2.76×10.71 **29.5596**

Practice 79

OBJECTIVE

To provide practice in multiplication

PACING

- Level A All the rows, but not necessarily all the examples
- Level B All the rows, but not necessarily all the examples
- Level C The even number rows, but not all the examples

SUGGESTIONS

Initial Activity On the board, with the students, do one example of each type of multiplication presented in the exercise. This will review the concepts taught thus far.

USING THE BOOK

Rows 1 and 2 may be done orally with the students. If all the examples are to be done by the students, do this page over 2 and maybe even 3 lessons, depending on the ability of your students. Alternatively, allow your students to choose any 2 questions to solve in each row. Ask them to write down in advance the number of questions they think they will answer correctly.

Provide appropriate remedial work for students who experience difficulty with any of the kinds of questions in this exercise.

ACTIVITIES

1. Provide graph paper. Have the students graph their results.
2. Copy some of the exercises onto cards. Use this deck to play "Football" as described in the Activity Reservoir.
3. As above, use some of the exercises printed onto cards to play "Road Rally" as described in the Activity Reservoir.

OBJECTIVE

To solve word problems involving multiplication

PACING

Level A 1-6

Level B All

Level C 2-8

VOCABULARY

contestant, spectator

MATERIALS

collection of stories and articles pertaining to sports, feats from ancient Greek myths, modern day events; Guinness Book of World Records

RELATED AIDS

BFA PROB. SOLVING LAB II — 89, 93, 95, 97, 101, 113

SUGGESTIONS

Initial Activity Tell students the story of Pheidippides the Athenian messenger who died after running from Marathon to Athens to tell of a victory over the Persians. His run is the origin of our marathon races. Ask students to contribute what knowledge they have of famous athletes and their achievements in the realm of sports.

USING THE BOOK

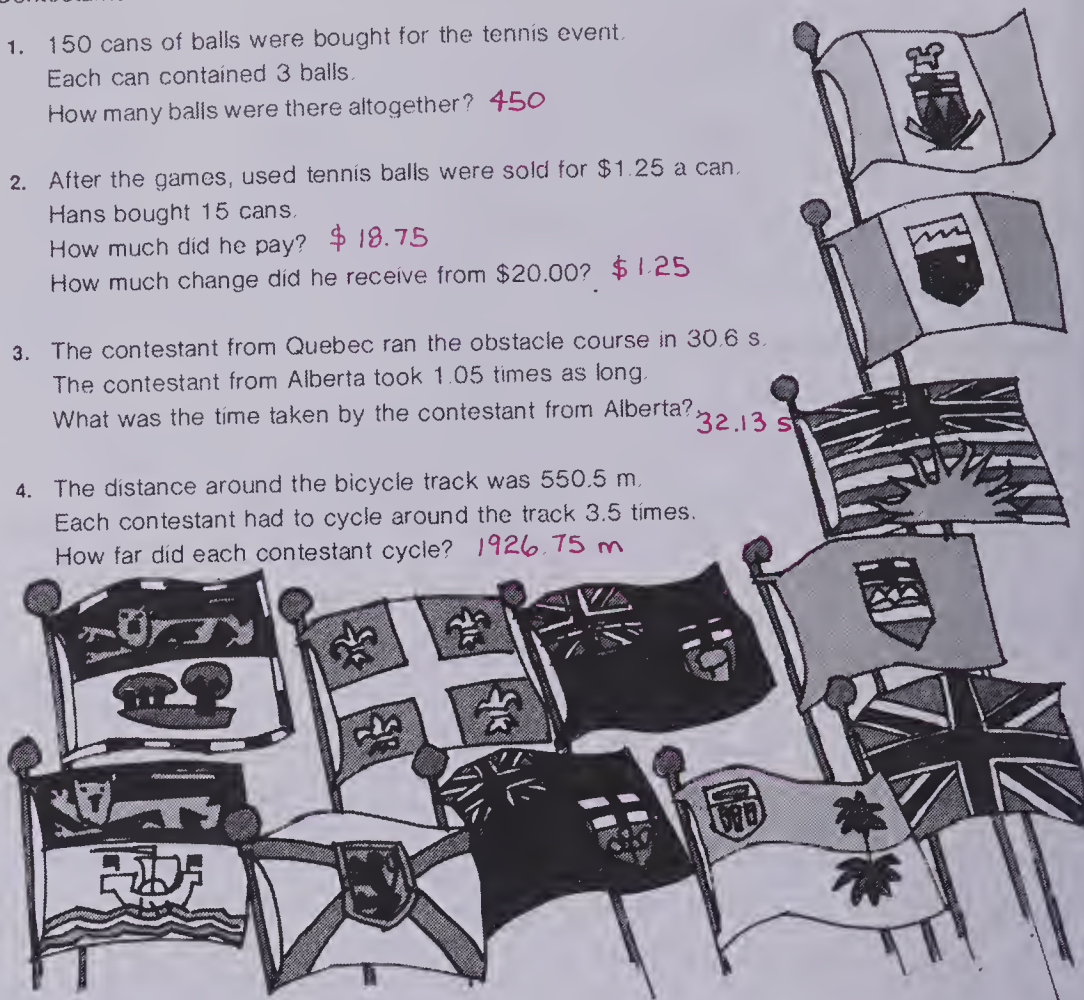
Ascertain that all students understand the words used in each problem before assigning them. You may wish to do the first problem on the board to review the format you may wish to have them follow for setting down word problems (see Professor Q, pages 17 and 22).

Exercise 7 involves the computation of average. You may wish to conduct a brief review of average as it pertains to average score over 3 or 4 attempts and how to calculate it (i.e., total number of points divided by number of tries equals average points per try).

Canada Games

Contestants from all the provinces were gathered to compete in a variety of events.

1. 150 cans of balls were bought for the tennis event.
Each can contained 3 balls.
How many balls were there altogether? **450**
2. After the games, used tennis balls were sold for \$1.25 a can.
Hans bought 15 cans.
How much did he pay? **\$ 18.75**
How much change did he receive from \$20.00? **\$ 1.25**
3. The contestant from Quebec ran the obstacle course in 30.6 s.
The contestant from Alberta took 1.05 times as long.
What was the time taken by the contestant from Alberta? **32.13 s**
4. The distance around the bicycle track was 550.5 m.
Each contestant had to cycle around the track 3.5 times.
How far did each contestant cycle? **1926.75 m**

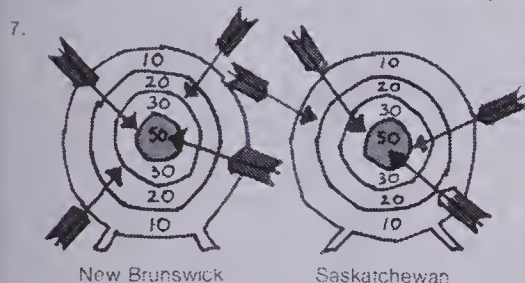


5. There were rows of seats for spectators to watch the swimming event.
There were 435 rows.
Each row had 26 seats.
How many seats altogether? **11 310**

6.

Province	Score			Total
Ontario	362	389	405	
B.C.	385	419	415	

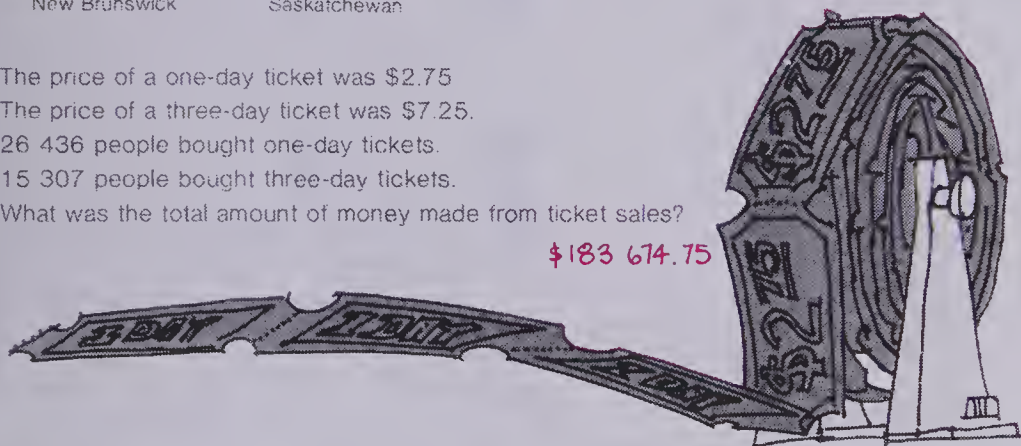
In bowling, what was the total score of the contestant from Ontario? **1156**
What was the total score of the contestant from B.C.? **1219**
Who won and by how much? **B.C. by 63**



What was the total score for the contestant from New Brunswick? **120**
What was the average score? **30**
What was the total score for the contestant from Saskatchewan? **120**
What was the average score? **30**
Who won? **It was a tie.**

8. The price of a one-day ticket was \$2.75
The price of a three-day ticket was \$7.25.
26 436 people bought one-day tickets.
15 307 people bought three-day tickets.
What was the total amount of money made from ticket sales?

\$183 674.75



ACTIVITIES

1. Make collages from cut-up magazine pictures depicting sports.
2. Students should be encouraged to do further reading on athletics and athletes from the collection of books you have assembled.
3. You may wish to have students compose short verses about athletes and their feats.

Example

A limerick:

There once was a sprinter from B.C.
Who developed severe "water on the knee"

My running days are done
Now I won't have any fun
Said the poor sprinter from B.C.

4. To provide practice in choosing the correct operation for word problems, see Activities 1 and 2 for pages 22 and 23.

OBJECTIVES

To show a model that illustrates the distributive property of multiplication

To provide practice in using the distributive property

PACING

Level A 1, 2

Level B 1, 2

Level C All

RELATED AIDS

CALC. ACTIVITY MASTERS — 68.

SUGGESTIONS

Initial Activity Study the display at the top of the page. Ask the students to indicate the relationships between Method 1 and Method 2, i.e., the 5 groups of 6 are now shown as 5 groups of 4 plus 5 groups of 2.

Ask students to draw an illustration showing 8 groups of 3. Have them draw an illustration showing 3 groups of 3 plus 5 groups of 3. Ask if they can suggest any other grouping of 3 (e.g., 4 groups of 3 plus 4 groups of 3, 2 groups of 3 plus 6 groups of 3, etc.).

With more able students you may wish to refer to the distributive property by name.

USING THE BOOK

You may wish to do this page over 2 days. Go over orally the example done in Exercise 1. Do parts (a), (b), and (c) on the board with the students. Assign the rest of the exercise.

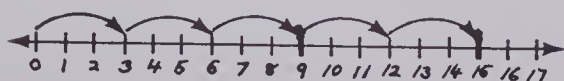
Go over the example done in Exercise 2. Remind students of the short method of multiplying by numbers such as 20, 30, etc. Do parts (a), (b), and (c) with the students. Assign the rest of the exercise.

Assign Exercise 3 to your more able students.

ACTIVITIES

1. Have students show the distributive property for various multiplication statements using the number line.

Example

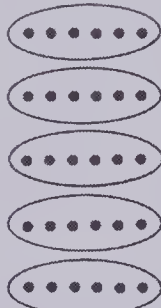


$$5 \times 3 = (3 \times 3) + (2 \times 3) = 15$$

Finding Products in a Different Way

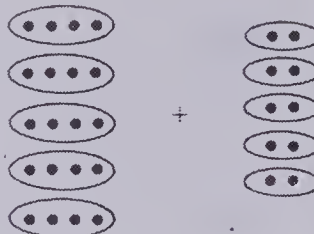
Here are two methods for finding a total number of marbles

Method 1

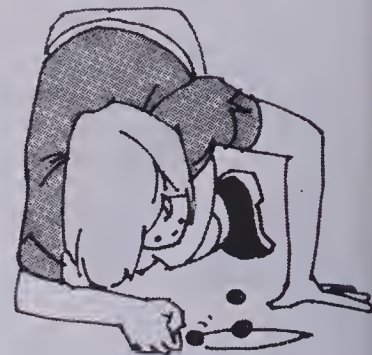


$$5 \times 6 = 30$$

Method 2



$$\begin{aligned} (5 \times 4) + (5 \times 2) \\ = 20 + 10 \\ = 30 \end{aligned}$$



Exercises

Use Method 2 to find each product

$$\begin{aligned} 1. \quad 9 \times 11 &= 9 \times (10 + 1) \\ &= (9 \times 10) + (9 \times 1) \\ &= 90 + 9 \\ &= 99 \end{aligned}$$

$$\begin{aligned} &8 \times 13 \quad 104 \quad 2 \times 14 \quad 28 \quad (c) \quad 4 \times 17 \quad 68 \\ (d) \quad 3 \times 15 \quad 45 \quad (e) \quad 7 \times 12 \quad 84 \quad (f) \quad 6 \times 18 \quad 108 \\ (g) \quad 5 \times 12 \quad 60 \quad (h) \quad 9 \times 17 \quad 153 \quad (i) \quad 4 \times 16 \quad 64 \\ (j) \quad 6 \times 11 \quad 66 \quad (k) \quad 3 \times 18 \quad 54 \quad (l) \quad 8 \times 15 \quad 120 \end{aligned}$$

$$\begin{aligned} 2. \quad 5 \times 364 &= 5 \times (30 + 64) \\ &= (5 \times 30) + (5 \times 64) \\ &= 150 + 320 \\ &= 470 \end{aligned}$$

$$\begin{aligned} &8 \times 42 \quad 336 \quad 7 \times 83 \quad 581 \quad (c) \quad 4 \times 76 \quad 304 \\ (d) \quad 6 \times 39 \quad 234 \quad (e) \quad 4 \times 72 \quad 288 \quad (f) \quad 8 \times 53 \quad 424 \\ &3 \times 67.3 \quad (h) \quad 9 \times 22.6 \quad (i) \quad 4 \times 64.1 \\ (j) \quad 7 \times 42.3 \quad (k) \quad 3 \times 35.6 \quad (l) \quad 6 \times 28.9 \end{aligned}$$

$$\begin{aligned} 3. \quad 23 \times 74 &= 23 \times (70 + 4) \\ &= (23 \times 70) + (23 \times 4) \\ &= 1610 + 92 \\ &= 1702 \end{aligned}$$

$$\begin{aligned} (a) \quad 53 \times 42 \quad (b) \quad 65 \times 27 \quad (c) \quad 31 \times 85 \\ (d) \quad 82 \times 49 \quad (e) \quad 68 \times 91 \quad (f) \quad 36 \times 36 \\ (g) \quad 76 \times 45 \quad (h) \quad 92 \times 81 \quad (i) \quad 77 \times 19 \end{aligned}$$

82 Finding products using the distributive property

ANSWERS:

$$\begin{aligned} 2. \quad (g) \quad 201.9 \quad (h) \quad 203.4 \quad (i) \quad 256.4 \quad (j) \quad 296.1 \quad (k) \quad 106.8 \quad (l) \quad 173.4 \\ 3. \quad (a) \quad 2226 \quad (b) \quad 1755 \quad (c) \quad 2635 \quad (d) \quad 4018 \quad (e) \quad 6188 \quad (f) \quad 1296 \\ (g) \quad 3420 \quad (h) \quad 7452 \quad (i) \quad 1463 \end{aligned}$$

2. Have the students use graph paper to show the distributive property for various multiplication facts.

Example

$$7 \times 3$$



$$(3 \times 3) + (4 \times 3) = 21$$

3. Have the children make up "fill-in-the-blank" examples of the

distributive property for exchange with classmates.

$$\begin{aligned} 26 \times 14.5 &= 26 \times (10 + 4.5) \\ &= (26 \times 10) + (26 \times 4.5) \\ &= \square + \square \\ &= \square \end{aligned}$$

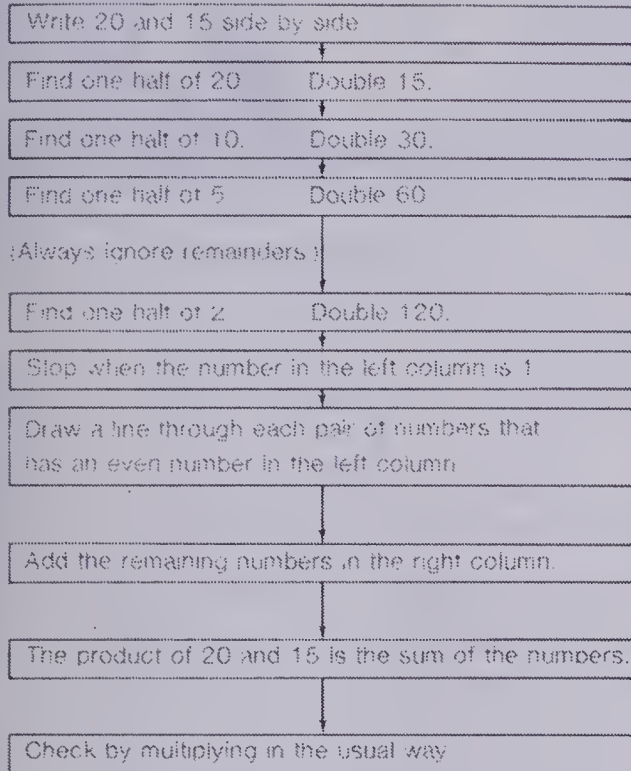
EXTRA PRACTICE

Calculate using Method 2.

$$\begin{aligned} (a) \quad 4 \times 19 [76] \quad (b) \quad 9 \times 36 [324] \\ (c) \quad 3 \times 48.4 [145.2] \quad (d) \quad 6 \times 52.8 [316.8] \end{aligned}$$

Another Way to Multiply

Here is an interesting way to find the product of two numbers.
To find the product of 20 and 15, follow these steps:



20 15
10 30
5 60
2 120
1 240

20 15
~~40 30~~
5 60
~~2 120~~
1 240

60
240
300
15
 $\times 20$
300

OBJECTIVE

To show another way to multiply

PACING

Level A Optional
Level B Optional
Level C Optional

USING THE BOOK

With your more able students you may wish to assign the page without preliminary explanations. Ask them to study the example in the book and then do Exercise 1. If they do this successfully, assign Exercises 2 to 6.

With less able students you may wish to do work on the example step by step on the board, having the students work along with you. Do Exercises 1 and 2 on the board, then assign the rest of the exercises.

Exercises

Use this method to find the following products.

- | | | |
|----------------------------------|--------------------------------|------------------------------------|
| 1. 4×26 104 | 2. 16×45 720 | 3. 28×56 1568 |
| 4. 34×385 13 090 | 5. 56×144 8064 | 6. 324×479 155 196 |

OBJECTIVE

To estimate products by rounding both factors, then multiplying

PACING

Level A All
Level B All
Level C All

VOCABULARY

professional, tournament, competitor, spectator

RELATED AIDS

CALC. ACTIVITY MASTERS — 17, 51.

SUGGESTIONS

Initial Activity Review rounding rules — less than 5, round to previous unit; 5 or more, round to the next unit (see pages 12 to 14).

Present a sample problem such as: T-shirts cost \$4.95 each. How much for 32 shirts?

Discuss estimates, how they are made, why we make them (they can help tell if answers are “about right”; we don’t always need exact answers, etc.); $\$4.95 \rightarrow \5 , $32 \rightarrow 30$, $\$5 \times 30 = \150 . They will cost *about* \$150.

USING THE BOOK

Read through the example situation at the top of the page. Note its similarity to the problem presented in the Initial Activity.

Assign the exercises. Be sure to clarify that estimated answers only are required. Be sure to compare the various answers to actual amounts at some point in the lesson (i.e., during or after marking). Emphasize how close the estimates and actual answers are. Note also that the estimates were (a) quickly done and (b) sufficient to give a “rough idea” of what the answers would be.

You may wish to have your less able students read the problems orally and ensure that they understand them before assigning the exercises.

ACTIVITIES

- See Activity 1 on page 12.
- See “Eraser” as described in the Activity Reservoir.

Fore!

A box of golf balls costs \$6.95.
The golf professional ordered 72 boxes.
How much did he pay?
Estimate by rounding before multiplying.

\$6.95 is rounded to \$ 7.00

72 is rounded to 70

\$490.00

Estimated answer is \$490.00.

Actual

\$6.95

$\times 72$

1 390

48 650

\$500.40

Actual answer is \$500.40.



Exercises

- Round the money to the nearest dollar. Round each number to the nearest 10. Then multiply to find the estimated product.

$\$4.68$	$\$5.00$	(b) $\$9.02$	(c) $\$8.86$	(d) $\$6.07$	(e) $\$5.97$
$\times 31$	$\times 30$	$\times 12$	$\times 49$	$\times 84$	$\times 68$
	\$150.00	\$90.00	\$450.00	\$480.00	\$420.00

- Round each number to the nearest hundred to find the estimated product. Then find the actual product.

793×84	$891 = 67\ 920\ 000$	(b) 389×75	$612 = 30\ 240\ 000$	(c) 304×38	$121 = 11\ 430\ 000$
	67 318 563		29 413 068		11 588 784

- 193 golfers paid an entry fee of \$15.75 each to compete in a golf tournament. Estimate the amount paid in entry fees. \$3040.00

- The average cost of each prize for the tournament was \$17.45. Estimate the total cost of 48 prizes. \$850.00

- The average distance travelled by each competitor to the tournament was 48.7 km. Estimate the total distance travelled by the 193 competitors. 9500 km

- One spectator's ticket costs \$6.25. 14 913 tickets were sold. Estimate the total amount collected in ticket sales. \$89 400.00

84 Rounding and estimating

- Using newspaper advertisements and catalogues, have students estimate the cost of (a) building a dog house, (b) planting a vegetable garden and/or (c) some other relevant project.

- Prepare (have the children help) Estimate the Product cards as shown.

Circle the best estimates.

(a)	28	85	90	95
	$\times 3$			
(b)	432	2000	3000	4000
	$\times 5$			
(c)	\$7.85	\$140	\$150	\$160
	$\times 19$			

EXTRA PRACTICE

- Books were on sale for \$2.95 each. 392 books were sold. What was the estimated amount received from the sale? What was the actual amount received?
- Decorated T-shirts are priced at \$7.95. About how much would it cost to buy a classroom set? Exactly how much would it cost?
- Gasoline costs Ms James 34.4 cents for each litre. About how much for 57 L? Exactly how much?
- There are about 32 children in each class in Sand St. School. There are 18 classes. About how many children in the school?

The Volleyball Tournament

There are 20 players on a team.
There are 5 teams.
How many players altogether?

Multiply. $20 \times 5 = N$
 $100 = N$

There are 100 players altogether.



A number sentence with an equals sign is called an equation.

Exercises

Write the value for each letter that makes the equation true.

- | | | |
|------------------------------------|------------------------------------|------------------------------------|
| 1. $4 \times 8 = N$ 32 | 2. $7 \times 9 = d$ 63 | 3. $10 \times 6 = n$ 60 |
| $? = N$ 32 | $? = d$ 63 | $? = n$ 60 |
| 4. $8 \times 7 = e$ 56 | 5. $8 \times 13 = B$ 104 | 6. $6 \times 15 = R$ 90 |
| 7. $3 \times 18 = N$ 54 | 8. $9 \times 16 = A$ 144 | 9. $7 \times 18 = B$ 126 |
| 10. $8 \times 14 = D$ 112 | 11. $5 \times 14 = E$ 70 | 12. $4 \times 46 = r$ 184 |
| 13. $6 \times 39 = n$ 234 | 14. $8 \times 64 = Z$ 512 | 15. $5 \times 348 = R$ 1740 |
| 16. $7 \times 272 = b$ 1904 | 17. $3 \times 460 = E$ 1380 | 18. $8 \times 507 = N$ 4056 |
| 19. $4 \times 3 = b \times 6$ | 20. $r \times 14 = 20 - 6$ | 21. $9 \times n = 63 + 9$ |
| $12 = b \times 6$ | $r \times 14 = 14$ | $9 \times n = 72$ |
| $? = b$ 2 | $r = ?$ 1 | $n = ?$ 8 |

Copy and complete.

- | | | |
|------------------------------|-------------------------------------|---|
| 22. $7 + 9 = R \times 4$ | 23. $3 \times (4 + 1) = 2 + E$ | 24. $(20 - 8) \times 2 = N$ 24 |
| 25. $5 + 6 + 9 = 4 \times B$ | 26. $9 \times 9 = 100 - Z$ | 27. $6 \times 5 = \frac{1}{y} \times 30$ |
| 28. $48 = (5 \times 10) - T$ | 29. $N + E + D = 3 \times 4$ | 30. $6 \times 3 = R + 12$ |
| 31. $42 - a = 9 \times 4$ | 32. $8 \times 5 = 60 - S$ 20 | 33. $(4 \times 7) + B = (6 \times 6) + 2$ |
| 34. $n + 5 + D = 5 \times 5$ | 35. $(8 \times 7) + 4 = 72 - h$ | 36. $E + (6 + 7) = 6 \times 4$ |
- Other possible answers.

Solving equations 85

OBJECTIVES

To review equations
To provide practice in working with equations in multiplication

PACING

Level A All
Level B All
Level C All

SUGGESTIONS

Initial Activity Review the concept of equations (see page 19). Be sure to mention (a) the use of letters of the alphabet to stand for unknowns and (b) the "balanced" feature of equations (i.e., each side of the "=" symbol is equal in value).

USING THE BOOK

Read through the display at the top of the page together. Point out that, in this example, we let "N" stand for the number of players altogether.

Complete Exercises 1, 2, and 19 to 21 orally before assigning the exercises. Be sure that the children are familiar with an accepted answer format (i.e., copy and complete?; show each step?; answers only?). Note that, for Exercises 1 to 18, the unknown remains by itself on the right side of the symbol. Exercises 19 to 21 are partially complete to serve as a thinking guide as to what to do when the unknown represents a missing factor, addend, subtrahend, etc. as in Exercises 22 to 36.

ACTIVITIES

1. Play the game "Simon Says", using the multiplication facts. The teacher says "Simon Says 4×3 "; students answer "12". If the teacher does not say Simon Says, then no answer should be given. Students who do respond are eliminated. The last student left wins.

2. See the "Coded Riddles" idea in the Activity Reservoir.

3. See "Number Sentence" as described in the Activity Reservoir.

OBJECTIVE

To write equations for word problems

PACING

Level A 1-3, 5

Level B 1-5

Level C All

RELATED AIDS

BFA PROB. SOLVING LAB II — 79.

SUGGESTIONS

Initial Activity Pose the problem:

There are 627 students in the school.
395 are girls.

How many boys? (Substitute your own school figures.)

Ask the questions:

1. What are we told? [627 students, 395 are girls]
2. What are we to find out? [how many boys]
3. How shall we do it? [subtract number of girls from total number of students]
4. How shall we write this in equation form? [$627 - 395 = N$]

Emphasize, if necessary, the components of the equation.

$627 - 395 = N$
 ↙ ↘ ↓
 Total Girls Must be all
 number of other students
 students in in school who
 school are not girls
 — that is, boys

USING THE BOOK

Go over the example in the display at the top of the page.

With less able students, you may wish to read the problems orally, before assigning them. Exercises 4 and 6 are 2-step problems and will therefore require more complicated equations. For these, you may wish to remind the students about the use of brackets and/or the order of operations rules.

ACTIVITIES

1. To provide practice in operations as they apply to word problems and equations, provide an exercise such as:

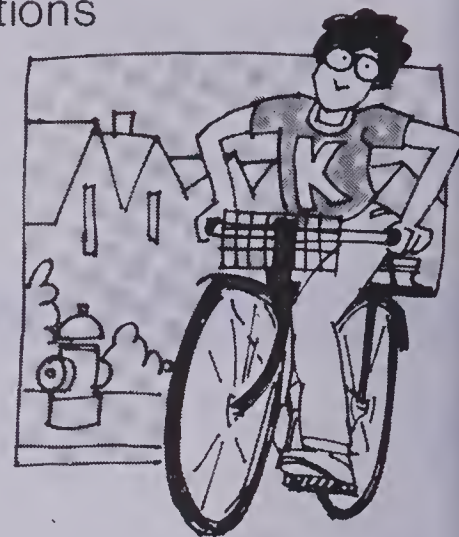
Using Equations

Karl rode his bicycle to and from work each day.
The round trip was 14.7 km.
He worked 227 d in a year.
How far did he ride in a year?

Write an equation: $14.7 \times 227 = n$

kilometres per day number of days stands for total number of kilometres

Solve the equation: $14.7 \times 227 = 3336.9$
Write a statement: Karl rode 3336.9 km in a year



Exercises

Write and solve an equation for each problem

1. A book has 211 pages.
There are 12 copies of the book.
How many pages altogether?
 $211 \times 12 = 2532$
2. Mario has \$275.
Benny has twice as much as Mario.
How much does Benny have?
 $\$275 \times 2 = \550
3. Wong ran 1500 m.
Lee bicycled 5 times that distance.
How far did Lee bicycle?
 $1500 \times 5 = 7500$
4. Tom saved \$5.75 a week for 7 weeks.
He spent \$20.00 of what he had saved for a birthday gift.
How much did he have left?
 $\$5.75 \times 7 - \$20.00 = \$20.25$
- ★ 5. Linda raised 23 sheep.
She sold each one for \$64.50.
How much did she receive?
 $23 \times \$64.50 = \1483.50
- ★ 6. Lois had \$35.00.
She paid bills of \$17.35 and \$12.08.
How much did she have left?
 $\$35.00 - (\$17.35 + \$12.08) = \5.57

86 Writing equations for problems

Fill in the empty spaces in this chart.
The first one has been done for you.

Facts	Solution	Operation
1. 7, 12	19	addition
2. 3.6, 5.8		addition
3. 726, 527		subtraction
4. 819, 20	799	
5. 0.5, 6.0	6.5	
6. 0.5, 6.0	3.0	
7. 27, 3		division
8. 100, 0.01		addition

2. See "Number Sentence" as described in the Activity Reservoir.

3. Have the students create their own "fill-in-the-blank" charts (as in Activity 1) for exchange with other groups or classmates.

EXTRA PRACTICE

Write equations for each problem.
Solve it.

1. There are seven days in each week.
How many days in 12 weeks?
2. There are twenty-four hours in one day.
How many hours are there in 1 week?
3. Marilynne has \$26.50 in the bank.
She spends \$8.99 for a record album.
How much does she have left?
4. Mary bought 6 cans of soup.
Each can cost 61¢.
How much change did she receive from \$5.00?
5. Mr. Hill works 8 h a day.
He works 5 d a week.
Last week he also worked 4 h overtime.
How many hours did he work last week?

Play Ball

The sports stadium has 3255 seats.
For 157 baseball games the stadium was filled.
How many people attended these games?

Write an equation: $3255 \times 157 = N$
 $511\ 035 = N$

511 035 people attended the games.

3255
 $\times 157$

22 785
162 750
325 500

511 035



Exercises

1. \bullet $\begin{array}{r} 3516 \\ \times 231 \\ \hline 812\ 196 \end{array}$ (b) $\begin{array}{r} 6857 \\ \times 473 \\ \hline 3\ 243\ 361 \end{array}$ (c) $\begin{array}{r} 7924 \\ \times 649 \\ \hline 5\ 142\ 676 \end{array}$ (d) $\begin{array}{r} 8923 \\ \times 917 \\ \hline 8182\ 391 \end{array}$ (e) $\begin{array}{r} 5374 \\ \times 753 \\ \hline 4\ 046\ 622 \end{array}$

2. (a) $\begin{array}{r} 476 \times 68\ 527 \\ \hline 32\ 618\ 852 \end{array}$ (b) $\begin{array}{r} 339 \times 84\ 359 \\ \hline 28\ 597\ 701 \end{array}$ (c) $\begin{array}{r} 763 \times 38\ 027 \\ \hline 29\ 014\ 601 \end{array}$

3. \bullet $\begin{array}{r} 3856 \\ \times 304 \\ \hline 1\ 172\ 224 \end{array}$ (b) $\begin{array}{r} 4823 \\ \times 408 \\ \hline 1\ 967\ 784 \end{array}$ (c) $\begin{array}{r} 9765 \\ \times 307 \\ \hline 2\ 997\ 855 \end{array}$ (d) $\begin{array}{r} 8237 \\ \times 4268 \\ \hline 35\ 155\ 516 \end{array}$ (e) $\begin{array}{r} 9876 \\ \times 4703 \\ \hline 46\ 446\ 828 \end{array}$

4. \bullet $\begin{array}{r} 8503 \\ \times 9830 \\ \hline 83\ 584\ 490 \end{array}$ (b) $\begin{array}{r} 8323 \\ \times 5006 \\ \hline 41\ 664\ 938 \end{array}$ (c) $\begin{array}{r} 7682 \\ \times 6003 \\ \hline 46\ 115\ 046 \end{array}$ (d) $\begin{array}{r} 6002 \\ \times 4007 \\ \hline 24\ 050\ 014 \end{array}$ (e) $\begin{array}{r} 3487 \\ \times 1002 \\ \hline 3\ 493\ 974 \end{array}$

5. \bullet $\begin{array}{r} 7682 \times 64\ 358 \\ \hline 494\ 398\ 156 \end{array}$ (b) $\begin{array}{r} 4765 \times 92\ 634 \\ \hline 441\ 401\ 010 \end{array}$ (c) $\begin{array}{r} 3076 \times 76\ 842 \\ \hline 236\ 365\ 992 \end{array}$

6. Multiply to find these peculiar products.

(a) $\begin{array}{r} 12\ 345\ 679 \\ \times 18 \\ \hline 222\ 222\ 222 \end{array}$ (b) $\begin{array}{r} 12\ 345\ 679 \\ \times 27 \\ \hline 333\ 333\ 333 \end{array}$ (c) $\begin{array}{r} 12\ 345\ 679 \\ \times 108 \\ \hline 1\ 333\ 333\ 332 \end{array}$

- ★ 7. 28 209 people attended each of the 102 baseball games at a baseball stadium.
Also, 34 008 people attended 28 of the baseball games.
How many people attended in all? $3\ 829\ 542$

OBJECTIVE

To provide practice in multiplying multidigit whole numbers

PACING

Level A 1, 3, 4
Level B 1-5
Level C 1, 4, 6, 7

RELATED AIDS

HMS — DM21.

USING THE BOOK

Go over the problem written in the display at the top of the page. You may also wish to do the first example of each row on the board with the students, before assigning the rest of the row.

ACTIVITIES

1. Play "Name Game". Each letter of the alphabet is given a value, e.g., A = 1, B = 2, C = 3, etc. Students find the products of their given names and surnames.

2. Students can find the value of oceans, countries, cities, planets, etc.

3. Challenge students to make up their own examples using larger numbers. Have them exchange their examples and mark the solutions.

EXTRA PRACTICE

1. $\begin{array}{r} 76\ 835 \\ \times 762 \\ \hline [58\ 548\ 270] \end{array}$ 2. $\begin{array}{r} 97\ 620 \\ \times 376 \\ \hline [36\ 705\ 120] \end{array}$

3. $\begin{array}{r} 43\ 017 \\ \times 4\ 682 \\ \hline [201\ 405\ 594] \end{array}$ 4. $\begin{array}{r} 99\ 485 \\ \times 8\ 301 \\ \hline [825\ 824\ 985] \end{array}$

OBJECTIVE

To solve word problems involving money

PACING

Level A All
Level B All
Level C All

RELATED AIDS

HMS — DM22.

BFA PROB. SOLVING LAB II —
105-113, 187, 188.

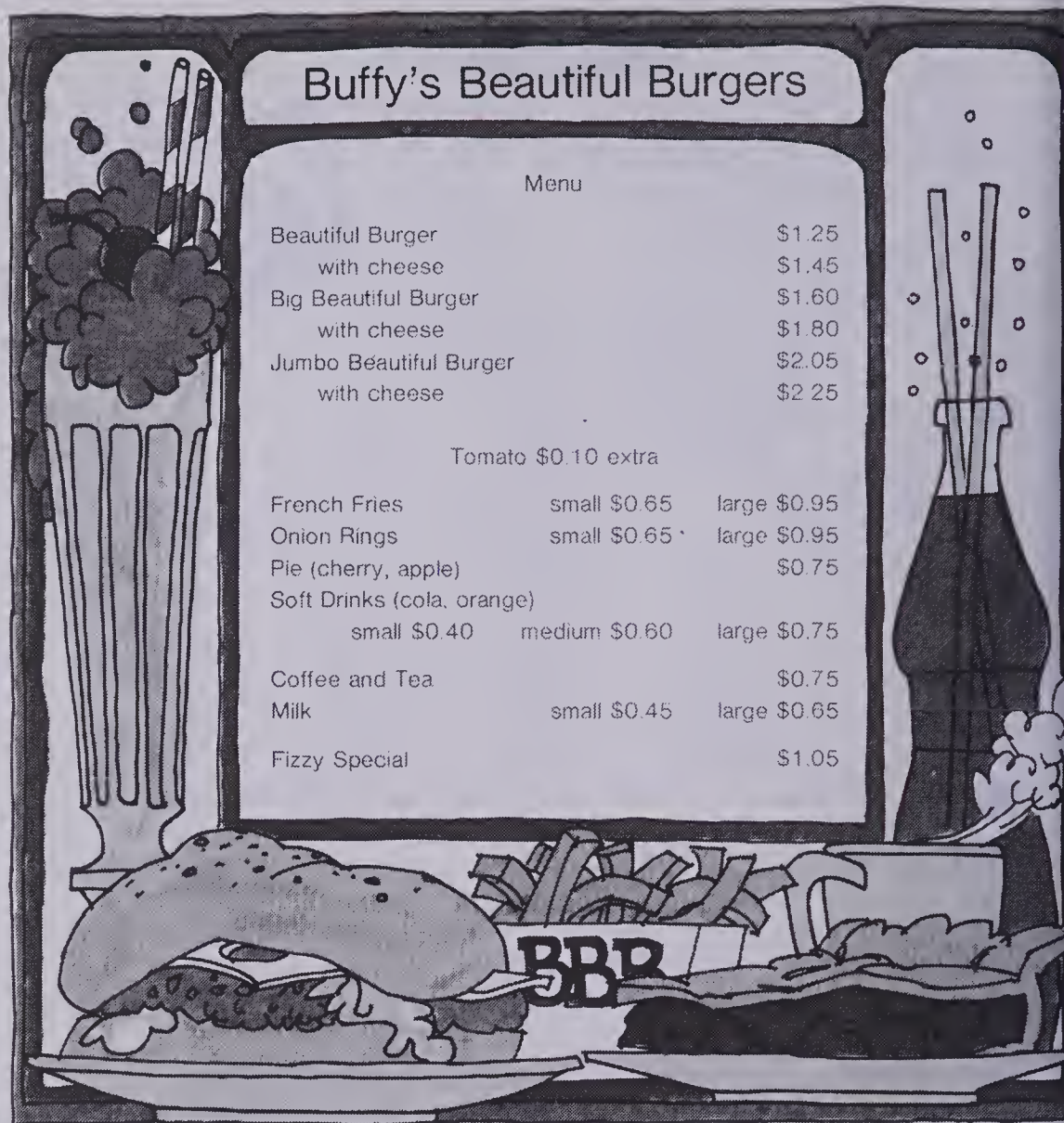
CALC. ACTIVITY MASTERS — 13.

SUGGESTIONS

Initial Activity Review the problem-solving technique that is used in your classroom (and/or see pages 17 and 22). Discuss waitressing (and waitering) and some of the skills it would require. See the Career Awareness notes in the Chapter Overview, page 56.

USING THE BOOK

Ensure that all students can read the problems. Help them decode words they do not know. You may wish to do the first problem on the board to review the format for presenting solutions to word problems. Students don't have to solve the problems in the order presented. Allow them to choose the ones they wish to solve first.



The Waitress

Marie is a waitress at Buffy's Beautiful Burgers

1. At table number five, 3 men each ordered.

1 big beautiful burger
1 small French fries
1 medium cola

What was the total cost of the three orders? **\$8.55**

2. On Saturday afternoon, Bob ordered:

1 jumbo beautiful burger with cheese and tomato
1 large onion rings
1 piece of cherry pie
1 coffee

Elaine had:

1 big beautiful burger
1 small French fries
1 large milk

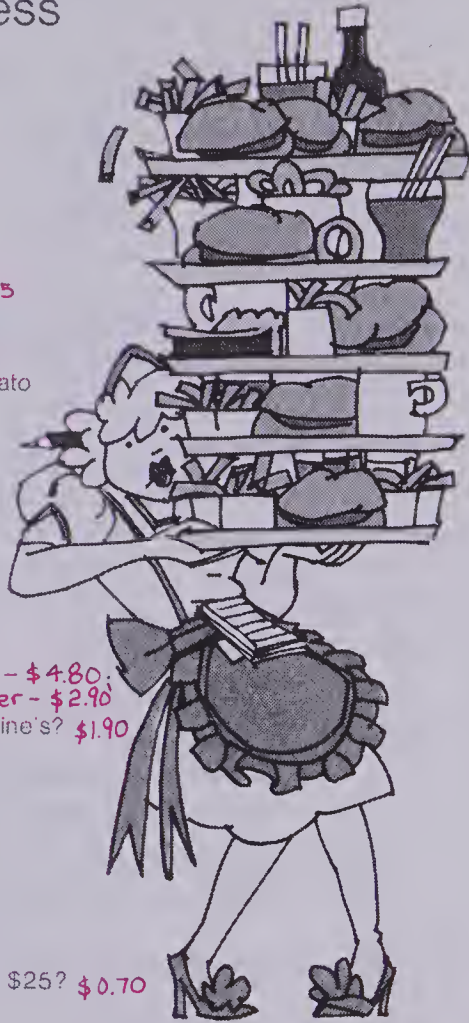
- (a) How much did each order cost? **Bob's order - \$4.80**
Elaine's order - \$2.90
- (b) How much more did Bob's order cost than Elaine's? **\$1.90**

3. The Paterson Family ordered:

6 jumbo beautiful burgers
6 large French fries
6 fizzy specials

- (a) What was the total cost? **\$24.30**
- (b) How much change did Mrs. Paterson get from \$25? **\$0.70**

4. (a) If you were at Buffy's Beautiful Burgers, what would your order be?
- (b) How much would it cost?



ACTIVITIES

1. Suggest that the students design a menu for an animal of their choice, e.g., an elephant, a mouse, etc. Having designed the menu with price list, they can make up problems based on the menu. Other students can solve the problems.

2. Students can make a list of the more popular fast-food items such as hotdogs, pizzas, hamburgers. Having done this they can collect data regarding the cost of the items in a number of local eating places. They can then compare the prices.

3. Make collages of cut-out pictures featuring food.

OBJECTIVE

To evaluate achievement of the chapter objectives

PACING

- Level A All
- Level B All
- Level C All

RELATED AIDS

HMS — DM23.

USING THE BOOK

Each student should do this test independently under supervision. Assistance should be given only when the instructions are not understood. After the work has been corrected, you should provide appropriate remedial work. You may wish to reteach if a large number of students had difficulty with a particular topic or concept.

The following chart will help in this regard. The specific objectives are listed in the Chapter Overview (see page 56).

An alternate Chapter Test can be found in the Holt Mathematics System Duplicating Masters available for use with this grade level.

Test Item	Objective	Text Page Number
1-7	A	61, 64-78
8	B	62, 76, 82
9	C	84
10	D, E	86

Chapter Test

1. (a) 76×63 **4788** (b) 63×308 **19404** (c) 634×409 **259 306** (d) 5831×6350 **37 026 850**
2. (a) 100×38 **3800** (b) 10×724 **7240** (c) 1000×542 **542 000** (d) 100×370 **37 000**
3. (a)
$$\begin{array}{r} 85.9 \\ \times 6 \\ \hline 515.4 \end{array}$$
 (b)
$$\begin{array}{r} 23.64 \\ \times 3 \\ \hline 70.92 \end{array}$$
 (c)
$$\begin{array}{r} 483 \\ \times 0.6 \\ \hline 289.8 \end{array}$$
 (d)
$$\begin{array}{r} 2039 \\ \times 0.4 \\ \hline 815.6 \end{array}$$
4. (a) 0.01×8 **0.08** (b) 0.001×374 **0.374** (c) 0.001×591 **0.591** (d) 0.01×3852 **38.52**
5. (a) 3.7×87 **321.9** (b) 9.1×4835 **43 998.5** (c) 5.38×290 **1560.2** (d) 0.05×3529 **176.45**
6. (a)
$$\begin{array}{r} 0.9 \\ \times 0.4 \\ \hline 0.36 \end{array}$$
 (b)
$$\begin{array}{r} 3.7 \\ \times 0.6 \\ \hline 2.22 \end{array}$$
 (c)
$$\begin{array}{r} 8.3 \\ \times 0.92 \\ \hline 7.636 \end{array}$$
 (d)
$$\begin{array}{r} 0.159 \\ \times 3.7 \\ \hline 0.5883 \end{array}$$
7. (a) $13.5 \times \$4.67$ **\$63.05** (b) $27.6 \times \$9.35$ **\$258.06** (c) $5.02 \times \$50.07$ **\$251.35** (d) $0.7 \times \$853.45$ **\$597.42**
8. Copy and complete.
(a) $29 \times 36 = \blacksquare \times 29$ **36** (b) $3 \times (15 + 4) = 3 \times 15 + \blacksquare \times \blacksquare$ **3 4**
9. Round each number to estimate the answer. Show your work.
Calculate the actual answer
(a) There are 72 pencils in each box. There are 450 boxes.
How many pencils altogether? **35 000; 32 400**
(b) Each crate of oranges has a mass of 28.75 kg
What is the total mass of 675 crates? **21 000 kg; 19 406.25 kg**
10. Write an equation for each problem and solve
(a) At a record sale, 52 albums were sold at \$4.99 each
How much money was received? **$52 \times \$4.99 = \259.48**
(b) Janice runs 0.2 km in 1 min.
How far will she run in 16.3 min? **$0.2 \times 16.3 = 3.26$**

Cumulative Review

1. Add.

(a)	1882	(b)	\$ 736.75	(c)	0.765	(d)	\$ 376.52
	903		127.02		1.276		92.01
	487		35.19		5.620		483.78
	+ 2170		+ 620.71		+ 0.761		+ 20.65
	<u>5442</u>		<u>\$1519.67</u>		<u>8.422</u>		<u>\$972.96</u>

2. Subtract.

(a)	48 276	(b)	\$6352.17	(c)	80 000	(d)	0.475
	- 19 028		- 981.08		- 14 635		- 0.297
	<u>29 248</u>		<u>\$ 5371.09</u>		<u>65 365</u>		<u>0.178</u>

3. Write in words.

- (a) 13 800 573 (b) 67 000 000 000 (c) 6 476 000 000

4. Write in decimal form.

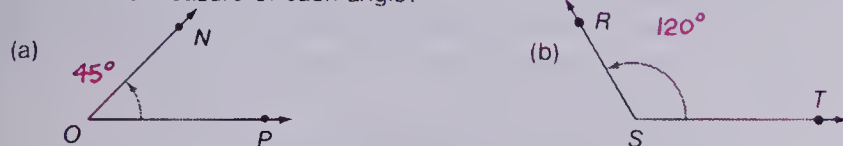
- (a) $12 \frac{7}{10}$ 12.7 (b) $409 \frac{27}{1000}$ 409.027 (c) $561 \frac{33}{100}$ 561.33 (d) $800 \frac{9}{1000}$ 800.009

5. Draw an acute angle and label it $\angle EFG$.

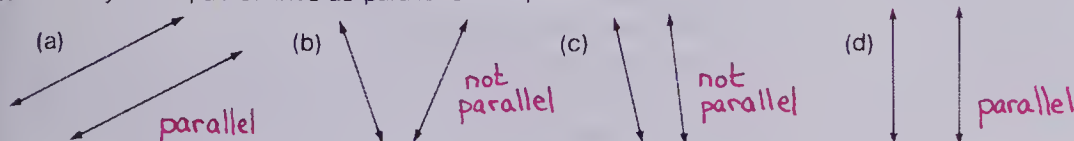
6. Draw a straight angle and label it $\angle PQR$.

7. Use your protractor.

What is the measure of each angle?



8. Identify each pair of lines as parallel or not parallel.



Chapters 1-3 cumulative review 91

OBJECTIVE

To review and test selected concepts and skills previously covered

PACING

Level A All
Level B All
Level C All

USING THE BOOK

This page may be used for diagnostic and remedial as well as review purposes. Students should check their work, correct any errors, and review the pages that contain any problems of the type they missed. Some students can do this on their own while others may need help. If a large number of students have a particular problem incorrect, you may want to reteach that topic to the groups, then assign a duplicated worksheet to reinforce that topic or, refer to an appropriate skill card in the BFA Computational Skills Kit II or BFA Problem Solving Kit II.

Test Item	Text Page Number
1	4, 6, 7
2	5, 8, 9
3	10, 11
4	2, 3
5, 6	36, 37
7	34, 35
8	44, 45

ANSWERS:

(a) thirteen million, eight hundred thousand, five hundred seventy-three

(b) sixty-seven billion

(c) six billion, four hundred seventy-six million

CHAPTER 4 OVERVIEW

This chapter reviews and extends division to two-digit divisors and multiple-digit quotients without and with remainders. Also decimal dividends are used.

Measurement is reviewed and extended in two areas: (1) the changes between measures of length, of capacity, of mass, and of relating the volume of a container to its capacity; (2) the development and use of formulas for the perimeter of rectangles; circumferences and areas of rectangles, parallelograms, triangles and circles; and the volume of rectangular prisms.

OBJECTIVES

- A To review the concept of division and the division algorithm
- B To divide by 10, 100, and 1000
- C To divide by up to 2-digit divisors, with and without remainders
- D To divide up to 2-decimal dividends by 1- and 2-digit divisors, without remainders
- E To use and identify relationships among common metric units
- F To find perimeter, circumference, area, and volume of regular shapes
- G To relate litres, millilitres, and cubic centimetres
- H To express metric linear units given in expanded form as metres

BACKGROUND

This 32-page chapter is actually 2 mini-chapters in one, containing pages on both division and measurement. This blend was necessary to relieve what would have been a formidable, perhaps overwhelming (to some individuals or groups) computational chapter if division had been presented as an uninterrupted unit.

Measurement not only offers a refreshing return to relevant, continuously hands-on learning opportunities but also can be easily related to concepts in division: metres can be *divided* into centimetres and millimetres; kilograms can be *divided* into grams; etc. The continuation of the division and measurement topics included in HMS Book 6 is presented in the same split fashion in Chapter 5.

Squared paper is suggested frequently to allow actual unit counting to find the area of various regular shapes (before formulas are introduced). You may wish to use squared paper even earlier in the chapter too, upon which the numerous division exercises can be completed. It is an easy way to keep computations organized and legible.

Collect, if possible (children and other staff members can help), a good number of common objects (containers, etc.) and pictures (larger objects, containers, etc.). These will serve not only as a basis for discussion of the various units of measure, but as an aid to establishing referents as well.

MATERIALS

counters
construction paper
flash cards
graph paper
abacus
metre sticks
rulers
tapes (showing millimetres, centimetres, decimetres)
various cylindrical cans
cardboard strips in various lengths (see page 106)
squared paper
scissors
trundle wheel (or long measuring tapes)
litre container (commercial or a milk container)
scales
centimetre cubes
cardboard
pictures of various tankers (truck, ocean-going, rail)
and of common objects labelled with their
approximate mass

CAREER AWARENESS

Truck Gardener [118 and 119]

Truck gardeners fulfil a very important and essential role — that of helping feed the nation. They operate large commercial gardens designed to grow and produce vegetables for sale and shipment to urban consumers. Since many people today do not grow enough garden produce to feed themselves, they rely on the truck gardeners as their year-round source of products.

Today, much of the work done in truck gardening is done by mechanical means, but there is a relatively high proportion of manual labour involved in those areas where machines just cannot do the work. In order to get this manual work done, extra “unskilled” workers are employed to do the back-breaking, manual jobs.

The truck gardener’s produce, in major part, is perishable: lettuce, tomatoes, corn, etc. do not have a long shelf life. It is necessary therefore that the truck gardener get the produce to market quickly and sell it immediately. Most truck gardeners operate near centres of large populations.

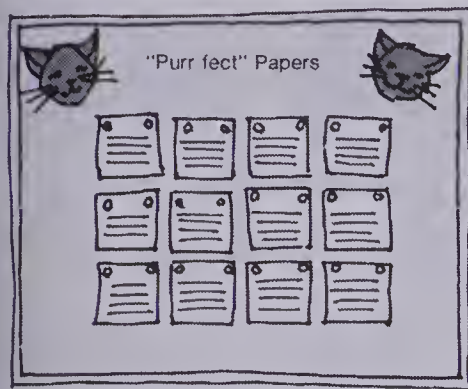
The Gardener [120]

The professional gardeners today usually work for large corporations that have extensive green areas or parks. Their green areas and parks are designed to beautify and to enhance the value of the property. Cities are using parks for the relaxation and entertainment of its residents.

Many private residents employ gardeners on a part-time basis. These private gardeners often contract to care for a number of residences.

(Continued on page 94)

Bulletin Boards



On the Grade 6 math bulletin board there were 12 "Purrfect" papers with 3 rows and 4 columns.

Dividend	Divisor	Quotient
12	3	4

means there are 4 groups of 3 in 12

Dividend	Divisor	Quotient
12	4	3

means there are 3 groups of 4 in 12.

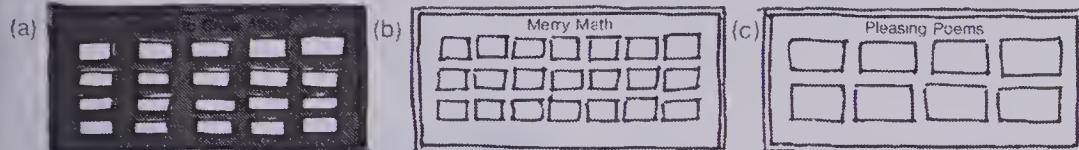
We can show division in other ways.

4 — Quotient
Divisor — 3 $\overline{)12}$ — Dividend

Dividend — 12
— = 3 — Quotient
Divisor — 4

Exercises

1. Write two division sentences for each bulletin board



2. Write each question in words, and then solve.

(a) $3 \overline{)15}$ How many groups of 3 in 15?
There are 5

(b) $6 \overline{)24}$ 4
(e) $32 \div 4$ 8
(h) $\frac{30}{6}$ 5

(c) $7 \overline{)49}$ 7
(f) $48 \div 6$ 8
(i) $\frac{20}{4}$ 5

(d) $5 \overline{)35}$ 7
(g) $72 \div 9$ 8
(j) $\frac{28}{7}$ 4

3. Find the quotient.

(a) $18 \div 3$ 6 (b) $64 \div 8$ 8 (c) $42 \div 7$ 6 (d) $9 \div 9$ 1 (e) $14 \div 2$ 7
(f) $8 \overline{)56}$ 7 (g) $9 \overline{)63}$ 7 (h) $6 \overline{)54}$ 9 (i) $9 \overline{)81}$ 9 (j) $5 \overline{)10}$ 2
(k) $\frac{6}{2}$ 3 (l) $\frac{16}{8}$ 2 (m) $\frac{24}{4}$ 6 (n) $\frac{63}{7}$ 9 (o) $\frac{48}{8}$ 6

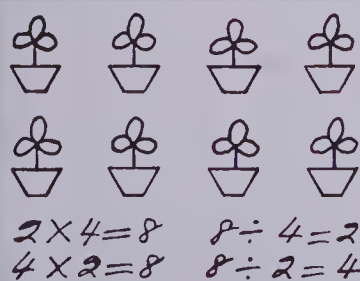
Meaning of division 93

ANSWERS:

(a) $20 \div 4 = 5$; $20 \div 5 = 4$ (b) $21 \div 3 = 7$; $21 \div 7 = 3$ (c) $8 \div 2 = 4$; $8 \div 4 = 2$

ACTIVITIES

On chart paper or construction paper cards, students make up fact cards. Cutouts from magazines and catalogues can be used or students can draw their own illustrations.



2. Students make up a personal set of flash cards with division facts written on them.

3. Students compile a list of situations where arrays are evident, e.g., eggs in a carton, teams formed for games in physical ed., cookies on a baking tray, etc.

OBJECTIVE

To review the concept of division

PACING

Level A All
Level B All
Level C All

VOCABULARY

divisor, dividend, quotient

MATERIALS

counters; labels made from construction paper with the names divisor, dividend, and quotient

RELATED AIDS

BFA COMP LAB II — 46, 47.
CALC. ACTIVITY MASTERS — 7.

SUGGESTIONS

Initial Activity Distribute counters. Students work with 12 counters. Instruct them to divide the counters into groups of 3. Ask: "How many groups?" Write on the board the division sentence $12 \div 3 = 4$. Instruct students to divide the 12 counters into groups of 4. Again ask: "How many groups?" Write the division sentence $12 \div 4 = 3$ on the chalkboard. Beside each number place the appropriate label. Instruct students to divide the 12 counters into groups of 6 and then into groups of 2. Ask: "How many groups?" Have them write the number sentence for each division situation and label with divisor, dividend, and quotient.

USING THE BOOK

Use the display at the top of the page to reinforce the concept and to review with students the different ways of writing a division question. With less able students, you may wish to do on the board Exercise 2, parts (b), (e), and (h) before assigning the page.

OBJECTIVE

To divide by a single-digit divisor

PACING

- Level A All
- Level B All
- Level C All

MATERIALS

flash cards

SUGGESTIONS

Initial Activity Conduct a drill using division-fact flash cards. Students quietly say the answers to the cards as they are shown one by one.

USING THE BOOK

Allow students to study the questions for a short period of time. Then allow them to project the number they think they will answer correctly. Ask students to write this number in their books or on a paper. When the questions are marked, students compare their projection and the number they get correct. You may or may not wish to set a time limit on the Tune Up.

ACTIVITIES

1. Have the students make charts with 2 columns.

Multiplication	Division
Sentence	Sentence
$5 \times 2 = 10$	$10 \div 2 = 5$
$3 \times 6 = 18$	$18 \div 6 = 3$
etc.	

2. Given sets of 3 numbers such as (48, 6, 8), students write the related multiplication and division facts.

3. Students make their own set of flash cards showing division facts. These are assembled into 2 bundles — “Facts I Know” and “Facts I Don’t Know”. As each unknown fact is mastered, it is transferred to the “Facts I Know” bundle.

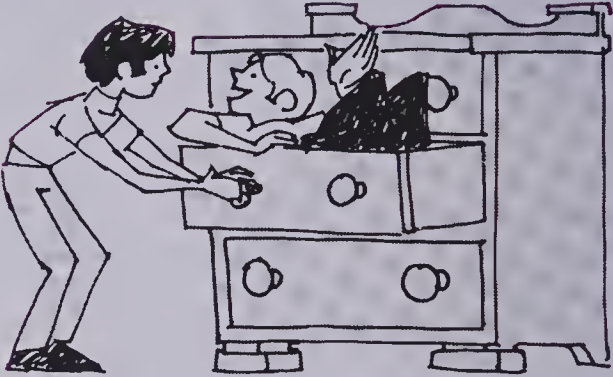
Tune Up

Divide.

1. $6 \overline{)36}$ ⁶
2. $8 \overline{)48}$ ⁶
3. $7 \overline{)56}$ ⁸
4. $9 \overline{)81}$ ⁹
5. $4 \overline{)16}$ ⁴
6. $30 \div 5$ ⁶
7. $42 \div 6$ ⁷
8. $18 \div 2$ ⁹
9. $12 \div 3$ ⁴
10. $72 \div 8$ ⁹
11. $\begin{smallmatrix} 24 \\ 8 \end{smallmatrix} \overline{)3}$ ³
12. $\begin{smallmatrix} 18 \\ 5 \end{smallmatrix} \overline{)3}$ ³
13. $\begin{smallmatrix} 49 \\ 7 \end{smallmatrix} \overline{)7}$ ⁷
14. $\begin{smallmatrix} 28 \\ 4 \end{smallmatrix} \overline{)7}$ ⁷
15. $\begin{smallmatrix} 54 \\ 6 \end{smallmatrix} \overline{)9}$ ⁹
16. $9 \overline{)72}$ ⁸
17. $7 \overline{)35}$ ⁵
18. $5 \overline{)40}$ ⁸
19. $6 \overline{)48}$ ⁸
20. $3 \overline{)18}$ ⁶
21. $42 \div 6$ ⁷
22. $16 \div 2$ ⁸
23. $20 \div 4$ ⁵
24. $24 \div 6$ ⁴
25. $27 \div 9$ ³
26. $6 \overline{)39}$ ^{6R3}
27. $9 \overline{)85}$ ^{9R4}
28. $8 \overline{)49}$ ^{6R1}
29. $7 \overline{)53}$ ^{7R4}
30. $8 \overline{)27}$ ^{3R3}
31. $\begin{smallmatrix} 355 \\ 5 \end{smallmatrix} \overline{)71}$ ⁷¹
32. $\begin{smallmatrix} 427 \\ 7 \end{smallmatrix} \overline{)61}$ ⁶¹
33. $\begin{smallmatrix} 549 \\ 9 \end{smallmatrix} \overline{)61}$ ⁶¹
34. $\begin{smallmatrix} 606 \\ 3 \end{smallmatrix} \overline{)202}$ ²⁰²
35. $\begin{smallmatrix} 648 \\ 8 \end{smallmatrix} \overline{)81}$ ⁸¹
36. $344 \div 4$ ⁸⁶
37. $460 \div 5$ ⁹²
38. $196 \div 7$ ²⁸
39. $804 \div 2$ ⁴⁰²
40. $937 \div 9$ ^{104R1}

Are you in the
“Top Drawer”?

Top Drawer	40-35
Middle Drawer	34-25
Bottom Drawer	24-20

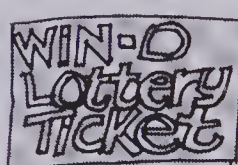
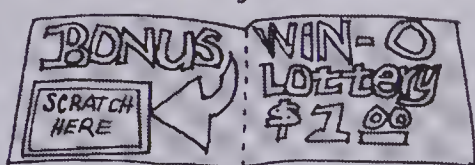
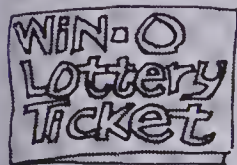


(Continued from Chapter Overview)

Much of the work done by gardeners is manual in nature, ranging from unskilled labour to highly specialized work. Mechanical tools are widely used for certain tasks.

To be a head gardener one must have training in a post high school institution in horticulture studying plants, care of plants, and plant diseases and their treatment. Following the schooling, an apprenticeship is served under the supervision of an experienced gardener.

Lottery Tickets



10 people shared a \$100 prize equally. How much did each receive?

$\$100 \div 10 = \10
Each received \$10.

100 people shared a \$2500 prize equally. How much did each receive?

$\$2500 \div 100 = \25
Each received \$25

1000 people shared a \$750 000 prize equally. How much did each receive?

$\$750\,000 \div 1000 = \750
Each received \$750

Study the examples carefully. What rule helps you to divide by 10, 100, and 1000?

Exercises

1. Divide.

$70 \div 10 = 7$ $90 \div 10 = 9$ (c) $660 \div 10 = 66$ (d) $760 \div 10 = 76$
 (e) $\$8000 \div 10 = \800 (f) $\$3770 \div 10 = \377 (g) $\$85\,630 \div 10 = \8563 (h) $\$52\,640 \div 10 = \5264

2. Divide.

$600 \div 100 = 6$ $900 \div 100 = 9$ (c) $4700 \div 100 = 47$ (d) $8300 \div 100 = 83$
 (e) $\$64\,800 \div 100 = \648 (f) $\$75\,500 \div 100 = \755 (g) $\$29\,600 \div 100 = \296 (h) $\$673\,000 \div 100 = \6730

3. Find the quotient.

$4000 \div 1000 = 4$ $6000 \div 1000 = 6$ (c) $78\,000 \div 1000 = 78$ (d) $92\,000 \div 1000 = 92$
 (e) $\$62\,000 \div 100 = \620 (f) $\$769\,000 \div 1000 = \769 (g) $\$685\,000 \div 1000 = \685 (h) $376\,111 \div 1000 = 376.111$

★ 4. (a) $7\,000\,000 \div 10\,000 = 700$ (b) $700\,000\,000 \div 100\,000 = 7000$
 (c) $7\,000\,000 \div 1\,000\,000 = 7$

Dividing by 10, 100, 1000 95

OBJECTIVE

To divide by 10, 100, and 1000

PACING

Level A All
Level B All
Level C All

MATERIALS

graph paper

RELATED AIDS

BFA COMP LAB II — 52.

BFA PROB. SOLVING LAB II — 98, 99, 102.

CALC. ACTIVITY MASTERS — 38.

SUGGESTIONS

Initial Activity Instruct students to outline on the graph paper a 10 by 10 array. How many squares altogether? [100] How many groups of 10 in 100? [10] Do the same for a 10 × 100 array. How many squares altogether? [1000] How many groups of 100 in 1000? [10] Ask students to write the number sentence $100 \div 10 = 10$. Instruct them to draw a line through the zero in the divisor and the last zero in the dividend, thus $10\cancel{0} \div 1\cancel{0} = 10$. Ask them to relate the quotient to the dividend. [They are now the same.] Repeat for the number sentence $1000 \div 100 = 10 \rightarrow 10\cancel{0}\cancel{0} \div 1\cancel{0}\cancel{0} = 10$. Expand to $10\,000 \div 1000 = 10 \rightarrow 10\,0\cancel{0}\cancel{0} \div 1\,0\cancel{0}\cancel{0} = 10$; $100\,000 \div 10000 = 100 \rightarrow 100\,0\cancel{0}\cancel{0} \div 10\,0\cancel{0}\cancel{0} = 100$.

USING THE BOOK

Use the examples at the top of the page to expand the procedure introduced in the Initial Activity. Illicit from the students the generalization that the number of zeros “stroked out” or “dropped” from the divisor must be the same number of zeros “dropped” from the dividend. You may wish to do the first two examples of each question with the students before assigning the page. Your more able students will be able to do most of the page writing answers only.

ACTIVITIES

1. Have students write questions in a different way.

Example

$9 \overline{) 90} \rightarrow 1 \text{ ten } \overline{) 9 \text{ tens}}$

$3 \overline{) 600} \rightarrow 2 \text{ hundreds } \overline{) 6 \text{ hundreds}}$

2. Students make lists of multiplication and division sentences such as:

$5 \times 10 = 50$ $50 \div 10 = 5$
 $40 \times 10 = 400$ $400 \div 10 = 40$
 $30 \times 100 = 3000$ $3000 \div 100 = 30$ etc.

3. Students make up problems similar to the ones in the display. They write them on chart paper and invite other students to solve them.

4. Play “Concentration” as described in the Activity Reservoir. Use matching cards which show basic division facts and division facts using divisors of 10, 100, and 1000.

OBJECTIVE

To review the division algorithm

PACING

- Level A 1-4(d)
- Level B 1-4
- Level C 2-6

MATERIALS

abacus

RELATED AIDS

- BFA COMP LAB II — 51.
- BFA PROB. SOLVING LAB II — 70, 74.
- CALC. ACTIVITY MASTERS — 8.

SUGGESTIONS

Initial Activity Write the division question $2\overline{)600}$ on the board. Have students show 6 hundred on an abacus. Have students show on the abacus the answer when 6 hundred is divided by 2. [3 hundred] Point out that in division they should focus on $6 \div 2$ first, but the answer is recorded as 3 hundred because of its place value. Expand on this with a few more division questions.

USING THE BOOK

Go over step by step the example developed in the book. Use of the phrases “divided part” and “undivided part” may prove useful when doing this.

With less able students you may wish to do the example in Exercise 1 on the board.

Baking

Mary baked 145 cookies. She wanted to divide them evenly among 6 friends.
How many cookies did she give each friend?
How many did she have left over?

Division Steps:

1. Divide.

Think: How many 6's in 14?

There are 2.

Write the 2 above the 4
2. Multiply.

Think: 2 times 6 is 12.

Write the 12 under the 14.
3. Subtract.

Think: 14 minus 12 is 2.

Write the 2.
4. Check.

Think: Is 2 less than the divisor 6? Yes.
5. Next number.

Write the next number.

5, beside the 2.

Begin again. Work through Steps 1 to 4.

$$\begin{array}{r} 2 \\ 6 \overline{)145} \end{array}$$

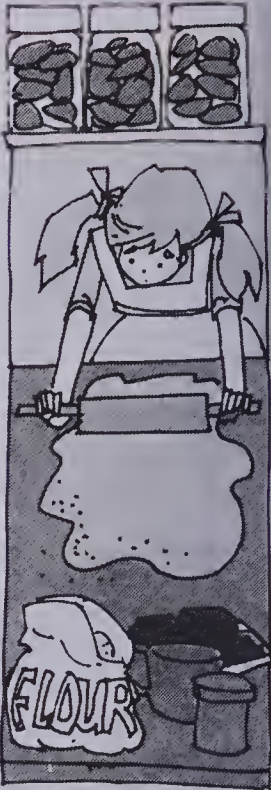
$$\begin{array}{r} 2 \\ 6 \overline{)145} \\ 12 \end{array}$$

$$\begin{array}{r} 2 \\ 6 \overline{)145} \\ -12 \end{array}$$

$$\begin{array}{r} 2 \\ 6 \overline{)145} \\ -12 \end{array}$$

$$\begin{array}{r} 2 \\ 6 \overline{)145} \\ -12 \end{array}$$

$$\begin{array}{r} 24 \text{ R } 1 \\ 6 \overline{)145} \\ -12 \end{array}$$



Exercises

1. Copy and complete.

(a)
$$\begin{array}{r} 29 \text{ R } 3 \\ 7 \overline{) 206} \\ \underline{14} \\ 66 \\ \underline{63} \\ 3 \end{array}$$

(b)
$$\begin{array}{r} 5 \\ 93 \blacksquare \\ 5 \overline{) 4675} \\ \underline{45} \\ 17 \\ \underline{15} \\ 25 \\ \underline{25} \\ 0 \end{array}$$

(c)
$$\begin{array}{r} 17 \\ 1 \blacksquare \blacksquare \text{ R } 5 \\ 8 \overline{) 941} \\ \underline{8} \\ 14 \\ \underline{14} \\ 0 \end{array}$$

2.
$$\begin{array}{r} 128 \\ 7 \overline{) 896} \end{array}$$

(b)
$$\begin{array}{r} 159 \text{ R } 1 \\ 4 \overline{) 637} \end{array}$$

(c)
$$\begin{array}{r} 131 \text{ R } 2 \\ 6 \overline{) 788} \end{array}$$

(d)
$$\begin{array}{r} 169 \\ 5 \overline{) 845} \end{array}$$

(e)
$$\begin{array}{r} 192 \\ 3 \overline{) 576} \end{array}$$

(f)
$$\begin{array}{r} 157 \\ 6 \overline{) 942} \end{array}$$

(g)
$$\begin{array}{r} 185 \text{ R } 1 \\ 2 \overline{) 371} \end{array}$$

(h)
$$\begin{array}{r} 118 \text{ R } 3 \\ 8 \overline{) 947} \end{array}$$

3.
$$\begin{array}{r} 229 \\ 6 \overline{) 1374} \end{array}$$

(b)
$$\begin{array}{r} 487 \\ 8 \overline{) 3896} \end{array}$$

(c)
$$\begin{array}{r} 787 \\ 3 \overline{) 2361} \end{array}$$

(d)
$$\begin{array}{r} 781 \\ 7 \overline{) 5467} \end{array}$$

(e)
$$\begin{array}{r} 756 \text{ R } 1 \\ 7 \overline{) 5293} \end{array}$$

(f)
$$\begin{array}{r} 751 \text{ R } 3 \\ 5 \overline{) 3758} \end{array}$$

(g)
$$\begin{array}{r} 956 \text{ R } 3 \\ 4 \overline{) 3827} \end{array}$$

(h)
$$\begin{array}{r} 781 \text{ R } 4 \\ 9 \overline{) 7033} \end{array}$$

4. (a)
$$\begin{array}{r} 7495 \text{ R } 1 \\ 5 \overline{) 37476} \end{array}$$

(b)
$$\begin{array}{r} 2176 \text{ R } 5 \\ 9 \overline{) 19589} \end{array}$$

(c)
$$\begin{array}{r} 7877 \text{ R } 2 \\ 6 \overline{) 47264} \end{array}$$

(d)
$$\begin{array}{r} 5940 \\ 4 \overline{) 23760} \end{array}$$

(e)
$$\begin{array}{r} 4498 \text{ R } 3 \\ 8 \overline{) 35987} \end{array}$$

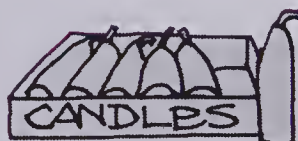
(f)
$$\begin{array}{r} 7557 \text{ R } 5 \\ 7 \overline{) 52904} \end{array}$$

(g)
$$\begin{array}{r} 15895 \text{ R } 1 \\ 3 \overline{) 47686} \end{array}$$

(h)
$$\begin{array}{r} 16649 \\ 5 \overline{) 83245} \end{array}$$

5. Each box holds 6 candles. There are 3684 candles.

How many boxes will be needed to pack the candles? **614**



★ 6. Five boys delivered 6340 handbills.

How many handbills did each boy deliver if each delivered the same number? **1268**

ACTIVITIES

1. To review place value, see "The P.V. Game" as described in the Activity Reservoir.

2. Make a chart with 3 columns.

Divisor	Dividend	Quotient
7	42 000	70
9	240	60
6	180	40
4	280	7000
2	630	90

The student matches up the 3 numbers which form 5 division questions.

3. Pose the problem:

You have won \$10 000.

You begin to spend it by spending \$5 a day.

How many days would it take to spend your money?

OBJECTIVE

To divide by 2-digit divisors, with no remainders

PACING

Level A 1-3, 5(a)-(d), 6(a)-(d)
Level B 2-6
Level C 5-8

RELATED AIDS

BFA PROB. SOLVING LAB II — 78, 82, 86.

CALC. ACTIVITY MASTERS — 52.

SUGGESTIONS

Initial Activity Write this problem on the board.

There are 312 bottles of pop.

24 bottles per case.

How many cases?

Write the question $24 \overline{)312}$. Solve the problem step by step, beginning: "Will 24 go into 3?" [No] "Will 24 go into 31?" [Yes] Then we think 2 into 3. Write the answer "1" above the "1" in the dividend. Multiply 1×24 and write "24" under "31". Subtract. The answer is 7. Write the "2" beside the "7". Begin again and work through the steps until the question is completed.

USING THE BOOK

Go over the problem in the display at the top of page 98, reinforcing the steps developed in the Initial Activity.

With less able students you may wish to do the first few examples of each exercise before assigning the rest. In Exercise 6, emphasize that when you divide and the answer is zero, the "0" must be shown in the quotient.

Parking Lots

Pete's Parking has room for 850 cars. There are 25 spaces for cars in each row. How many rows are there in Pete's Parking?

Divide.

$$\begin{array}{r} 34 \\ 25 \overline{)850} \\ \underline{75} \\ 100 \\ \underline{100} \\ 0 \end{array}$$

Check by multiplying:

$$\begin{array}{r} 34 \\ \times 25 \\ \hline 170 \\ 680 \\ \hline 850 \end{array}$$

There are 34 rows in Pete's Parking.



Exercises

1. Divide. Check by multiplying.

(a) $\begin{array}{r} 21 \\ 30 \overline{)630} \\ \underline{60} \\ 30 \\ \underline{30} \\ 0 \end{array}$

$\bullet \begin{array}{r} 13 \\ 70 \overline{)910} \\ \underline{70} \\ 210 \\ \underline{210} \\ 0 \end{array}$

(c) $\begin{array}{r} 13 \\ 40 \overline{)520} \end{array}$

(d) $\begin{array}{r} 8 \\ 90 \overline{)720} \end{array}$

(e) $\begin{array}{r} 13 \\ 50 \overline{)650} \end{array}$

(f) $\begin{array}{r} 8 \\ 70 \overline{)560} \end{array}$

(g) $\begin{array}{r} 12 \\ 20 \overline{)240} \end{array}$

(h) $\begin{array}{r} 11 \\ 60 \overline{)660} \end{array}$

2. Divide.

(a) $\begin{array}{r} 6 \\ 47 \overline{)282} \\ \underline{282} \\ 0 \end{array}$

$\bullet \begin{array}{r} 5 \\ 58 \overline{)290} \\ \underline{290} \\ 0 \end{array}$

(c) $\begin{array}{r} 4 \\ 82 \overline{)328} \end{array}$

(d) $\begin{array}{r} 2 \\ 54 \overline{)108} \end{array}$

(e) $\begin{array}{r} 8 \\ 29 \overline{)232} \end{array}$

(f) $\begin{array}{r} 6 \\ 71 \overline{)426} \end{array}$

(g) $\begin{array}{r} 4 \\ 56 \overline{)224} \end{array}$

(h) $\begin{array}{r} 4 \\ 62 \overline{)248} \end{array}$

3. At Civic Parking, a row holds 20 cars.

How many rows would be needed to park 360 cars? **18**

4. Centre City Parking Lot holds 576 cars.

There are 18 rows.

How many spaces are there in each row? **32**



5. Find the quotient.

$$\begin{array}{r} 36 \\ 43 \overline{)1548} \\ \underline{129} \\ 258 \\ \underline{258} \\ 0 \end{array}$$

$$\begin{array}{r} 25 \\ 75 \overline{)1875} \\ \underline{150} \\ 375 \\ \underline{375} \\ 0 \end{array}$$

$$\begin{array}{r} 64 \\ 33 \overline{)2112} \\ \underline{66} \\ 112 \\ \underline{112} \\ 0 \end{array}$$

$$\begin{array}{r} 64 \\ 62 \overline{)3968} \\ \underline{124} \\ 272 \\ \underline{272} \\ 0 \end{array}$$

6. Divide.

$$\begin{array}{r} 302 \\ 27 \overline{)8154} \\ \underline{81} \\ 05 \\ \underline{0} \\ 54 \\ \underline{54} \\ 0 \end{array}$$

$$\begin{array}{r} 206 \\ 32 \overline{)6592} \\ \underline{64} \\ 192 \\ \underline{192} \\ 0 \end{array}$$

$$\begin{array}{r} 105 \\ 44 \overline{)4620} \\ \underline{44} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

$$\begin{array}{r} 103 \\ 56 \overline{)5768} \\ \underline{56} \\ 8 \\ \underline{8} \\ 0 \end{array}$$

7. Find the quotient.

$$\begin{array}{r} 194 \\ 56 \overline{)10864} \\ \underline{56} \\ 526 \\ \underline{504} \\ 224 \\ \underline{224} \\ 0 \end{array}$$

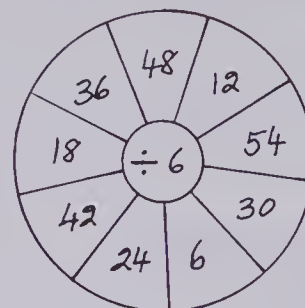
$$\begin{array}{r} 356 \\ 64 \overline{)22784} \\ \underline{192} \\ 358 \\ \underline{352} \\ 64 \\ \underline{64} \\ 0 \end{array}$$

$$\begin{array}{r} 287 \\ 47 \overline{)13489} \\ \underline{94} \\ 408 \\ \underline{401} \\ 9 \\ \underline{9} \\ 0 \end{array}$$

8. The City Music Centre parking lot was full every evening for 35 d straight. During this time it held a total of 14 420 cars. How many cars each evening? **412**

ACTIVITIES

1. Do a clock drill orally with the class.



2. See "Football" as described in the Activity Reservoir. Use question cards which drill division (and related multiplication) facts.

3. See "Itza Fact!" as described in the Activity Reservoir.

OBJECTIVE

To divide by 2-digit divisors, with remainders

PACING

Level A 1-4(d), 5(a)-(e)
Level B 1, 3-5
Level C 4-7

RELATED AIDS

HMS — DM24.
BFA COMP LAB II — 53-56.
CALC. ACTIVITY MASTERS — 39, 84.

SUGGESTIONS

Initial Activity On the board write this problem:
There are 685 chairs to be arranged in rows.
Each row has 25 chairs.
How many rows and how many chairs left over?

Write the division question $25 \overline{)685}$.
Solve it step by step with the students.
Check by multiplying.

USING THE BOOK

Go over the rolling-coins problem step by step. With less able students you may wish to do the first few examples of each exercise before assigning the remainder of the exercises. In Exercise 5, emphasize that when a zero occurs in the quotient it must be recorded. Remind students that once a figure is placed in the quotient every digit in the dividend must have a corresponding digit in the quotient, zero included.

Rolling Coins

Pat saved 736 pennies. He rolled them in brown paper rolls with 50 pennies in each roll.
How many rolls did he have? **14**
How many pennies did he have left over? **36**

Divide. $50 \overline{)736}$

$$\begin{array}{r} 14 \text{ R } 36 \\ 50 \overline{)736} \\ \underline{50} \\ 236 \\ \underline{200} \\ 36 \end{array}$$

Check by multiplying:

$$\begin{array}{r} 14 \\ \times 50 \\ \hline 700 \\ + 36 \text{ (add the remainder)} \\ \hline 736 \end{array}$$



He had 14 rolls and 36 pennies left over.

Exercises

1. Divide.

(a) $40 \overline{)207}$

$$\begin{array}{r} 5 \text{ R } 7 \\ 40 \overline{)207} \\ \underline{200} \\ 7 \end{array}$$

(b) $70 \overline{)569}$

$$\begin{array}{r} 8 \text{ R } 9 \\ 70 \overline{)569} \\ \underline{560} \\ 9 \end{array}$$

(c) $30 \overline{)246}$

$$\begin{array}{r} 8 \text{ R } 6 \\ 30 \overline{)246} \\ \underline{240} \\ 6 \end{array}$$

(d) $60 \overline{)542}$

$$\begin{array}{r} 9 \text{ R } 2 \\ 60 \overline{)542} \\ \underline{540} \\ 2 \end{array}$$

(e) $20 \overline{)165}$

$$\begin{array}{r} 8 \text{ R } 5 \\ 20 \overline{)165} \\ \underline{160} \\ 5 \end{array}$$

(f) $90 \overline{)724}$

$$\begin{array}{r} 8 \text{ R } 4 \\ 90 \overline{)724} \\ \underline{720} \\ 4 \end{array}$$

(g) $80 \overline{)481}$

$$\begin{array}{r} 6 \text{ R } 1 \\ 80 \overline{)481} \\ \underline{480} \\ 1 \end{array}$$

2. Find the quotient and the remainder

(a) $73 \overline{)456}$

$$\begin{array}{r} 6 \text{ R } 18 \\ 73 \overline{)456} \\ \underline{438} \\ 18 \end{array}$$

(b) $64 \overline{)273}$

$$\begin{array}{r} 4 \text{ R } 17 \\ 64 \overline{)273} \\ \underline{256} \\ 17 \end{array}$$

(c) $52 \overline{)475}$

$$\begin{array}{r} 9 \text{ R } 7 \\ 52 \overline{)475} \\ \underline{468} \\ 7 \end{array}$$

(d) $94 \overline{)599}$

$$\begin{array}{r} 6 \text{ R } 35 \\ 94 \overline{)599} \\ \underline{564} \\ 35 \end{array}$$

(e) $81 \overline{)408}$

$$\begin{array}{r} 5 \text{ R } 3 \\ 81 \overline{)408} \\ \underline{405} \\ 3 \end{array}$$

(f) $72 \overline{)219}$

$$\begin{array}{r} 3 \text{ R } 3 \\ 72 \overline{)219} \\ \underline{216} \\ 3 \end{array}$$

(g) $56 \overline{)397}$

$$\begin{array}{r} 7 \text{ R } 5 \\ 56 \overline{)397} \\ \underline{392} \\ 5 \end{array}$$

3. Mrs. Blackwood has 185 coins in her collection.

She uses 15 coins to fill a page in her album.

How many full pages? **12**

How many coins left over? **5**



4. Find each quotient and remainder

- (a) $93 \overline{)5591}$ **60 R11** (b) $42 \overline{)5782}$ **137 R28** (c) $86 \overline{)6331}$ **73 R53** (d) $99 \overline{)7185}$ **72 R57**
 (e) $83 \overline{)5939}$ **71 R46** (f) $39 \overline{)1434}$ **36 R30** (g) $31 \overline{)6549}$ **211 R8** (h) $22 \overline{)4895}$ **222 R11**

5. Divide.

- (a) $42 \overline{)12946}$ **308 R10**
 $\begin{array}{r} 126 \\ \hline 34 \\ 00 \\ \hline 346 \\ 336 \\ \hline 10 \end{array}$
10
 (b) $93 \overline{)46769}$ **502 R83**
 (c) $34 \overline{)35351}$ **1039 R25**
 (d) $78 \overline{)31649}$ **405 R59**
 (e) $45 \overline{)91376}$ **2030 R26**
 (f) $62 \overline{)89321}$ **1440 R41**
 (g) $81 \overline{)80117}$ **989 R8**
 (h) $48 \overline{)50800}$ **1058 R16**
 (i) $56 \overline{)90000}$ **1607 R8**

6. Find each quotient and remainder

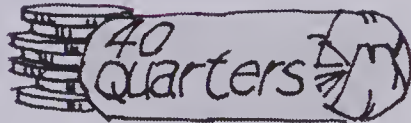
- (a) $54 \overline{)53794}$ **996 R10**
 $\begin{array}{r} 486 \\ \hline 519 \\ 486 \\ \hline 334 \\ 324 \\ \hline 10 \end{array}$
 (b) $23 \overline{)20976}$ **912**
 (c) $65 \overline{)64682}$ **995 R7**
 (d) $33 \overline{)32487}$ **984 R15**
 (e) $95 \overline{)93658}$ **985 R83**
 (f) $84 \overline{)72600}$ **864 R24**
 (g) $42 \overline{)88000}$ **2095 R10**
 (h) $19 \overline{)86008}$ **4526 R14**
 (i) $77 \overline{)10007}$ **129 R74**

7. At the bank, Mrs. Wilson has 865 quarters to roll.

She packs 40 coins to a roll.

How many full rolls is this? **21**

How many quarters will be left over? **25**



ACTIVITIES

1. Play the game "Bingo" as described in the Activity Reservoir. Use it to review division facts.

2. To provide practice using a frequently-required skill for the long division algorithm, have students multiply 2 digits by 1 digit in *horizontal* fashion.

$$78 \times 4 =$$

$$26 \times 9 =$$

$$33 \times 5 =$$

etc.

3. Have the students make up some "fill-in-the-blank" type puzzlers such as:

$$\begin{array}{r} \square 0 \square R \square \square \\ 78 \overline{)31649} \\ \underline{31 \square} \\ \square \square \\ \square \\ \square \square 9 \\ \underline{3 \square 0} \\ \square \square \end{array}$$

OBJECTIVE

To divide 1-decimal dividends by 1- and 2-digit whole-number divisors

PACING

Level A 1-4
Level B 1-5
Level C All

RELATED AIDS

BFA COMP LAB II — 110.
BFA PROB. SOLVING LAB II — 139.

SUGGESTIONS

Initial Activity Have the students estimate the quotient for $5 \overline{)22.5}$; it is greater than 4 since $5 \times 4 = 20$ but less than 5 since $5 \times 5 = 25$.

Repeat for similar questions. Have students give their estimate in this form:

It is greater than ____
since ____ \times ____ = ____.
It is less than ____
since ____ \times ____ = ____.

USING THE BOOK

The point to be emphasized here is that division with a decimal in the dividend is computed in the same way as regular division. However, a decimal point must be placed in the quotient immediately above the decimal point in the dividend. Go over the question in the display at the top of the page and highlight this point. The check by multiplying will help convince students that this procedure is valid.

With less able students you may wish to do the first example of Exercises 1, 2, and 3 on the board before assigning the rest of the exercises.

ACTIVITIES

1. Using division problems play the game "Eraser" as outlined in the Activity Reservoir.

2. Have the students prepare decks of cards which show numerals 0 to 9 (5 of each numeral so that each deck = 50 cards). Prepare blank division problems as shown so that there are enough problems for each player per round.

$$\begin{array}{r} \square.\square \\ 28 \overline{)95.2} \\ \underline{\square\square} \\ \square 12 \\ \underline{\square 12} \\ 0 \end{array}$$

Students take turns (a) drawing 1 card from the shuffled, face-down deck; (b) placing it in an appropriate (if possible) blank space on their

Skipping Ropes

The total length of 5 skipping ropes is 22.5 m.
What is the length of 1 skipping rope?

Divide $5 \overline{)22.5}$ Check by multiplying.

$$\begin{array}{r} 4.5 \\ 5 \overline{)22.5} \\ \underline{20} \\ 25 \\ \underline{25} \\ 0 \end{array}$$

$$\begin{array}{r} 4.5 \\ \times 5 \\ \hline 22.5 \end{array}$$



The length of one skipping rope is 4.5 m

Exercises

1. Find the quotient

(a) $3 \overline{)70.8}$ **23.6** (b) $4 \overline{)67.2}$ **16.8** (c) $7 \overline{)30.1}$ **4.3** (d) $5 \overline{)48.0}$ **9.6**
(e) $9 \overline{)78.3}$ **8.7** (f) $4 \overline{)19.2}$ **4.8** (g) $8 \overline{)47.2}$ **5.9** (h) $3 \overline{)23.1}$ **7.7**

2. Divide

(a) $23 \overline{)13.8}$ **0.6** (b) $62 \overline{)18.6}$ **0.3** (c) $14 \overline{)50.4}$ **3.6** (d) $28 \overline{)95.2}$ **3.4**

3. Find the quotient.

(a) $25.2 \div 28$ **0.9** (b) $79.2 \div 36$ **2.2** (c) $40.5 \div 81$ **0.5** (d) $98.4 \div 41$ **2.4**

4. The total length of 8 skipping ropes is 16.8 m.

What is the length of 1 skipping rope? **2.1 m**

5. For a class party, 33 students want to make a paper chain. They want the chain to be 75.9 m long. How much should each person make? **2.3 m**

★6. Patricia, Jane, and Sonia collected equal amounts of apples. Altogether they gathered 98.4 kg. How many kilograms of apples did each of them pick? **32.8 kg**



problem; (c) keeping unused cards in hand. First player to correctly complete his/her problem wins. All other players score a number of points equal to the number of unused cards they hold in their hand. Player with the *least* point total after 5 rounds is the overall winner.

3. To drill division and other facts, see "Itza Fact!" as described in the Activity Reservoir.

"Rainy Recess" Games

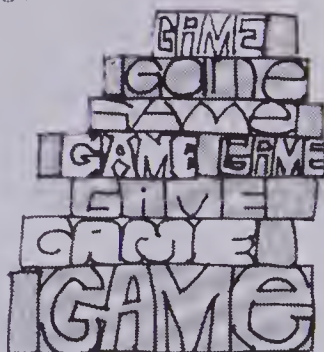
Mr. Richards' Grade 6 class at Young St. School collected \$63.84 to buy "Rainy Recess" games. They bought 8 different games. Each one was the same price. What was the price of each game?

Divide $8 \overline{) \$63.84}$ Check by multiplying:

$$\begin{array}{r} 7.98 \\ 8 \overline{) \$63.84} \\ \underline{56} \\ 78 \\ \underline{72} \\ 64 \\ \underline{64} \\ 0 \end{array}$$

$$\begin{array}{r} \$7.98 \\ \times 8 \\ \hline \$63.84 \end{array}$$

The price of each game was \$7.98



Exercises

- Divide.

(a) $9 \overline{) \$58.32}$ $\$6.48$	(b) $5 \overline{) \$32.85}$ $\$6.57$	(c) $4 \overline{) \$21.36}$ $\$5.34$	(d) $8 \overline{) \$55.36}$ $\$6.92$
(e) $\$69.12 \div 8$ $\$8.64$	(f) $\$27.84 \div 3$ $\$9.28$	(g) $\$11.55 \div 7$ $\$1.65$	(h) $\$49.15 \div 5$ $\$9.83$
- Find the quotient.

(a) $9 \overline{) 208.53}$ 23.17	(b) $5 \overline{) 119.45}$ 23.89	(c) $8 \overline{) 600.32}$ 75.04	(d) $6 \overline{) 147.9}$ 24.65
(e) $4 \overline{) 251.24}$ 62.81	(f) $7 \overline{) 597.52}$ 85.36	(g) $8 \overline{) 451.92}$ 56.49	(h) $6 \overline{) 411.96}$ 68.66
- Divide.

(a) $27 \overline{) 64.53}$	(b) $35 \overline{) 222.25}$	(c) $62 \overline{) 517.08}$	(d) $45 \overline{) 373.95}$
(e) $89 \overline{) \$529.55}$	(f) $74 \overline{) \$319.68}$	(g) $87 \overline{) \$749.07}$	(h) $26 \overline{) \$203.58}$
- Grandma Murphy bought 6 games for her grandchildren. Each game was the same price. She spent \$23.94 altogether. What was the price of one? $\$3.99$
- Dana, George, and Pierre saved \$61.50 to buy a TV computer game. They each saved the same amount. How much did each save? $\$20.50$

2-decimal dividends, whole-number divisors 103

ANSWERS:

3. (a) 2.39 (b) 6.35 (c) 8.34 (d) 8.31 (e) \$5.95 (f) \$4.32
(g) \$8.61 (h) \$7.83

OBJECTIVE

To divide 2-decimal dividends by 1- and 2-digit whole-number divisors

PACING

Level A All
Level B All
Level C 3-5

RELATED AIDS

HMS — DM25.
BFA COMP LAB II — 111.
BFA PROB. SOLVING LAB II — 102, 114.
CALC. ACTIVITY MASTERS — 53.

SUGGESTIONS

Initial Activity Repeat the Initial Activity described on page 102 to estimate the quotient. Use the format shown there.

On the board write the following:

$$2 \overline{) \$4.50} \quad 3 \overline{) \$6.75} \quad 4 \overline{) \$8.44}$$

Have the students mentally compute the quotients. Write the quotients in their proper place, e.g., \$2.25

$2 \overline{) \$4.50}$. In each example, have the students find the relationship between the placement of the decimal point in the quotient and the location of the decimal point in the dividend.

Reinforce the concept learned in the previous lesson:

"When a question contains a decimal point in the dividend, divide in the regular way. However, place a decimal point in the quotient directly above the decimal point in the dividend." Check by estimating the quotients.

USING THE BOOK

You will probably wish to have students complete this page over 2 or 3 periods. Students often make errors, not through lack of understanding, but through tedium, doing too much of the same kind of computation at one time. Having divided up the page, as an added incentive to compute directly, have students estimate the number of questions they will compute correctly. This will give them a goal to achieve or surpass.

ACTIVITIES

1. Using catalogues and magazines, have students make up a list of the games they would like for "rainy recesses".

2. Using the catalogues and magazines mentioned above, have some students (a) cut out pictures of 5 different-priced games and mount them on cardboard with prices showing; (b) choose 3 of the games and multiply the price of each by a different number (i.e., multiply the

\$7.98 game by 6; multiply the \$12.44 game by 15; multiply the \$9.95 game by 8); (c) challenge fellow students with an accompanying card such as:

6 _____ games cost \$47.88.

15 _____ games cost \$186.60.

8 _____ games cost \$79.60.

Divide to find which games I chose.

OBJECTIVES

To estimate lengths

To convert between certain units of measure:

- millimetres, centimetres, and metres
- metres, hectometres, and kilometres

PACING

Level A All

Level B All

Level C All

MATERIALS

metre sticks or tapes in millimetres and centimetres

RELATED AIDS

BFA PROB. SOLVING LAB II — 97.

SUGGESTIONS

Initial Activity Review the prefixes milli-, centi-, hecto-, kilo-.

USING THE BOOK

Assign the exercises. If the students require it, have them make the measurements for Exercises 13 to 16. Encourage the students to look at a metre stick for Exercises 1, 2, 5, 7, and 8 to see if their answers are reasonable.

ACTIVITIES

1. Play "Metric Snap". Prepare 3 sets of cards consisting of the symbols for the prefixes; the prefixes; and the numerals, both fraction and decimal, for each prefix. (Use only the symbols used on this page — the balance will be included later.) The cards are shuffled and dealt to two players. Then the game is played like "Snap".

Example

c	centi-	$\frac{1}{100}$	0.01
---	--------	-----------------	------

2. Have a scavenger hunt. Each student is to find 2 objects or lengths that are: 1 mm, 1 cm, 1 m, 1 hm, and 1 km. They need only make a list for each length.

3. Have some of the unit pairs from Exercises 1 to 12 written on cards.

Example

1 cm	10 mm	1 m	100 cm
------	-------	-----	--------

Use the cards to play "Concentration" as described in the Activity Reservoir.

Units of Length

10 mm = 1 cm

100 cm = 1 m

100 m = 1 hm

1000 m = 1 km

A centimetre is 10 times longer than a millimetre.

A metre is 100 times longer than a centimetre.

A hectometre is 100 times longer than a metre.

A kilometre is 1000 times longer than a metre.

Exercises

Copy and complete.

- 1 cm = $\frac{10}{45}$ mm 1 m = $\frac{100}{760}$ cm 1 hm = $\frac{10000}{0.65}$ cm 4. 1 km = $\frac{1000}{7.8}$ m
5. 4.5 cm = $\frac{0.95}{7}$ mm 6. 7.6 m = $\frac{7}{7}$ cm 7. 65 cm = $\frac{0.5}{0.5}$ m 8. 78 mm = $\frac{4.5}{4.5}$ cm
9. 950 m = $\frac{0.95}{7}$ km 10. 700 m = $\frac{7}{7}$ hm 11. 50 m = $\frac{0.5}{0.5}$ hm 12. 450 m = $\frac{4.5}{4.5}$ hm

Choose the best unit of measure.

13.  14.  15.  16. 

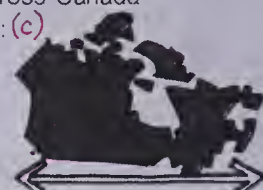
Estimate the length in centimetres.

Use your ruler to check.

17. 17 cm
18. 6 cm
19. 9 cm

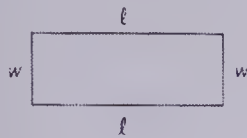
Choose the best answer.

20. The distance around the earth at the equator is: (a)
- (a) 40 000 km (b) 40 000 m (c) 40 km.
21. The distance across Canada is approximately: (c)
- (a) 5000 m (b) 5000 hm (c) 5000 km.



Perimeter

Perimeter is the distance around a shape

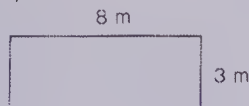


The length is l units.

The width is w units

$$\text{Perimeter} = l + w + l + w$$

$$P = 2l + 2w$$



$$\text{Perimeter} = l + w + l + w$$

$$P = 8 \text{ m} + 3 \text{ m} + 8 \text{ m} + 3 \text{ m}$$

$$P = 22 \text{ m}$$

Or

$$\text{Perimeter} = 2l + 2w$$

$$P = 2 \times 8 + 2 \times 3$$

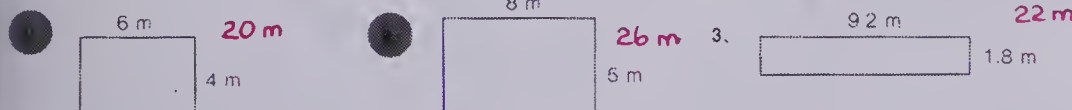
$$P = 22 \text{ m}$$

The perimeter is 22 m.

Exercises

Calculate the perimeter of each rectangle using the formula:

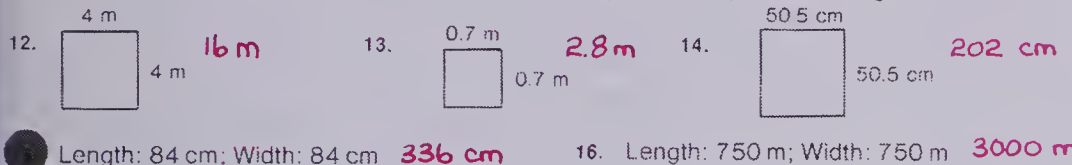
$$P = l + w + l + w \text{ or } P = 2l + 2w.$$



Draw a rectangle and label. Calculate the perimeter.

	Length	Width	Perimeter		Length	Width	Perimeter
1.	8 cm	4 cm	24 cm	5.	16 cm	12 cm	56 cm
6.	13 cm	11 cm	48 cm	7.	22 cm	19 cm	82 cm
8.	45 cm	28 cm	146 cm	9.	17.8 m	23.3 m	82.2 m
10.	192.6 m	86.9 m	559 m	11.	1.7 km	0.8 km	5 km

Calculate the perimeter of each rectangle. *Hint: A square is a special rectangle.*



Perimeter formula for a rectangle 105

OBJECTIVE

To calculate the perimeter of a rectangle using the formula

$$P = l + w + l + w$$

$$\text{or } P = 2l + 2w.$$

PACING

Level A 1-8, 12-16

Level B All

Level C 4-16

MATERIALS

metre tapes

RELATED AIDS

BFA PROB. SOLVING LAB II — 125, 129.

CALC. ACTIVITY MASTERS — 12.

BACKGROUND

There are three formulas commonly used for the perimeter of a rectangle. Each has its advantages. Some students may prefer to stay with $P = l + w + l + w$ since it relates most closely to the basic steps in calculating the perimeter. The more able students will see the advantage of $P = 2l + 2w$. Still other students may want to use a third formula $P = 2(l + w)$. All are equally acceptable. The perimeter of the special rectangle — the square — is $P = 4l$. Depending on your class, you may wish to use only one formula or you may wish to provide some students with them all.

SUGGESTIONS

Initial Activity Ask a student or students to measure each side of a rectangular surface such as the teacher's desk, small chalkboard, etc. Write these measures on the chalkboard. Discuss with the class how to find the distance around (perimeter) the rectangle. From this you can develop $P = l + w + l + w$.

USING THE BOOK

Review with the students the development of the one (or more) formulas for perimeter. Emphasize that $P = 2l + 2w$ is just a short-cut way of writing $P = l + w + l + w$.

Ask the students to do Exercise 1. Check to see which formula each student uses and that the student uses it correctly. Assign the balance of the exercises.

ACTIVITIES

1. Have students use geo-boards to make as many different rectangles as they can. Then record the size of the rectangle and the perimeter.

Example

Length	Width	Perimeter
6 cm	4 cm	20 cm

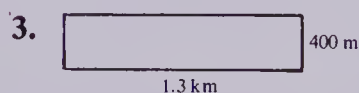
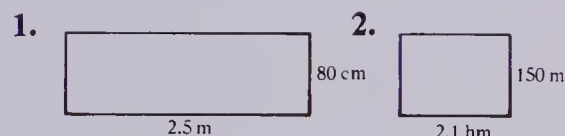
2. Prepare in advance a set of rectangles made from tag. Number them. The students are to measure and calculate the perimeter of each.

3. See the "Tangram" ideas in the Activity Reservoir.

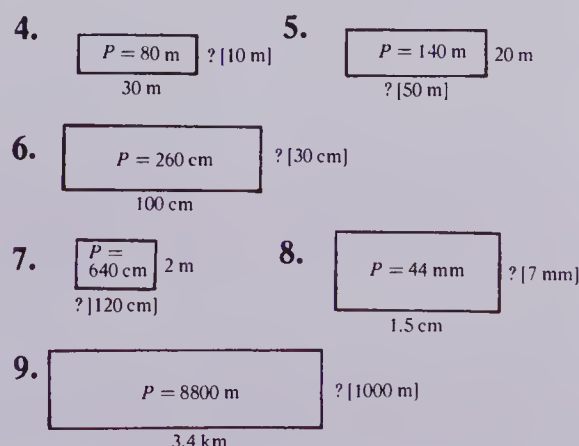
4. Start a "Math Formulas" page in the back of the students' notebooks. You may also wish to have a "Math Formulas" Corner in your classroom where formulas are written on library cards and tacked up on the bulletin board.

EXTRA PRACTICE

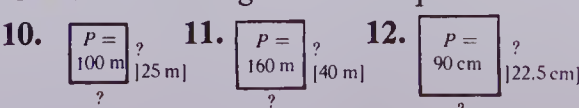
Calculate the perimeter of the rectangles.



Calculate the missing length or width of the rectangles.



Calculate the length of each square.



OBJECTIVES

To discover the relationship for a circle:

$$\frac{\text{circumference}}{\text{diameter}}$$

To use an approximation for pi to calculate an approximation for the circumference of a circle given the diameter

PACING

Level A Activity, 1-6

Level B All

Level C All

VOCABULARY

circumference, diameter

MATERIALS

various cylindrical cans, tape measures in millimetres, rulers in millimetres, cardboard strips in various sizes (see Initial Activity)

RELATED AIDS

HMS — DM26.

CALC. ACTIVITY MASTERS — 86.

BACKGROUND

The relation between the diameter of a circle and its circumference is not difficult when time is spent on developing it. The student must have ample opportunity in real situations to measure to see that the circumference is a little more than three times the diameter. When this is done numerous times, students readily accept the meaning of pi as the number of times larger that the circumference is than the diameter.

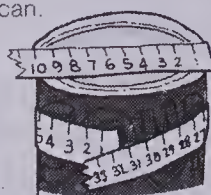
Remember to emphasize that measurements are approximate; they are not exact. Therefore the values obtained for circumference ÷ diameter or π may vary from experiment to experiment. Students should be made aware of the reasons the values may vary.

Circumference

Circumference is the distance around a circle.

Henry measured the circumference and diameter of a circular can. He calculated the ratio:

$$\frac{\text{circumference}}{\text{diameter}} = \frac{31.5 \text{ cm}}{10 \text{ cm}} = 3.15$$



Mary Lou repeated the experiment with another can.

$$\frac{\text{circumference}}{\text{diameter}} = \frac{25.6 \text{ cm}}{8 \text{ cm}} = 3.2$$

The circumference of a circle is a little more than 3 times its diameter.

Activity

1. Use a circular can. Repeat Henry's experiment

$$\frac{\text{circumference}}{\text{diameter}} = \frac{\triangle}{\blacksquare}$$

$$\text{Calculate } \frac{\text{circumference}}{\text{diameter}} = \frac{\triangle}{\blacksquare}$$

Copy and complete.

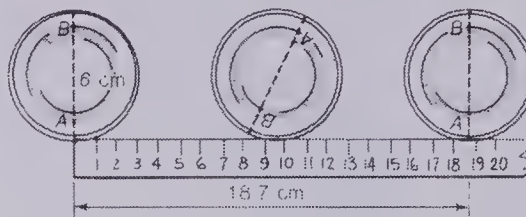
The circumference is a little more than $\frac{3}{\blacksquare}$ times longer than its diameter.

2. Repeat Henry's experiment with another circular can. Calculate: $\frac{\text{circumference}}{\text{diameter}} = \frac{\triangle}{\blacksquare}$

Copy and complete

The circumference is a little more than $\frac{3}{\blacksquare}$ times longer than its diameter.

3. Tom marked and rolled a can to measure the circumference



$$\frac{\text{circumference}}{\text{diameter}} = \frac{18.7}{6} = 3.12$$

Repeat Tom's experiment for several cans. Calculate this ratio for each: $\frac{\text{circumference}}{\text{diameter}}$

Pi

Mathematicians write the relations:

$$\frac{\text{circumference of a circle}}{\text{diameter}} = \pi \text{ (Pi)}$$

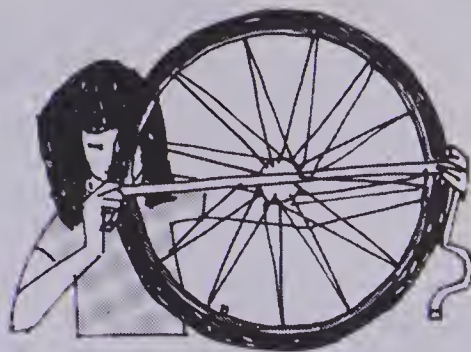
The number that π represents cannot be written exactly in decimal form.

We use $\pi = 3.14$.

We write for a circle:

$$C = \pi \times d$$

$$C = 3.14 \times d$$



Exercises

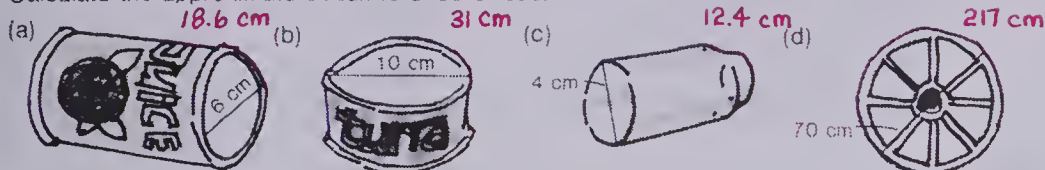
- Copy and complete the chart.

Circular Can	Circumference (C)	Diameter (d)	$C \div d$ (2 decimals)
a	28.3 cm	9 cm	3.14 cm
b	19.2 cm	6 cm	3.20 cm
c	37.2 cm	12 cm	3.10 cm

- Henry measured the diameter of 4 circular objects.

He knew the circumference was a little more than 3 times the diameter.

Calculate the approximate circumference of each.



Calculate the circumference of each circle. Use $\pi = 3.14$.

$$C = \pi \times d$$

$$C = 3.14 \times 12$$

$$C = \blacksquare \text{ cm } 37.68$$

Circumference is \blacksquare cm.

6.



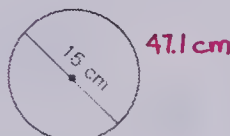
7.



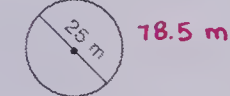
8.



5.



9.



Circumference using π 107

SUGGESTIONS

Initial Activity Review measuring distances in centimetres to one decimal place. Prepare a set of strips of cardboard, e.g., 8.9 cm \times 3 cm; 6.2 cm \times 4.5 cm, etc. Label each A, B, C, etc. and keep a record of each size for easy settling of disputes. Give each group of students several strips. Each student is to measure the length of each and check with the rest of the group.

USING THE BOOK

If tape measures are not available, stout cord may be used and measurements be made with a ruler. The Activity can be done in millimetres, but when done in centimetres to one decimal place, a real practical need for dividing decimals is apparent. Spend at least one day — preferably two days — doing the Activities on Page 106, repeating them for as many tin cans as you have in the class. After doing a couple of cans, in the manner indicated in the Activities on pupil page 106, ask the students to record the data for the remaining cans in a chart like that in Exercise 1, page 107. After the Activities, the exercises can be assigned.

Emphasize constantly that the ratio of the circumference to the diameter is important and that the ratio is the same for all circles. Any differences in the ratios the students find are due only to the approximation of measurements.

ACTIVITIES

- Bring a bicycle into the classroom. Have the students calculate:

- how far the bicycle wheel goes in one revolution;
- how many turns the bicycle wheel makes in going 1 km.

2. Ask the students to research what differences it makes to have small wheels versus large wheels on a car. Actual diameters should be measured and calculations made.

3. *Extension:* Ask some students to research pi.

OBJECTIVE

To calculate the area of a rectangle using the formula $A = l \times w$

PACING

Level A 1-11
Level B All
Level C 1-3, 7-15

VOCABULARY

square metres, square centimetres

RELATED AIDS

BFA PROB. SOLVING LAB II — 126, 130.

CALC. ACTIVITY MASTERS — 70, 71.

BACKGROUND

The development of the formula for the area of a rectangle is based on the number of squares in 1 row *times* the number of rows. This is numerically consistent with length \times width since when we measure the length of a rectangle and find it is 6 cm we determine that 6 centimetre squares will fit in each of the rows. Hence we only need multiply the length and the width.

SUGGESTIONS

Initial Activity Use unit squares (about 10 cm \times 10 cm) to cover a student's desk. Ask how many squares are required. Then place the unit squares along two edges of the teacher's desk (or other suitable surface) as in the display.

Ask: "How many units in one row?"

"How many rows if the surface was covered?"

"How many units altogether to cover the desk?"

"Can you give me a short-cut way to get the answer?" Elicit the answer:

Area = length \times width.

USING THE BOOK

Introduce the symbol for square centimetre (cm^2) and square metre (m^2).

Assign Exercises 1 to 6 emphasizing:

Area = no. in 1 row \times no. of rows.

Assign Exercises 7 to 15 emphasizing the short cut: Area = length \times width.

Area of Rectangles

5 m
3 m

Area of 1 row = 5 m^2
Area of 3 rows = $5 \times 3 \text{ m}^2$
Area = length \times width
 $A = l \times w$

f units
w units
 $A = l \times w$

The area is 15 m^2 . Note: m^2 is read square metres.

Exercises
Find the area.

1. 6 m
3 m
Area is $\blacksquare \text{ m}^2$
18

2. 7 m
5 m
35 m²

3. 8 cm
4 cm
32 cm²

4. 7 cm
3 cm
21 cm²

5. 6 cm
4 cm
24 cm²

6. 7 m
7 m
49 m²

Calculate the area of each rectangle.

7. 16 cm
8 cm
128 cm²

8. 8 m
8 m
64 m²

9. 16.8 m
6.5 m
109.2 m²

10. Length: 8 cm
Width: 5 cm
40 cm²

11. Length: 24 cm
Width: 11 cm
264 cm²

12. Length: 8.3 cm
Width: 6.1 cm
50.63 cm²

13. Length: 18.6 cm
Width: 5.5 cm
102.3 cm²

14. Length: 22.2 cm
Width: 22.2 cm
492.84 cm²

15. Length: 10 m
Width: 10 m
100 m²

108 Area of rectangles using formula

ACTIVITIES

1. Have students write the formula on the "Math Formulas" page in their notebooks.

2. Ask students to find the area of: the chalkboard, the windows, the door, the classroom, etc.

Ask the students to decide what units will be used when measuring the length and width of each surface.

3. Prepare in advance a set of rectangles of different sizes. Number them. The students are to measure them and find the area of each.

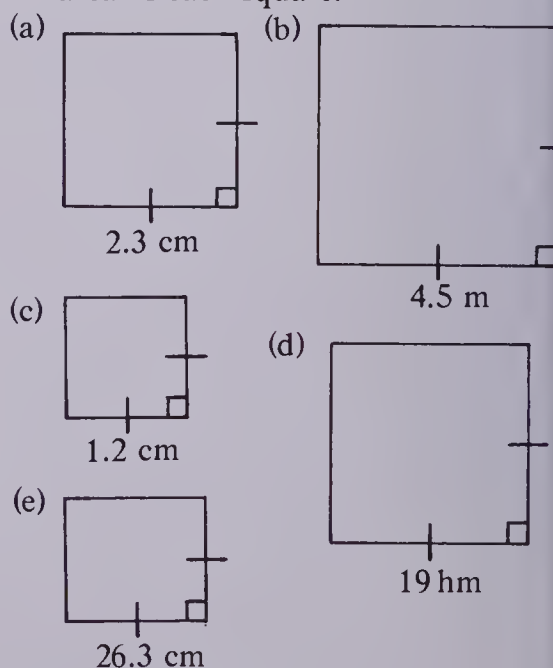
4. *Extension:* You may wish to introduce the formula for the area of a square: $A = s \times s$ where s is the length of each side.

5. Select 5 rectangular areas in the classroom. Have the class estimate the area of each, then measure the sides and calculate the area to check.

6. See the "Tangram" ideas in the Activity Reservoir.

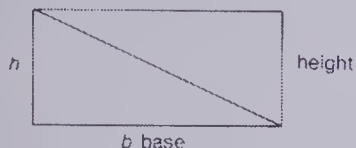
EXTRA PRACTICE

- Calculate the area of each rectangle in Exercises 1 to 16, page 105.
- Calculate the area of each rectangle in Extra Practice, page 105.
- Calculate the perimeter and the area of each square.

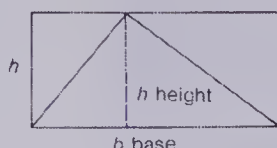


(Continued on page 119)

Area of Triangles



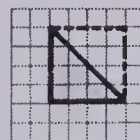
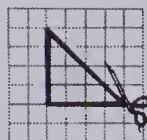
Area of rectangle = length \times width.
Area of triangle is $\frac{1}{2}$ of the rectangle.
Area of triangle = $\frac{1}{2} \times b \times h$.



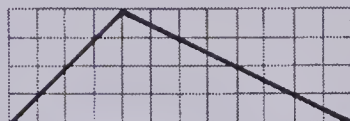
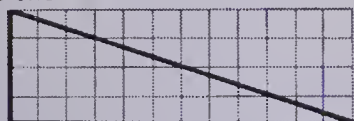
Area of the triangle is one half of the area of the rectangle with the same base and height.
Area of triangle = $\frac{1}{2} \times b \times h$.

Activity

1. Cut out a right triangle from squared paper. Place it on a new piece of squared paper. Compare the areas of the triangle and the parallelogram you can make from it. Repeat for any triangle.

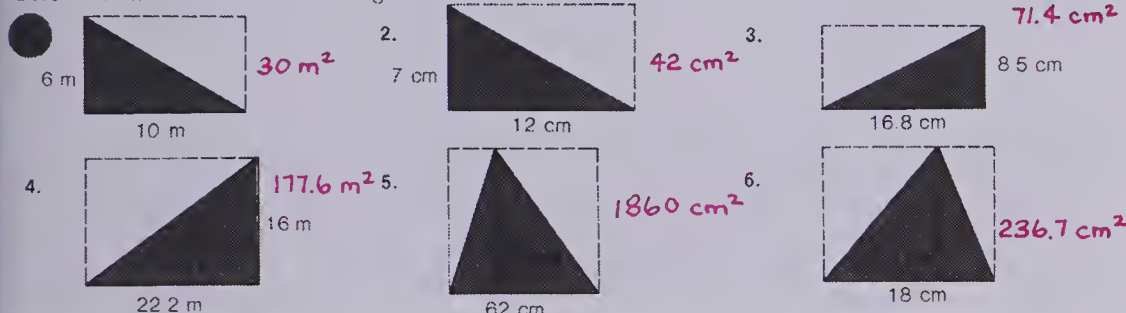


2. Count the squares. Is it true that the area of the triangle is one half the area of the rectangle? **Yes**



Exercises

Calculate the area of these triangles.



Area of triangles using formula 109

OBJECTIVE

To calculate the area of a triangle using the formula $A = \frac{1}{2} \times b \times h$

PACING

Level A All
Level B All
Level C All

MATERIALS

squared paper, scissors, rulers

BACKGROUND

The area of a triangle is developed from the fact that a triangle has an area equal to half that of a rectangle of the same base and same height.

SUGGESTIONS

Initial Activity Illustrate by means of the overhead projector the relationship between a triangle and its related rectangle. Do this once for the right triangle and once for the general triangle. Then say, "Does this relationship hold for triangles of different sizes? I would like you to do the Activity on page 109." (You may wish to have the students do the Activity under your direction rather than from the text where the relationship is stated.)

USING THE BOOK

Following the Initial Activity, discuss the conclusions the students reached. Elicit: "The area of a triangle is equal to half the area of the rectangle with the same base and same height. Then ask, "What is the formula for the area of a rectangle?" [$A = \ell \times w$]

Ask, "What, then, is the formula for the area of a triangle?" [$A = \frac{1}{2} \times \ell \times w$] But, for a triangle we call the length *base*, and the width *height*.

Hence $A = \frac{1}{2} \times b \times h$. Then show how to make replacements for *b* (base of triangle) and *h* (height or altitude of the triangle).

Emphasize the terms base and altitude. Some students may find this form of the formula easier to work with: $A = \frac{b \times h}{2}$.

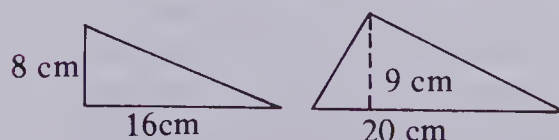
ACTIVITIES

1. Have students put the formula in their "Math Formulas" list.

2. Provide each pair of students with a geo-board and elastic bands. One student makes a triangle and challenges the other student to find its area. Then the two reverse roles. The student with the most correct after 10 turns each is the winner.

3. Prepare a set of 20 triangles (as illustrated). Do not put the measurements on the triangle. Number each triangle and record for

your records their dimensions (for easy corrections). Students are to measure the base and height and calculate the area of each triangle.



4. See the "Tangram" ideas in the Activity Reservoir.

OBJECTIVE

To calculate the area of a parallelogram using the formula $A = b \times h$

PACING

Level A All

Level B All

Level C All

MATERIALS

squared paper for the students, large display parallelogram constructed as described in the Suggestions

BACKGROUND

The area of a parallelogram is developed through its relationship to the related rectangle — the rectangle with the same base and same height.

The length of the base of a parallelogram is represented by b and the height by h . Hence: $A = b \times h$.

SUGGESTIONS

Initial Activity Prepare this demonstration parallelogram with congruent ends that fold as illustrated.



This piece is folded behind so it can be folded back into place.



Explain that the triangle at the right end can be cut off and placed at the left end to form a rectangle. The area of the new rectangle is equal to the area of the original parallelogram. Since we can calculate the area of a rectangle, we can calculate the area of the parallelogram.

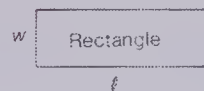
Elicit: "The area of a parallelogram is the same as for a rectangle with the same length and width."

USING THE BOOK

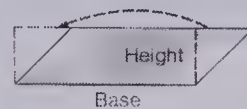
You may wish to have the students do the Activities without opening their texts. In which case, you can direct the Activities orally.

Assign the exercises. Discuss what the students learned by asking the students to tell what they have discovered about parallelograms and

Area of Parallelograms



$$A = \text{length} \times \text{width}$$



Area of the parallelogram is the same as the area of the rectangle with same base and height

$$\text{Area} = \text{base} \times \text{height}$$



$$A = b \times h$$

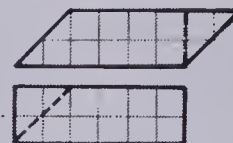
Activities

1. Use squared paper.

Draw and cut out a parallelogram.

Cut off the one end and place it on the other end.

What is the length of the rectangle? the width? the area?

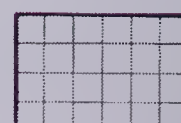
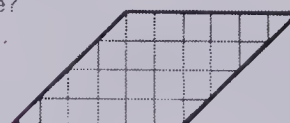


They are the same as the parallelogram.

2. Count the squares of the rectangle and the parallelogram.

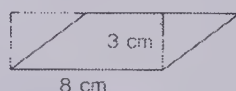
How do they compare?

They are the same.



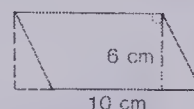
Exercises

Calculate the area of each parallelogram.



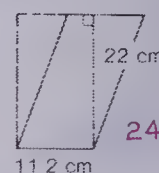
Area is 24 cm^2

2.



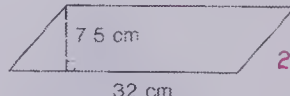
60 cm^2

3.



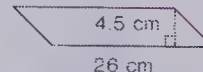
246.4 cm^2

4.



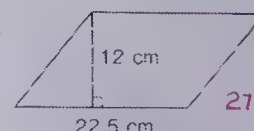
240 cm^2

5.



117 cm^2

6.



270 cm^2

110 Area of parallelograms using formula

rectangles. Emphasize the terms "base" and "height".

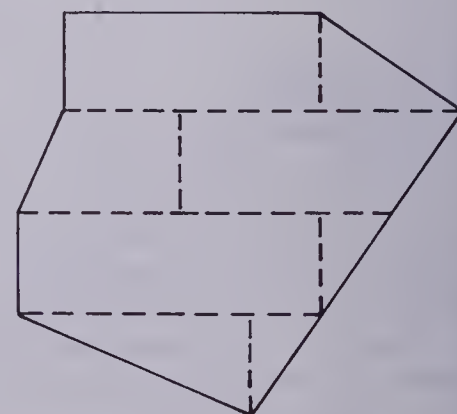
ACTIVITIES

1. Have the students put the formula for the area of a parallelogram in their "Math Formulas" list. Also, have students add all the formulas to date to the bulletin board.

2. Prepare a set of parallelograms suitable for measuring. Number each and record for your records the sizes and areas. Students are to measure the base and height and calculate the area of each.

3. Prepare a set of oddly-shaped polygons similar to the one shown. Students are to calculate the area of

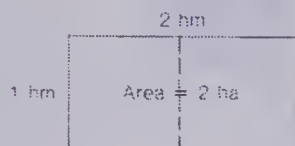
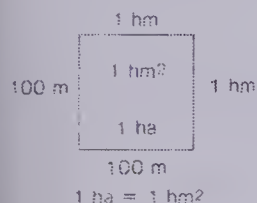
each. The dotted lines help the students.



4. See the "Tangram" ideas in the Activity Reservoir.

The Hectare

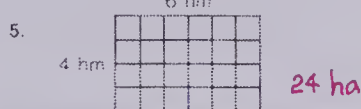
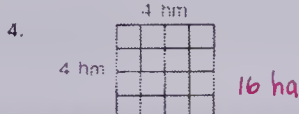
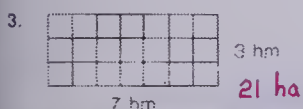
The square metre (m^2) is too small to measure land area.
The square kilometre (km^2) is too large for small land areas.
Therefore the **hectare** (ha) is used for small land areas.



Exercises

- A city block is 3 hm long and 1 hm wide.
How many hectares in the lot? **3 ha**
- A city block is 3 hm long and 2 hm wide.
How many hectares in the lot? **6 ha**

Calculate the area in hectares for each land area.



BRAINTICKLER

Draw the pattern shown.

Continue to number around the spiral.

- Is there a ray that contains:
 - only the squares? **Yes**
 - only primes? **Yes**
 - only squares of prime numbers? **Yes**
- Predict what the corner numbers will be.
- Note:

19	6
20	7

 $19 + 7 = 20 + 6$

Does this pattern always hold? **No**

17	16	15	14	13
18	5	4	3	12
19	6	1	2	11
20	7	8	9	10
21	22	23		

Hectares 111

OBJECTIVE

To calculate area in hectares

PACING

Level A Optional
Level B All
Level C All

VOCABULARY

hectare

MATERIALS

trundle wheel or long measuring tapes

BACKGROUND

A hectare is a unit area of land equivalent to 100 m (one hectometre) by 100 m or one square hectometre. The easiest way to calculate the area in hectares is to convert (if necessary) the linear measures to hectometres then multiply as usual.

SUGGESTIONS

Initial Activity Use a trundle wheel or long metric tapes to measure off a 100 m by 100 m area in the school yard. Once measured and marked, have the students calculate the perimeter and area in square metres. Identify the area as 1 hm^2 or 1 ha (hectare). Use the perimeter as a course for relay races. Record the various running times.

USING THE BOOK

Explain the meaning of a hectare. Review how area is calculated:
 $\text{Area} = l \text{ (in hectometres)} \times w \text{ (in hectometres)}$.

Assign the exercises.

ACTIVITIES

- Find the approximate number of hectares in your school yard.
- Find the number of hectares in a city block or nearby field.
- Provide a set of squares 1 cm \times 1 cm cut from cardboard. (Geo-boards may be used also.)
 - Use 24 squares to form a rectangle.
 - Make as many different rectangles as you can.
 - Record the sizes of each in a chart.

Rectangle	Length	Width	Perimeter	Area
1	12	2	28	24
2				
3				

EXTRA PRACTICE

In a city's residential area there are about 16 single-family dwellings per hectare.

- In a city of 0.6 million single-family dwellings, how many hectares are used?
- By using multi-family dwellings, the number of families per hectare rises to 44. How many hectares of land can be saved if all the dwellings are multi-family rather than all single-family dwellings?

OBJECTIVE

To use the relationship: area of a circle is just less than four times the radius times radius

PACING

Level A All
Level B All
Level C All

VOCABULARY

approximation

BACKGROUND

It is important that the student experiences, in various contexts, the meaning of pi. Here pi is the relationship between the area of a circle and the radius squared; its value is less than 4. Earlier, its value was established as being greater than 3.

SUGGESTIONS

Initial Activity Before the students open their books to this page, you may wish to direct them orally to do the display as an activity. This allows the students to discover the relationship themselves. After the students have identified the desired relationship shown here, tell them that pi is the name for the relationship between the area of a circle and "radius \times radius". Remind them that we earlier determined pi to be greater than 3 and that during these exercises, they should keep this in mind. Tell the students we will use the value 3.14 later, but not with this page.

USING THE BOOK

After developing the display, assign the exercises. You may wish the more capable math students to include these two statements for each area:

$A > 3 \times r \times r$.

$A < \underline{\hspace{2cm}}$.

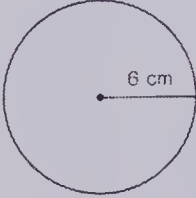
ACTIVITIES

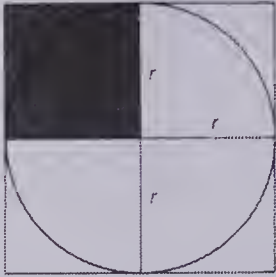
1. Prepare in advance, a set of circles with radii different than those shown. Then to emphasize approximations, have an oral "flash-card" game in which students reply when shown a circle: "The area is less than _____."

2. Identify circular regions in the school or on the playground (i.e., circles on the gym floor). Ask the students to complete the statement for each circle:

Area of a Circle

Compare the area of this square and the circle shown.





Area of shaded square is $r \times r$.

Area of large square is $4 \times r \times r$.

Area of circle is less than the area of large square.

$\text{Area of circle} < 4 \times r \times r$

An approximation for the area of this circle is:

Area of circle $< 4 \times r \times r$. (r is the radius of the circle.)

Area of circle $< 4 \times 6 \times 6$.


Area of circle $< 144 \text{ cm}^2$.


Exercises


- (a) What is the area of the shaded square? **25 square units**


(b) What is the area of the total large square? **100 square units**

(c) How does the area of the circle compare with the area of the large square? **Area of circle is less than the area of the large square.**
- Use the relation $\text{Area of circle} < 4 \times r \times r$ to find an approximation for the area of each circle









$A < \underline{\hspace{1cm}} \text{ 64 cm}^2$

$A < \underline{\hspace{1cm}} \text{ 16 cm}^2$

$A < \underline{\hspace{1cm}} \text{ 400 cm}^2$

$A < \underline{\hspace{1cm}} \text{ 256 cm}^2$

112 Area of Circle approximations

"The area is less than _____."

3. Ask the students to draw in their exercise books three circles using compasses and/or circular objects. They are then to write a statement about the area of each.

More About Pi

Mathematicians write the relation:

$$\frac{\text{area of a circle}}{\text{radius} \times \text{radius}} = \pi \text{ (pi).}$$

The number that π represents is too long to write exactly.

$$\pi = 3.141\ 592\ 653\ 589\ 793\ 238\ 462\ 643\ 383\ 279 \dots$$

We use $\pi = 3.14$ For a circle we write: $A = \pi \times r \times r$
or $A = 3.14 \times r \times r$.



Exercises

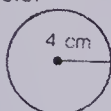
Calculate the area of each circle.

1. $A = \pi \times r \times r$

$$A = 3.14 \times 4 \times 4$$

$$A = \blacksquare \text{ cm}^2 \quad 50.24$$

Area is $\blacksquare \text{ cm}^2$. 50.24

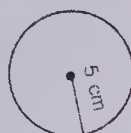


2.



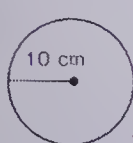
$$12.56 \text{ cm}^2$$

3.



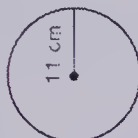
$$78.5 \text{ cm}^2$$

4.



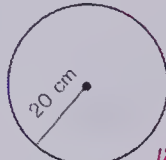
$$314 \text{ cm}^2$$

5.



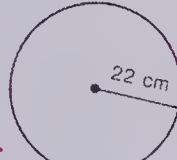
$$379.94 \text{ cm}^2$$

6.



$$1256 \text{ cm}^2$$

7.



$$1519.76 \text{ cm}^2$$

8. A plate has a radius of 10 cm.

What is its area? 314 cm^2

9. An outdoor fire pit used for bonfires has a radius of 0.8 m.

What is the area of its circular cover? 2.0096 m^2

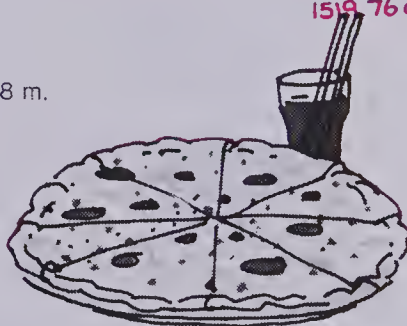
10. A circular dining room table has a radius of 0.4 m.

What is the area of the table top? 0.5024 m^2

★11. Which is the better buy?

(a) A pizza with radius 8 cm for \$5.00.

(b) A pizza with radius 10 cm for \$6.50 (b)



Area using π 113

OBJECTIVE

To use $\pi = 3.14$ to calculate the area of a circle

PACING

Level A 1-10

Level B All

Level C All

VOCABULARY

pi (π), mathematician

RELATED AIDS

HMS — DM27.

CALC. ACTIVITY MASTERS — 87.

BACKGROUND

An approximation for pi is 3.14.

Sometimes $3\frac{1}{7}$ is used as an approximation. Pi is an irrational number. That is, it cannot be expressed exactly as a fraction or decimal.

SUGGESTIONS

Initial Activity To develop a better approximation for pi, lead the students through this activity. Emphasize we can only get an approximation for the value of pi. In this activity, pi is greater than 3.

USING THE BOOK

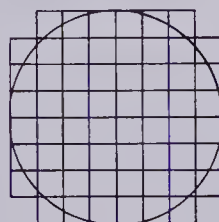
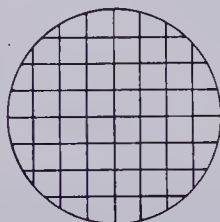
Explain that we have a special name for the ratio of the area of a circle to the square of the radius. (You may need to explain the square of a radius is radius \times radius.) Also explain that this ratio is the same ratio as the length of the circumference to the diameter of the circle. The special name is pi and is written using the Greek letter π . Also explain that the value of this ratio cannot be written exactly as a number. The values 3.14 and $\frac{22}{7}$ ($3\frac{1}{7}$) are only approximations we will use. For simplicity we write $\pi = 3.14$.

Then help the students through Exercises 1 and 2. Assign the rest of the page.

ACTIVITIES

1. Use the circles prepared in Activity 1 on page 112. The students can find the area of each using $\pi = 3.14$.

2. Prepare an activity sheet such as:



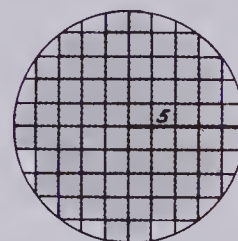
Copy and complete these statements:
Area of the circle is between \blacksquare and \blacksquare square units.

An approximate area is:
the average of the two numbers: \blacksquare .

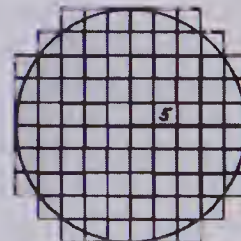
Calculate the ratio: $\frac{\text{approximate area}}{\text{radius} \times \text{radius}}$.

Area of a circle is about \blacksquare times the product radius \times radius.

3. Have some of your Level C students prepare a bulletin-board display showing diagrams and the various formulas used thus far.



68 square units



88 square units

Area of circle is between 88 square units and 68 square units.

$$\text{Average: } \frac{88 + 68}{2} = \frac{156}{2} = 78$$

$$\frac{\text{approximate area}}{\text{radius} \times \text{radius}} = \frac{78}{5 \times 5} = \frac{78}{25} = 3.12$$

The area is about 3 times the product radius \times radius.

Area of a circle $\approx 3 \times \text{radius} \times \text{radius}$ (square units).

(\approx means is approximately.)

OBJECTIVE

To use the formula $V = l \times w \times h$ for the volume of a rectangular prism

PACING

- Level A 1-9
- Level B 1-9
- Level C All

MATERIALS

centimetre cubes

RELATED AIDS

- BFA PROB. SOLVING LAB II — 127, 131.
- CALC. ACTIVITY MASTERS — 29, 72, 73.

SUGGESTIONS

Initial Activity Direct the students in a drill exercise practising multiplication of 3 numbers.

Example

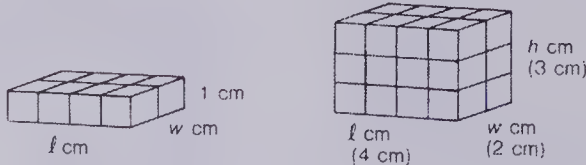
- Oral: $2 \times 8 \times 5$, $3 \times 4 \times 6$, $6 \times 8 \times 10$.
- Pencil and Paper: $7 \times 8 \times 6$, $4 \times 9 \times 8$, $13 \times 4 \times 9$.

With students working in groups and each group with a set of centimetre cubes, direct them to make a layer of blocks 3 cm by 4 cm. Ask, "How many blocks?" [12] Ask them to write a number sentence for this. [$3 \times 4 = 12$ or $3 \times 4 \times 1 = 12$] Encourage them to use the 1 to mean "1 layer 3 cm by 4 cm". Then ask them to make 2 layers the same size — one layer on top of the other. "How many blocks now?" [24] "Write a number sentence for how you would calculate this." [$3 \times 4 \times 2 = 24$] Repeat for 3 layers and 4 layers. Then, if you feel it necessary, repeat the process for layers 2 cm \times 3 cm, 2 cm \times 6 cm, 3 cm \times 5 cm, etc. Ask the students to write the number sentence for 25 layers, 45 cm by 37 cm (they can use \square to represent the answer). [$45 \times 37 \times 25 = \square$] Then for those who are ready, ask "What number sentence can you write for the volume if there are h layers, and each layer is l units long and w units wide. [$V = l \times w \times h$]

USING THE BOOK

Review the development of calculating volume by:
number of cubes in 1 layer ($l \times w$)
number of layers (h)

Volume of Rectangular Prisms



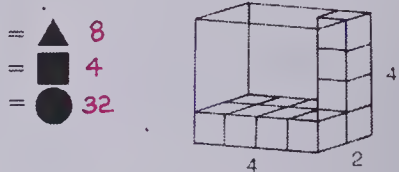
Number of cubes in 1 layer = $l \times w$.
Number of cubes in h layers = $l \times w \times h$.

$V = l \times w \times h$
 $V = 4 \times 2 \times 3$
 $V = 24$
Volume is 24 cm³.
Note: cm³ is read cubic centimetres.

$V = l \times w \times h$

Exercises

- 1. Find the volume of the box.
Number of cubic centimetres in 1 layer = 8
Number of layers = 4
Number of cubic centimetres in box = 32
Compare with this solution:
 $V = l \times w \times h$
 $V = 4 \times 2 \times 4$
 $V = 32$ cm³



The volume of the box is 32 cm³

Calculate the volume of each box.

2. 1920 cm³

3. 34.5 cm³

4. 3.84 m³

5. 1000 cm³

6. 1000 cm³

7. 1 000 000 cm³

8.

Length	Width	Height
12.3 cm	8 cm	6.2 cm

 610.08 cm³

9.

Length	Width	Height
2.8 m	4.5 m	10 m

 126 m³

10.

Length	Width	Height
1 m	56 cm	40 cm

 224 000 cm³

11.

Length	Width	Height
1.1 m	1.1 m	1.1 m

 1.331 m³

114 Volume of rectangular prisms using formula

number of cubes altogether ($l \times w \times h$)
This in brief is: $V = l \times w \times h$.
Do exercise 1 orally.
Assign Exercises 2 to 11.

ACTIVITIES

- 1. Have students collect a number of different kinds of cereal boxes. Students can record their guess for the volume and then measure and record the dimensions and calculate the volume. They should do one at a time so that their guesses will improve with practice.

Box	Guess Volume	Length	Width	Height	Calculated Volume
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Points may be given for the best guesses and correctly-calculated volumes.

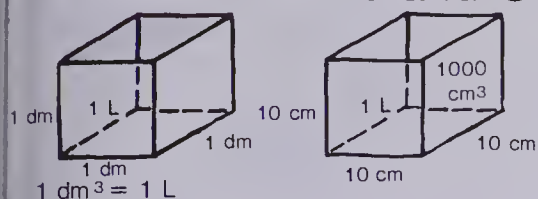
- 2. Identify rectangular prisms in the classroom such as aquarium, chalk box, book box, etc. Repeat Activity 1.
- 3. Some students may be ready

for the volume of a cube of side s .
 $V = l \times w \times h$
 $V = s \times s \times s$
 $V = s^3$
Explain $s^3 = s \times s \times s$.
Then ask them to find the volume of cubes with sides
(a) 2 cm (b) 4 cm (c) 5 cm (d) 10 cm

EXTRA PRACTICE

- 1. A moving van: Length: 12 m
Width: 2.4 m
Height: 2.1 m
Volume: ?
- 2. Railway box car: Length: 14.6 m
Width: 3.0 m
Height: 3.1 m
Volume: ?
- 3. Commercial wheelbarrow: Length: 1.4 m
Width: 0.8 m
Depth: 0.4 m
Volume: ?
- 4. Packing crate: Length: 1.2 m
Width: 0.8 m
Height: 1.1 m
Volume: ?

Litres and Cubic Centimetres



A container with a volume of 1000 cm³ will hold 1 L of water.

$$1000 \text{ cm}^3 = 1 \text{ L}$$

Exercises

(a) 1000 cm³ (b) 2 dm³

- What is the volume of each container?
How many litres of water will each hold?

(a) 1 L (b) 2 L

Copy and complete.

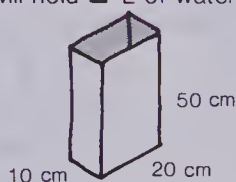
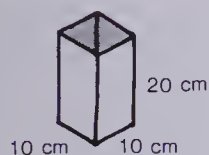
- 1 L of water fills \blacksquare cm³.
- 4 L of ginger ale fills \blacksquare cm³.

Copy and complete.

- 1000 cm³ will hold \blacksquare L of water.
- 2500 cm³ will hold \blacksquare L of water.
- $V = \blacksquare$ cm³. It will hold \blacksquare L of water.

- 2 L of milk fills \blacksquare cm³.
- 3.5 L of water fills \blacksquare cm³.

- 5000 cm³ will hold \blacksquare L of water.
- 6500 cm³ will hold \blacksquare L of water.
- $V = \blacksquare$ cm³. It will hold \blacksquare L of water.



Calculate the number of cubic centimetres (cm³) and the number of litres (L).

- Diagram of a rectangular container with dimensions 10 cm by 10 cm by 40 cm.
- Diagram of a rectangular container with dimensions 15 cm by 10 cm by 40 cm.
- Diagram of a rectangular container with dimensions 20 cm by 10 cm by 20 cm.
- Diagram of a rectangular container with dimensions 15 cm by 15 cm by 20 cm.

- One litre of water has a mass of 1 kg.
What is the mass of water that will fill a rectangular can that is 60 cm long, 30 cm wide, and 20 cm high?

Litres and cubic centimetres 115

OBJECTIVE

To calculate the number of litres in a rectangular container using the relation 1 L = 1000 cm³

PACING

- Level A 1-11
Level B 1-15
Level C 2-16 (even)

MATERIALS

cardboard, litre container (such as a milk container), scale

RELATED AIDS

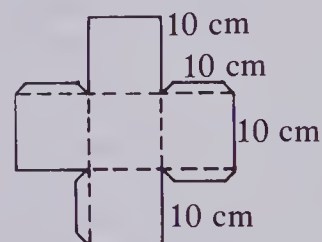
BFA PROB. SOLVING LAB II — 98.

BACKGROUND

The quantity of liquid a container can hold is its capacity. Capacity is commonly measured in litres (or millilitres). Volume is the amount of space filled by an object and is measured in cubic units, i.e., cubic centimetres and cubic metres. One litre of water at maximum density has a mass of one kilogram. (Emphasize that this is true for water only.) Hence one cubic centimetre of water has a mass of one gram.

SUGGESTIONS

Initial Activity To emphasize the size of a litre, have the students draw a net for the litre container (shown at the top of the page), assemble, and glue.



Ask the students to calculate the number of cubic centimetres (volume) of this container. Then point out that 1000 cm³ holds 1 L of liquid.

Have the students demonstrate the mass of 1 L of water: Use the scale to find the mass of the litre container empty (A). Fill the container with one litre of water and obtain its mass again (B). Subtract A from B to get the mass of one litre of water. Remember: measurements are approximate!

USING THE BOOK

Discuss liquids sold in litres. Have students name different liquids.

Review: 1 L = 1000 cm³.

The shape of 1000 cm³ need not be fixed. The volume is calculated using $V = l \times w \times h$ (cm).

Explain or summarize the result of the demonstration:

$$1 \text{ L of water} = 1 \text{ kg}$$

$$1000 \text{ cm}^3 \text{ of water} = 1000 \text{ g}$$

Assign Exercises 1 to 16. You may wish to explain that these questions can be done using ratios.

ACTIVITIES

- Assemble a number of containers with capacities of 1 L, 2 L, 3 L, and 4 L. Provide the students with a 1 L container and ask them to find the capacity of each container after first estimating the capacity.

- Ask the students to find the measures of 5 different rectangular tanks that hold 1 L. (Example: $5 \text{ cm} \times 20 \text{ cm} \times 10 \text{ cm} = 1000 \text{ cm}^3$)

- Ask the Level C students to calculate the mass of water that would fill each of the containers shown in Exercises 10 through 15.

OBJECTIVES

To calculate the number of millilitres in a container
To convert litres to millilitres

PACING

Level A 1-5
Level B All
Level C All

VOCABULARY

millilitre (mL)

RELATED AIDS

HMS — DM28.

SUGGESTIONS

Initial Activity Point out that:
Since 1 L of water has a mass of 1 kg,
1000 cm³ of water has a mass of 1000 g,
1 cm³ of water has a mass of 1 g.
Also 1 L of water fills 1000 cm³,
1 mL of water fills 1 cm³.
(mL = millilitre)

USING THE BOOK

Explain the display at the top of page.
The symbol mL is new. Explain L is
the symbol for litre and the prefix m
means milli (one thousandth) so mL
means millilitre or one thousandth of
a litre.

Assign Exercises 1 to 6.

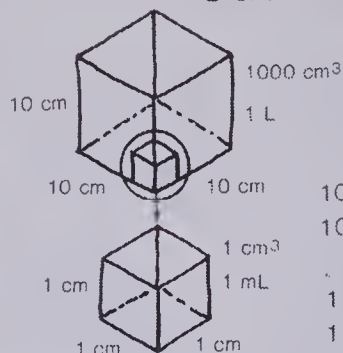
ACTIVITIES

1. Assemble a number of containers with various capacities from 15 mL to 2000 mL. Provide the students with graduated cylinders or graduated beakers. Students are to estimate the capacity, then measure it by pouring from a full container into a graduated cylinder. The estimates and capacities should be recorded in chart form.

2. Have the students collect and measure the water from a leaking faucet for 15 min. They are then to calculate the amount of water wasted in one day, one week, and 52 weeks.

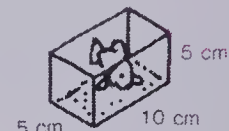
3. Have the students prepare a report on the history of measuring capacity. The report may be written or in chart form.

Cubic Centimetres and Millilitres



1000 cm³ = 1 L
1000 cm³ = 1000 mL
1 cm³ = 1 mL
1 cm³ will hold
1 mL of water.

How many litres of water will this small aquarium hold? **0.25 L**



Volume = $l \times w \times h$
Volume = $10 \times 5 \times 5$

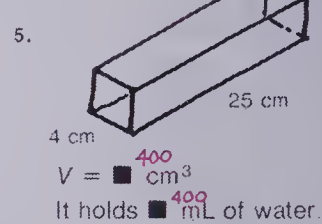
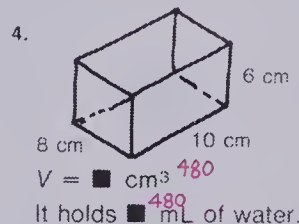
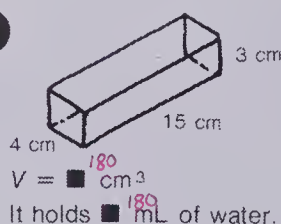
Exercises

Copy and complete.

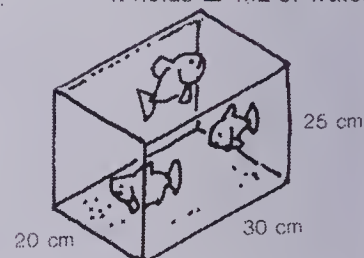
1. 1 cm³ will hold **1** mL of water.
3 cm³ will hold **3** mL of water.
500 cm³ will hold **500** mL of water.

2. 1 mL of water fills **1** cm³.
10 mL of water fills **10** cm³.
600 mL of water fills **600** cm³.

How many cubic centimetres (cm³)? How many millilitres (mL)?



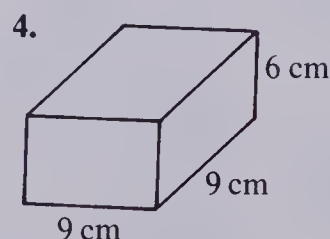
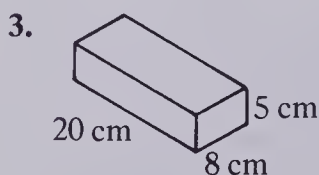
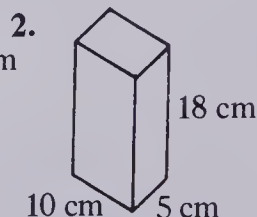
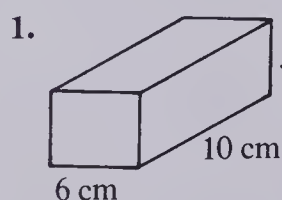
- ★6. What is the mass of water in this aquarium?
Remember, 1 L of water has a mass of 1 kg. **15 kg**



116 Cubic centimetres and millilitres

EXTRA PRACTICE

How many millilitres of water will each container hold?





More About Litres

Oil is moved in huge tankers.
A tanker may hold thousands of kilolitres.

$$1000 \text{ L} = 1 \text{ kL}$$



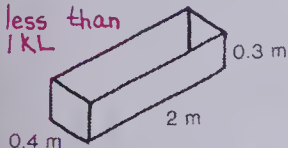
150 000 kL of oil

Milk is sold
in litre containers.

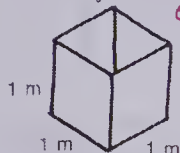
Exercises

Tell whether each holds more, less, or exactly 1 kL.

1. *less than 1 kL*

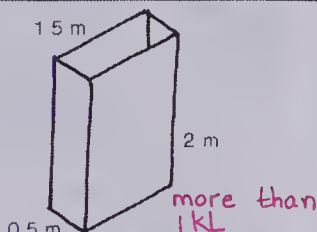


2.



exactly 1 kL

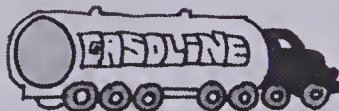
3.



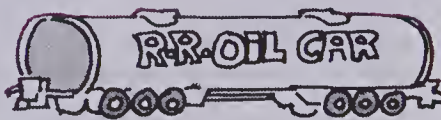
more than 1 kL



About 30 kL of milk



About 100 kL of gasoline



About 1000 kL of oil

4. How many milk-truck tanks would be needed to have a capacity equal to:
 - (a) one gasoline-truck tank? *3.3*
 - (b) one railway-car tank? *33.3*
5. How many gasoline-truck tanks would be needed to have a capacity equal to:
 - (a) one railway-car tank? *10*
 - (b) the tanker at the top of the page? *1500*
6. A family uses 4 L of milk daily. *280 L*
How many litres of milk do they use in 10 weeks? Do they use 1 kL in 10 weeks? *No*
7. A family uses 5 L of milk daily.
How many litres of milk do they use in January, February, March, and April? *600 L*
Do they use 1 kL in the 4 months? *No*
8. A tanker carrying 8940 kL of oil split in half off the coast of Japan. It lost its cargo.
How many litres of oil were spilled? *8 940 000 L*

Kilolitres 117

OBJECTIVES

To convert litres to kilolitres and
kilolitres to litres
To use kilolitres in problems

PACING

Level A 1-5
Level B All
Level C 1-5 (odd), 6-8

VOCABULARY

kilolitres (kL), capacity

BACKGROUND

Kilo- is the prefix meaning one thousand. Therefore a kilolitre is one thousand litres. Large quantities of liquids are measured in kilolitres.

SUGGESTIONS

Initial Activity Collect and bring to class, pictures of tankers: truck tankers, rail tankers, and seagoing tankers. Approximate capacities of each are shown on page 117. Discuss the pictures.

USING THE BOOK

Explain the relation between litre and kilolitre emphasizing the meaning of kilo-. Emphasize that a kilolitre is the capacity of a cubic metre.

You may do Exercises 1 to 5 in class orally. Assign the balance.

ACTIVITIES

1. Discuss the need, advantages, and disadvantages of huge oil tankers. Discuss problems existent if a tanker is sunk or wrecked on rocks — effect of oil spills on the ocean, ocean life, and short life. Discuss alternatives — overland pipelines — and potential problems therein. Emphasize: We need the oil; we need the wildlife (flora and fauna); we need extreme caution and care.

2. Ask students to make a collection of pictures of liquids being transported. Each picture should be accompanied by a statement of approximate capacity of each tank. Post on the bulletin board.

3. Arrange three containers: 800 mL, 500 mL, and 300 mL. Fill the 800 mL container with water. Challenge the class to get 400 mL in one container.

One answer: (8, 0, 0) → (5, 0, 3) → (5, 3, 0) → (2, 3, 3) → (2, 5, 1) → throw out 1 L → (2, 5, 0) → (2, 2, 3) → (0, 4, 3)]

EXTRA PRACTICE

1. A family uses 4 L of milk daily. How many litres of milk will they use in 10 weeks? [280 L]
Do they use more or less than 1 kL in 10 weeks? [Less]
2. A family uses 5 L of milk daily. How many litres of milk do they use from April 1 to August 31? [765 L]
3. A cow produces 28 L of milk daily. How many kilolitres of milk would the cow produce in one year? [10.22 kL]
4. If a farmer received 18¢ for each litre of milk the cow in Exercise 3 produced, how much would the farmer receive for the year's production? [\$1839.60]
5. The Torrey Canyon tanker grounded and spilled its cargo. It carried 191 000 kL of oil. How many litres of oil were spilled? [191 000 000 L]

OBJECTIVES

To solve problems involving grams, kilograms, and tonnes
To convert among grams, kilograms, and tonnes

PACING

Level A 1-11, Tune Up
Level B 1-13, Tune Up
Level C 1-6, 8, 10, 12-16, Tune Up is optional

VOCABULARY

tonne

RELATED AIDS

BFA PROB. SOLVING LAB II — 32, 103.

BACKGROUND

A tonne is a unit of mass equivalent to one thousand kilograms. Mass is not equal to weight. Mass is the quantity of matter in an object whereas weight is the force of gravity on the object. Mass always remains constant, whereas the weight of an object may vary depending on its location (gravity varies on Earth and on different planets).

SUGGESTIONS

Initial Activity Discuss the work of a truck gardener. See the Career Awareness notes in the Chapter Overview, page 92. Review the relations among grams, kilograms, and tonnes. Collect and bring to class a set of pictures of common objects that will serve as referents for certain masses. On each write the approximate mass of each object.

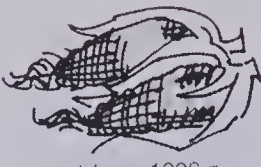
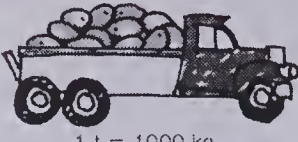
USING THE BOOK

Do Exercises 1 to 6 orally in class. Use Exercise 7 to illustrate the problem-solving techniques recommended:

- Calculate the mass in grams of 10 apples (ask how to do this).
- Convert the mass in grams of 10 apples to mass in kilograms (ask how to do this).
- Write a sentence answering the question.

Point out that in some exercises, it may be necessary to change the kilograms to grams first — or the grams to kilograms, in which cases decimals are used.

Truck Gardener

$1000 \text{ g} = 1 \text{ kg}$
 $1000 \text{ kg} = 1 \text{ t}$
 $1\,000\,000 \text{ g} = 1 \text{ t}$

$1 \text{ kg} = 1000 \text{ g}$

$1 \text{ t} = 1000 \text{ kg}$

Exercises

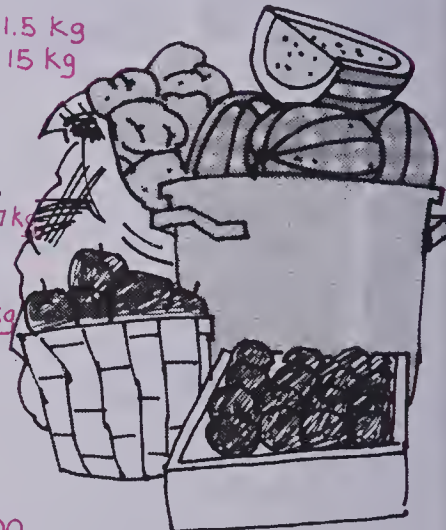
Copy and complete.

1. $1000 \text{ g} = \blacksquare \text{ kg}$ 1	2. $1000 \text{ kg} = \blacksquare \text{ t}$ 1	3. $1\,000\,000 \text{ g} = \blacksquare \text{ t}$ 1
$500 \text{ g} = \blacksquare \text{ kg}$ 0.5	$500 \text{ kg} = \blacksquare \text{ t}$ 0.5	$500\,000 \text{ g} = \blacksquare \text{ t}$ 0.5
$3000 \text{ g} = \blacksquare \text{ kg}$ 3	$8000 \text{ kg} = \blacksquare \text{ t}$ 8	$2\,000\,000 \text{ g} = \blacksquare \text{ t}$ 2

Copy and complete.

- $2 \text{ kg} = \blacksquare \text{ g}$ **2000**
 $2.5 \text{ kg} = \blacksquare \text{ g}$ **2500**
 $7.2 \text{ kg} = \blacksquare \text{ g}$ **7200**
- $1 \text{ t} = \blacksquare \text{ kg}$ **1000**
 $0.5 \text{ t} = \blacksquare \text{ kg}$ **500**
 $3.4 \text{ t} = \blacksquare \text{ kg}$ **3400**
- $5 \text{ kg} = \blacksquare \text{ g}$ **5000**
 $4 \text{ t} = \blacksquare \text{ kg}$ **4000**
 $2500 \text{ g} = \blacksquare \text{ kg}$ **2.5**

- The mass of an apple is about 150 g.
 - What is the mass in kilograms of 10 apples? **1.5 kg**
 - What is the mass in kilograms of 100 apples? **15 kg**
 - A box of apples has a mass of 20 kg. About how many apples are in the box? **133**
- The mass of a large tomato is about 170 g.
 - What is the mass in kilograms of 10 tomatoes? **1.7 kg**
 - About how many tomatoes are in 5 kg? **29**
- The mass of an ear of corn is about 650 g. What is the mass in kilograms of 1000 ears? **650 kg**
- A large potato has a mass of about 0.5 kg.
 - How many potatoes are in a 50 kg bag? **100**
 - How many bags are in 1 t? **20**
- An average watermelon has a mass of 15 kg. A truck delivers 6 t of watermelons. How many watermelons does the truck deliver? **400**



118 Problems using mass

While certain answers can be left in the smaller unit such as grams, students should be encouraged to express answers like 15 000 g in the larger unit — 15 kg.

The Tune Up can be assigned or done orally. If done orally, ask questions like: "Name an object with a mass of about

- (a) 1 kg (b) 2 kg (c) 100 g."

ACTIVITIES

1. Play "Concentration" as described in the Activity Reservoir. Use cards which reinforce the relationships between various units of measure. Example

100 cm	1 m
500 mL	0.5 L
5000 g	5 kg

2. Have the students start and build a "Mass" chart. Students can add an object or item and its mass under each heading. Pictures can be added instead of words.

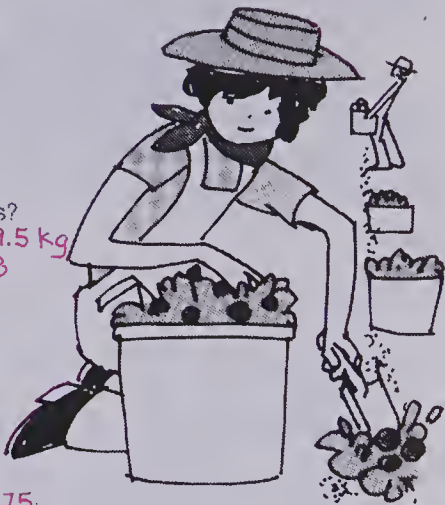
MASS		
Items measured in		
grams	kilograms	tonnes
bar of soap — 200 g	cows — 1600 kg	trucks — 3 t

3. For those students who need it, you may prepare a set of objects that students are to hold in their hands. Estimate the mass then check by placing on a scale.

EXTRA PRACTICE

Puzzle:
A Raja sent his Maharaja 5 identical gold coins — by messenger. The Maharaja had a balance. He wished to determine in the least number of weighings possible, if the messenger had substituted an identical but light coin for one of the real gold coins.
(a) How many weighings are necessary? [Max: 3]
(b) How many are necessary if there were 7 identical coins? [Max: 4]
(c) How many are necessary if there were 6 identical coins? [Max: 3]

- 12. An average turnip has a mass of 1.8 kg.
(a) How many turnips are in a 25 kg bag? (b) How many bags are in 1 t? 40
- 13. Watermelons sell for 35¢/kg
The average mass of each watermelon is 18 kg.
A truck has 260 melons.
What is the load worth? \$1638
- 14. The mass of a large orange is about 130 g.
(a) What is the mass in kilograms of 150 oranges?
(b) How many oranges are in a 40 kg carton? 19.5 kg 308
- 15. Peaches sell for 85¢/kg.
The average peach has a mass of 160 g.
One carton has 60 peaches.
What is the price of the carton? \$8.16
- 16. The mass of the average pumpkin is 8 kg
A truck delivers 4.6 t of pumpkins.
(a) How many pumpkins are in the shipment? 575
(b) Pumpkins sell for 16¢/kg. What is the load worth? \$736



Tune Up

- 1. A ping-pong ball has a mass of about: (c)
(a) 1 kg (b) 2 kg (c) 1 g (d) 100 g.
- 2. A small car has a mass of about: (c)
(a) 1 kg (b) 100 kg (c) 1 t (d) 10 t.
- 3. This text has a mass of about: (c)
(a) 1 g (b) 10 g (c) 1 kg (d) 100 g.
- 4. Your mass is closest to: (c)
(a) 50 g (b) 500 g (c) 50 kg (d) 500 kg.
- 5. The mass of a large carrot is about: (b)
(a) 1 g (b) 100 g (c) 1 kg (d) 100 kg.

Continued from page 108)

- Use a set of squares about 1 cm × 1 cm cut from cardboard. (Geo-boards may be used also.)
(a) Use squares to form a rectangle with a perimeter of 40 units.
(b) Make as many different rectangles as you can.
(c) Record the sizes in a chart.

Rectangle	Length	Width	Area	Perimeter
1	10	10	100	40
2				

- (d) Is the perimeter always the same? the area?
(e) Repeat for 30 units, 12 units, 36 units, etc. (Teacher: always choose an even number of units.)

OBJECTIVE

To solve problems involving metric measurements

PACING

Level A 1-6
Level B 1-6
Level C All

RELATED AIDS

HMS — DM29.

BFA PROB. SOLVING LAB II —
29-31, 34, 35, 41, 65, 73, 77, 93, 102,
103.

CALC. ACTIVITY MASTERS — 57.

SUGGESTIONS

Initial Activity Discuss the various jobs that a gardener performs. See the Career Awareness notes in the Chapter Overview, page 92. In doing so, indicate wherein mathematics is used. The problems illustrate just a few uses of mathematics.

USING THE BOOK

Some students will need little if any help. For the others, you may wish to analyze the first problem or two.

Exercise 1: Since the lawn is not rectangular in shape, can we make two rectangles out of it? Where would we draw a line? (two places possible) What are the dimensions of each rectangle? Can you find the amount of sod needed now? (You may need to explain what sod is and how it is delivered.)

Point out that some questions require the answers to the preceding questions. Therefore, it may be advisable to list the correct answers on the side chalkboard so the students can check in order to proceed.

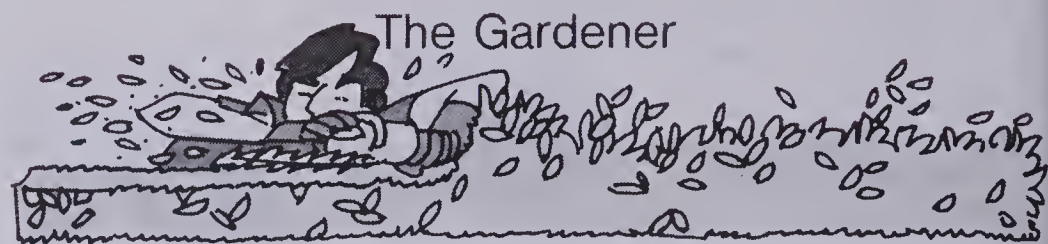
Remind the students of the steps in problem solving. (See page 22.)



ACTIVITIES

1. Students may collect pictures to prepare a collage or mural on the work of gardeners.

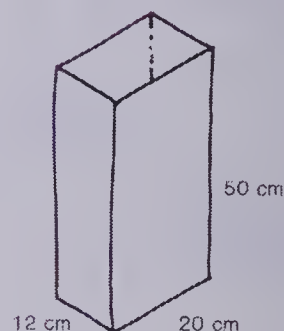
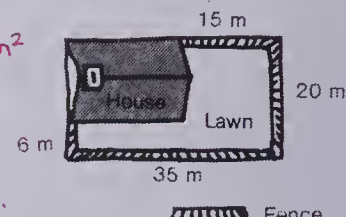
2. Choosing the correct operation is often the most difficult of the problem-solving steps. See Activity 1 for pages 22 and 23 for an idea



Gardeners care for lawns and plants. They mow, clip, prune, water, and spray as needed.

Exercises

- The lawn needs sod.
What is the area of the lawn, in square metres? **420 m²**
- Anna, the gardener, ordered 430 m² of sod.
It costs \$2.15/m² (per square metre).
What is the total price? **\$ 924.50**
- Ted, Anna's partner, is building a fence around the lawn.
How many metres of fence are required? **76 m**
- Ted calculated a profit of \$2.75/m on the fence.
What is the profit? **\$ 209**
- To fertilize the lawn, they use 1 kg of fertilizer for each 10 m².
How much fertilizer is required? **42 kg**
- They need 11 L of water to spray the plants.
Does this container hold 11 L? **Yes**
- ★ To mix the spray, Ted puts 20 mL of concentrate in 1 L of water.
How many millilitres of concentrate must he use with 11 L of water? **220 mL**
- ★ An empty container has a mass of 2 kg.
What is the mass of the container filled with the spray liquid in Exercise 6? **13 kg**



120 Problems using units of measure

designed to practise this skill.

3. Bring a flyer from a supplier of garden supplies. Then ask the students to calculate the cost of fencing a rectangular lot 20 m by 50 m. Have the students make up a problem or two of their own using actual prices, quantities, etc. Have these exchanged with classmates.

EXTRA PRACTICE

- A car uses 1 L of gas to go 8 km.
How much gas is needed to go 176 km? [22 L]

- An airplane flies 980 km/h.
How long does it take to go 3920 km? [4 h]
- At a cattle auction, the auctioneer said, "This animal is 165 kg more than 1 t."
What was the mass in kilograms? [1165 kg]
- Jill tied her horse to a post in the centre of a field with a 10 m rope.
Over what area can the horse eat? [314 m²]

More About Metres

$$1 \text{ km} = 1000 \text{ m}$$

$$1 \text{ m} = 10 \text{ dm}$$

$$1 \text{ hm} = 100 \text{ m}$$

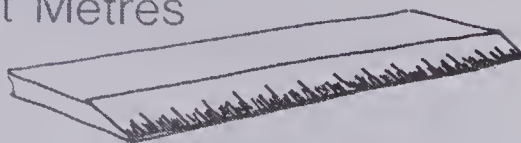
$$1 \text{ m} = 100 \text{ cm}$$

$$1 \text{ dam} = 10 \text{ m}$$

$$1 \text{ m} = 1000 \text{ mm}$$

$$2 \text{ km} + 3 \text{ hm} + 7 \text{ dam} + 8 \text{ m} = 2378 \text{ m}$$

$$\begin{array}{r} +8 \text{ m} \\ +70 \text{ m} \\ +300 \text{ m} \\ +2000 \text{ m} \\ \hline 2378 \text{ m} \end{array}$$



$$3 \text{ m} + 9 \text{ dm} + 1 \text{ cm} + 6 \text{ mm} = 3.916 \text{ m}$$

$$\begin{array}{r} 3 \text{ m} \\ 0.9 \text{ m} \\ 0.01 \text{ m} \\ 0.006 \text{ m} \\ \hline 3.916 \text{ m} \end{array}$$

Exercises

Write as metres.

1. $3 \text{ km} + 4 \text{ hm} + 5 \text{ dam} + 2 \text{ m}$

$$3 \text{ km} = 3000 \text{ m}$$

$$4 \text{ hm} = 400 \text{ m}$$

$$5 \text{ dam} = 50 \text{ m}$$

$$2 \text{ m} = 2 \text{ m}$$

$$3452 \text{ m}$$

2. $6 \text{ km} + 8 \text{ hm} + 9 \text{ dam} + 1 \text{ m}$

$$6 \text{ km} = 6000 \text{ m}$$

$$8 \text{ hm} = 800 \text{ m}$$

$$9 \text{ dam} = 90 \text{ m}$$

$$1 \text{ m} = 1 \text{ m}$$

$$6891 \text{ m}$$

3. $7 \text{ km} + 0 \text{ hm} + 0 \text{ dam} + 3 \text{ m}$

$$7003 \text{ m}$$

5. $2 \text{ km} + 0 \text{ hm} + 0 \text{ dam} + 0 \text{ m}$

$$2000 \text{ m}$$

7. $4 \text{ m} + 2 \text{ dm} + 3 \text{ cm} + 5 \text{ mm}$

$$4.235 \text{ m}$$

$$4 \text{ m} = 4 \text{ m}$$

$$2 \text{ dm} = 0.2 \text{ m}$$

$$3 \text{ cm} = 0.03 \text{ m}$$

$$5 \text{ mm} = 0.005 \text{ m}$$

4. $8 \text{ km} + 1 \text{ hm} + 2 \text{ dam} + 0 \text{ m}$

$$8120 \text{ m}$$

6. $1 \text{ km} + 2 \text{ hm} + 1 \text{ dam} + 3 \text{ m}$

$$1213 \text{ m}$$

8. $1 \text{ m} + 3 \text{ dm} + 8 \text{ cm} + 9 \text{ mm}$

$$1.389 \text{ m}$$

$$1 \text{ m} = 1 \text{ m}$$

$$3 \text{ dm} = 0.3 \text{ m}$$

$$8 \text{ cm} = 0.08 \text{ m}$$

$$9 \text{ mm} = 0.009 \text{ m}$$

9. $9 \text{ m} + 3 \text{ dm} + 0 \text{ cm} + 4 \text{ mm}$

$$9.304 \text{ m}$$

11. $0 \text{ m} + 0 \text{ dm} + 4 \text{ cm} + 5 \text{ mm}$

$$0.045 \text{ m}$$

13. $9 \text{ m} + 9 \text{ dm} + 9 \text{ cm} + 3 \text{ mm}$

$$9.993 \text{ m}$$

15. $2 \text{ km} + 1 \text{ hm} + 2 \text{ dam} + 1 \text{ m}$

$$2121 \text{ m}$$

10. $6 \text{ m} + 9 \text{ dm} + 2 \text{ cm} + 1 \text{ mm}$

$$6.921 \text{ m}$$

12. $5 \text{ m} + 0 \text{ dm} + 8 \text{ cm} + 0 \text{ mm}$

$$5.080 \text{ m}$$

14. $6 \text{ km} + 3 \text{ hm} + 4 \text{ dam} + 5 \text{ m}$

$$6345 \text{ m}$$

16. $4 \text{ m} + 0 \text{ dm} + 1 \text{ cm} + 2 \text{ mm}$

$$4.012 \text{ m}$$

Meanings of metric prefixes 121

OBJECTIVE

To write measures in metres given the expanded form

PACING

Level A All

Level B All

Level C 1-16 (odd)

MATERIALS

metre tapes marked in millimetres and centimetres

RELATED AIDS

BFA PROB. SOLVING LAB II — 97, 98.

BACKGROUND

The metric system with its prefixes is based on the powers of 10. Therefore the number of metres automatically gives the number of kilometres, hectometres, and so on to centimetres and millimetres. All one has to remember is the order of the units.

The order is given by the meaning of the prefixes: kilo (k) — 1000; hecto (h) — 100; deca (da) — 10; deci (d) — 0.1; centi (c) — 0.01; milli (m) — 0.001.

SUGGESTIONS

Initial Activity Review the prefixes, their meanings, and the symbols:

kilo — k; hecto — h; deca — da;

deci — d; centi — c; milli — m.

Explain that these prefixes are used with the basic metric units: metres, grams, and litres. Here we limit their use to metres. Demonstrate the relationships using the tapes.

USING THE BOOK

You may wish to ask questions such as: "How many metres in 3 hm? in 5 dam? in 300 cm? in 4 cm? in 9 mm?" etc. Then explain the second part of the display at the top of the page; assign Exercises 1 to 6 and correct. Then explain part three of the display (under the ruler) and assign Exercises 7 to 16.

ACTIVITIES

1. "Metric Snap". Prepare 3 sets of cards consisting of the symbols for the prefixes and the numeral for each prefix. The cards are shuffled and dealt to two or three players. Then the game is played like "Snap".

2. Form two sets (A and B) of four teams. Set A teams: metre, decimetre, centimetre, and millimetre teams. Set B teams: metre, decametre, hectometre, and kilometre teams.

Set A teams using rulers or tapes locate, measure, and record the measures of 5 objects. Each team then challenges the other teams to change its measures into the other units, i.e., the centimetre team challenges the other teams to change the centimetre measures to millimetres, decimetres, and metres.

Set B teams collect pictures of distances or objects *suitable* to be measured in their units (even though these may not be customarily

measured in those units). The same type of challenge as in Set A is issued.

3. Have students go on a scavenger hunt for names of objects or distances. Make up a list of 5 objects or distances for each team to find in the schoolyard. (Pictures may also be used.) Specify a range of sizes for each: (1) an object 3-4 m long, (2) a distance 2-4 dam long, etc.

4. Have some of your students make up a bulletin-board display explaining the meanings and use of metric prefixes and providing some local referents for each.

EXTRA PRACTICE

Write as metres.

1. $1 \text{ m} + 2 \text{ km} + 2 \text{ hm} + 4 \text{ dam}$

2. $2 \text{ hm} + 5 \text{ dam} + 3 \text{ km} + 4 \text{ m}$

3. $5 \text{ dam} + 2 \text{ m} + 3 \text{ km} + 8 \text{ hm}$

4. $6 \text{ km} + 2 \text{ m} + 5 \text{ dam} + 4 \text{ hm}$

5. $2 \text{ cm} + 3 \text{ mm} + 4 \text{ dm} + 5 \text{ m}$

6. $2 \text{ m} + 7 \text{ mm} + 5 \text{ cm} + 4 \text{ dm}$

7. $3 \text{ dm} + 5 \text{ mm} + 9 \text{ cm} + 2 \text{ m}$

8. $2 \text{ cm} + 4 \text{ dm} + 3 \text{ mm} + 5 \text{ m}$

OBJECTIVE

To evaluate achievement of the chapter objectives

PACING

Level A All
Level B All
Level C All

RELATED AIDS

HMS — DM30.

USING THE BOOK

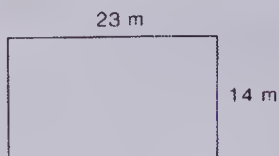
Each student should do this test independently under supervision. Assistance should be given only when the instructions are not understood. After the work has been corrected, you should provide appropriate remedial work. You may wish to reteach if a large number of students had difficulty with a particular topic or concept.

The following chart will help in this regard. The specific objectives are listed in the Chapter Overview (see page 92).

An alternate Chapter Test can be found in the Holt Mathematics System Duplicating Masters available for use with this grade level.

Chapter Test

1. Calculate the perimeter. **74 m**

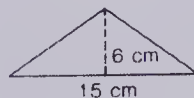


2. Calculate:

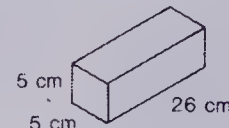
- (a) the circumference **50.24 cm**
(b) the area (Use $\pi = 3.14$) **200.96 cm²**



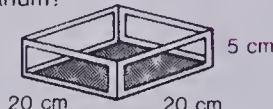
3. Calculate the area. **45 cm²**



4. Calculate the volume. **650 cm³**



5. How many litres in this aquarium? **2 L**



6. The word meaning "1000 g" is

kilogram

7. The word meaning "one hundredth of a metre" is **centimetre**

Copy and complete.

8. 3000 g = **3** kg

10. 500 L = **0.5** kL

9. 1.5 t = **1500** kg

11. 3 km + 8 hm + 0 dam + 4 m = **3804** m

Divide.

12. $\frac{3700}{10}$ **370**

13. $\frac{6370}{100}$ **63.7**

14. $\frac{90\ 000}{1000}$ **90**

15. $4 \overline{)10.4}$ **2.6**

16. $\frac{372}{6}$ **6.2**

17. $6 \overline{)318}$ **53**

18. $4 \overline{)8832}$ **2208**

19. $9 \overline{)738}$ **82**

20. $8 \overline{)8968}$ **1121**

21. $100 \overline{)\$3700}$ **\\$ 37**

22. $52 \overline{)6448}$ **124**

23. $68 \overline{)2100}$ **30 R60**

24. $31 \overline{)3131}$ **101**

25. $44 \overline{)8888}$ **202**

26. $38 \overline{)87.4}$ **2.3**

27. $27 \overline{)97.2}$ **3.6**

28. $68 \overline{)\$653.48}$ **9.61**

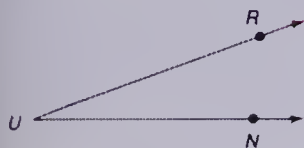
29. $33 \overline{)\$153.12}$ **4.64**

122 Chapter 4 test

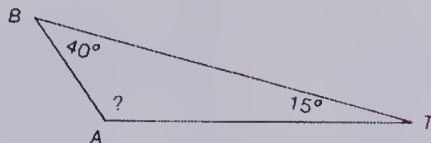
Test Item	Objective	Text Page Number
12-29	A	93, 96, 97
12-14, 21	B	95
17-20, 22-25	C	96-101
15, 16, 26-29	D	102, 103
6-10	E	104, 111, 115, 117, 118, 121
1-4	F	105-109, 112-114
5	G	115, 116
11	H	121

Cumulative Review

1. Use a protractor. What is the measure of $\angle RUN$? 20°



2. Calculate the measure of $\angle BAT$. 125°



Calculate.

3.
$$\begin{array}{r} 45.6 \\ 3.76 \\ + 431. \\ \hline 480.36 \end{array}$$

4.
$$\begin{array}{r} 76\ 030 \\ + 14\ 909 \\ \hline 90\ 939 \end{array}$$

5.
$$\begin{array}{r} \$408.00 \\ - 124.46 \\ \hline \$283.54 \end{array}$$

6.
$$\begin{array}{r} 361 \\ \times 7 \\ \hline 2527 \end{array}$$

7.
$$\begin{array}{r} 764 \\ \times 51 \\ \hline 38\ 964 \end{array}$$

8.
$$\begin{array}{r} 15\ 050 \\ - 7462 \\ \hline 7588 \end{array}$$

9. $123 \times 0 = 0$

10.
$$\begin{array}{r} 14.3 \\ \times 10 \\ \hline 143 \end{array}$$

11.
$$\begin{array}{r} 174 \\ \div 100 \\ \hline 1.74 \end{array}$$

12.
$$\begin{array}{r} 5 \times (7 \times 8) \\ \hline 280 \end{array}$$

13. $6 \times 5 = 40 - 10 = 30$

14.
$$\begin{array}{r} 3427 \\ \times 0.2 \\ \hline 685.4 \end{array}$$

15.
$$\begin{array}{r} 0.3 \\ \times 0.8 \\ \hline 0.24 \end{array}$$

16.
$$\begin{array}{r} 84 \\ \times 0.75 \\ \hline 63 \end{array}$$

17.
$$\begin{array}{r} 1.24 \\ \times 0.25 \\ \hline 0.31 \end{array}$$

18.
$$\begin{array}{r} 0.03 \\ \times 0.02 \\ \hline 0.0006 \end{array}$$

19.
$$\begin{array}{r} 93\ 000 \\ 1000 \\ \hline 93 \end{array}$$

20.
$$\begin{array}{r} 34 \overline{)76\ 194} \\ \underline{2241} \end{array}$$

21.
$$\begin{array}{r} 7 \overline{)3200} \\ \underline{4571} \end{array}$$

22.
$$\begin{array}{r} 3 \overline{)12.6} \\ \underline{4.2} \end{array}$$

23.
$$\begin{array}{r} 28 \overline{)19.6} \\ \underline{0.7} \end{array}$$

24. A tailor bought 250 cm of velvet cloth for \$15.50. How many metres of cloth did he buy? $2.5\ m$

25. At a scout rally, 104 scouts came from Manitoba, 285 came from Ontario, and 8 came from British Columbia.

Give an estimate of the number of scouts at the rally. 400

Chapters 1-4 cumulative review 123

OBJECTIVE

To review and test selected concepts and skills previously covered

PACING

Level A All
Level B All
Level C All

USING THE BOOK

This page may be used for diagnostic and remedial as well as review purposes. Students should check their work, correct any errors, and review the pages that contain any problems of the type they missed. Some students can do this on their own while others may need help. If a large number of students have a particular problem incorrect, you may want to reteach that topic to the groups, then assign a duplicated worksheet to reinforce that topic or, refer to an appropriate skill card in the BFA Computational Skills Kit II or BFA Problem Solving Kit II.

Test Item	Text Page Number
1	35
2	42
3	7
4	4
5	8
6	58
7	65
8	5
9	164
10	60
11	95
12	63
13	18
14	71
15	75
16	74
17, 18	77
19	95
20	98
21	100
22, 23	102
24	104
25	14, 15

CHAPTER 5 OVERVIEW

This chapter further develops the division procedures begun in Chapter 4. It examines dividing whole numbers by decimals, dividing decimals by decimals, and dividing when the divisor is larger than the dividend. Also in this chapter, the student studies the interrelationships between capacity of a container, and the mass and volume of water the container can hold. Time zones and temperature are also presented.

OBJECTIVES

- A To introduce and provide a short method for division by 0.1, 0.01, and 0.001
- B To divide whole numbers by divisors from 0.2 to 0.9
- C To divide with decimals in the divisors and in the dividend (up to and including 3 decimal places)
- D To divide when the divisor is larger than the dividend (annexation of zeros)
- E To discover and use the relationship between capacity, mass, and volume of water
- F To compute time after and time in between using the 24 h clock; to explore time zones, daylight saving time, and to relate a specific year to its correct century
- G To read temperature including minus readings and to compute the temperature change between two readings
- H To provide practice in writing and solving equations, estimating for reasonableness of answers, identifying missing and extraneous information, and drawing simple diagrams to help solve word problems

BACKGROUND

The division lessons presented in this chapter presume a reasonable mastery of the concepts introduced in the first part of Chapter 4. (See Chapter 4 Overview, Background notes on page 92; also pages 93 to 103.)

The relationship among capacity of a container, the mass, and the volume of water the container can hold is a very difficult one for most students of this age. While technically the relationship exists only under very precise conditions, it is not necessary to make this point at this time except for those students cognitively prepared. It is recommended that the students be afforded the actual experiences of measuring the approximate mass of 1000 cm³ of water. Emphasize that the data collected are approximate but that under very specific and exact conditions:

1 L of water has a mass of 1 kg.

Solving word problems can be a much more meaningful and successful experience if opportunities for "exploration" are presented. Be sure to include

topics such as explaining a problem in own words, finding alternate solutions, estimating, checking for reasonableness, and checking over an answer that has been calculated. This checking is a "looking back" to reflect on a problem in order to help generalize a technique for future similar situations.

Another aspect is that of identifying extraneous and missing information. Having to supply logical and relevant data requires an awareness of problem-solving techniques that constantly needs development.

Students also need practice in illustrating a problem with quick, meaningful sketches. Drawing a picture to represent data can help simplify a confusing issue that otherwise may have remained incomprehensible. The problems that appear in the displays, the exercises, and on the problem pages present an on-going emphasis on some of these problem-solving techniques.

MATERIALS

construction paper
small amount of artificial or real money
graph paper
Bristol board
pennies
litre containers (graduated if possible)
scales
light plastic (sandwich wrap)
demonstration-sized twenty-four hour clock

CAREER AWARENESS

Airplane Pilot [151]

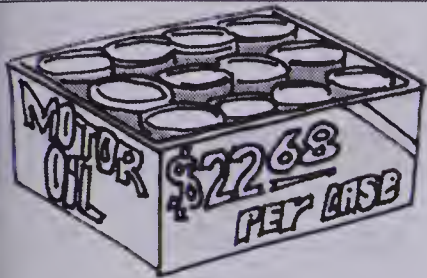
Airplane pilots are employed by the Canadian Armed Forces, major airlines, and small private industries or companies. The type of airplane flown will vary from the extremely fast fighter type used by the armed forces to the heavy "workhorse" freight planes used by the Canadian Armed Forces.

Pilots must take very intensive training and retraining programs, the length and technical degree depending on the type of plane the trainee expects to be qualified to fly.

A pilot assumes a great deal of responsibility. An error on his/her part can be fatal to him/herself as well as to all passengers and innocent earthbound bystanders.

The working conditions vary greatly. While the pilot of a major commercial airline has definite rules and regulations about how long and often, and in what kind of weather he/she may work, the "bush" pilot for private industry may at times be asked to fly long hours, everyday, and in all kinds of weather.

Division



What is the cost of 1 can? **\$1.89**



Tape Length	Price
45 min	\$2.79
60 min	\$2.99
90 min	\$3.99
120 min	\$4.39

What is the average price of the tapes? **\$3.54**



There are 50 dimes in 1 roll.
There are 2857 dimes.
How many rolls? **57**
How many dimes left over? **7**



There are 3476 chairs for the concert.
There are 27 chairs in each row.
How many rows? **128**
How many chairs left over? **20**

Phone bill for:	Amount:
March	\$12.96
April	7.59
May	22.63
June	8.27
July	17.30

What is the average phone bill for these months? **\$13.75**
How much less than \$20.00 is the average phone bill? **\$6.25**



How much is taken off the price of the bird?
What is the price of the bird now? **\$8.64**

OBJECTIVE

To solve division problems

PACING

Level A All
Level B All
Level C All

SUGGESTIONS

Initial Activity Write on the board the following problem.

"There are 7 books on one shelf, 9 on a second shelf, and 5 on a third shelf. What is the total number of books altogether?"

What is the average number of books?"

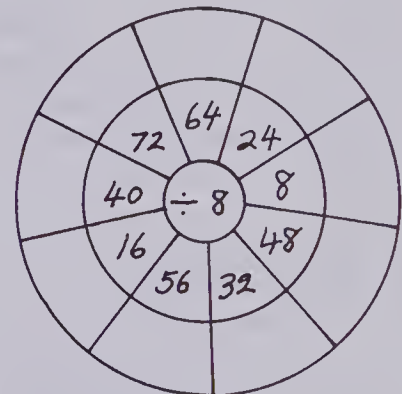
Solve the problem with the students to quickly review the concept of average and the procedures involved therein.

USING THE BOOK

With the students, read over the 6 problems presented on the page. Since the last 2 problems are "2-step problems" you may wish to talk about the steps with your less able students before assigning them.

ACTIVITIES

1. "Round the clock" drill:



2. Have students use the supermarket advertisements in the newspaper as a source for making up problems similar to the ones on the page.

3. See "Itza Fact!" as described in the Activity Reservoir.

OBJECTIVE

To introduce division by 0.1 and 0.01 and their relationship to multiplication by 10 and 100 respectively.

PACING

Level A All
Level B All
Level C All

BACKGROUND

Dividing by a number is the same as multiplying by its reciprocal. 10 is the reciprocal of 0.1 and 100 is the reciprocal of 0.01.

SUGGESTIONS

Initial Activity Draw the following diagram on the board.



Explain that the rectangle has been divided up into tenths.
Each section = 0.1.

How many 0.1's are in 1?

$$1 \div 0.1 = \blacksquare \quad [10]$$

$$1 \times 10 = \blacksquare \quad [10]$$

Draw another rectangle on the board similar to the first.

How many 0.1's are in 2?

$$2 \div 0.1 = \blacksquare \quad [20]$$

$$2 \times 10 = \blacksquare \quad [20]$$

Have students study the two sets of answers carefully and elicit from them that dividing by 0.1 is the same as multiplying by 10.

USING THE BOOK

Go over the first example in the display at the top of the page and have the students explain what the set of balances shows. (Dividing by 0.1 is the same as multiplying by 10.) Examine the second set of balances. Have students explain what this set of balances shows. (Dividing by 0.01 is the same as multiplying by 100.)

Do Exercise 1 orally with the students. Have students write answers only for Exercises 2 to 4.

ACTIVITIES

1. Play "Concentration" with questions and answers similar to the ones in Exercise 4. An explanation of the game is outlined in the Activity Reservoir.

In the Balance

Explain what each set of balances shows.

What rule helps you to divide by 0.1? by 0.01?

Exercises

- Complete these "balances". Use your rules.

(a) $18 \div 0.1 = \blacksquare$ 180	(b) $9 \div 0.1 = \blacksquare$ 90	(c) $152 \div 0.1 = \blacksquare$ 1520
$18 \times 10 = \blacksquare$ 180	$9 \times 10 = \blacksquare$ 90	$152 \times 10 = \blacksquare$ 1520
(d) $35 \div 0.01 = \blacksquare$ 3500	(e) $7 \div 0.01 = \blacksquare$ 700	(f) $289 \div 0.01 = \blacksquare$ 2890
$35 \times 100 = \blacksquare$ 3500	$7 \times 100 = \blacksquare$ 700	$289 \times 100 = \blacksquare$ 28900
- Divide by 0.1.

(a) 24 240	(b) 126 1260	(c) 3 30	(d) 17 170	(e) 458 4580
(f) 19 190	(g) 5 50	(h) 60 600	(i) 1 10	(j) 10 100
- Divide by 0.01.

(a) 16 1600	(b) 5 500	(c) 219 21900	(d) 28 2800	(e) 186 18600
(f) 2 200	(g) 67 6700	(h) 70 7000	(i) 10 1000	(j) 101 10100
- Divide.

(a) $37 \div 0.01$ 3700	(b) $507 \div 0.1$ 5070	(c) $281 \div 0.1$ 2810	(d) $54 \div 0.01$ 5400
--------------------------------	--------------------------------	--------------------------------	--------------------------------

126 Patterning

2. On a bulletin board, students make a display similar to the display in the book. Have them write the rules and show them on the display. This will help reinforce the idea.

3. Some of the students might enjoy investigating further

relationships with 10, 0.1 and 100, 0.01 such as:

$$1\ 463\ 904 \div 0.01 = \underline{\hspace{2cm}} \times 100$$

$$512\ 215 \times 100 = \underline{\hspace{2cm}} \div \underline{\hspace{2cm}}$$

etc.

Have them spot-check their findings with a calculator.

Patterns in Division

Investigate these pattern boards.

$$\begin{array}{l} 53 \div 1 = 53 \\ 53 \div 0.1 = 530 \\ 53 \div 0.01 = 5300 \end{array}$$

$$\begin{array}{l} 4 \div 1 = 4 \\ 4 \div 0.1 = 40 \\ 4 \div 0.01 = 400 \end{array}$$

$$\begin{array}{l} 2.5 \div 1 = 2.5 \\ 2.5 \div 0.1 = 25 \\ 2.5 \div 0.01 = 250 \end{array}$$

What happens to the decimal point in the quotient when you divide by 1? by 0.1? by 0.01? Make a "decimal rule"

Exercises

Complete these patterns.

1. $26 \div 1 = 26$	2. $9 \div 1 = \blacksquare 9$	3. $5.8 \div 1 = \blacksquare 5.8$
$26 \div 0.1 = \blacksquare 260$	$9 \div 0.1 = \blacksquare 90$	$5.8 \div 0.1 = \blacksquare 58$
$26 \div 0.01 = \blacksquare 2600$	$9 \div 0.01 = \blacksquare 900$	$5.8 \div 0.01 = \blacksquare 580$
4. $30 \div 1 = \blacksquare 30$	5. $1 \div 1 = \blacksquare 1$	6. $10 \div 1 = \blacksquare 10$
$30 \div 0.1 = \blacksquare 300$	$1 \div 0.1 = \blacksquare 10$	$10 \div 0.1 = \blacksquare 100$
$30 \div 0.01 = \blacksquare 3000$	$1 \div 0.01 = \blacksquare 100$	$10 \div 0.01 = \blacksquare 1000$

Divide by 0.1.

7. $83 \div 0.1 = 830$ 8. $6 \div 0.1 = 60$ 9. $50 \div 0.1 = 500$ 10. $392 \div 0.1 = 3920$ 11. $100 \div 0.1 = 1000$

Divide by 0.01.

12. $41 \div 0.01 = 4100$ 13. $2 \div 0.01 = 200$ 14. $70 \div 0.01 = 7000$ 15. $505 \div 0.01 = 50500$ 16. $100 \div 0.01 = 10000$

Complete these patterns.

★17. $116 \div 0.1 = \blacksquare 1160$	★18. $74 \div 0.01 = \blacksquare 7400$
$16 \div 0.1 = \blacksquare 160$	$7 \div 0.01 = \blacksquare 700$
$16 \div 0.1 = \blacksquare 16$	$7.4 \div 0.01 = \blacksquare 740$
$1.16 \div 0.1 = \blacksquare 11.6$	$0.7 \div 0.01 = \blacksquare 70$

Patterning 127

OBJECTIVE

To introduce patterns in division by 1, 0.1, and 0.01

PACING

Level A 1-16

Level B 1-16

Level C All

MATERIALS

coloured dots made out of construction paper to represent decimal points

SUGGESTIONS

Initial Activity Write the following questions on the board:

$9 \div 1 = 9$. (coloured dot

representing decimal point)

$9 \div 0.1 = 90$. (move decimal point

1 place to right)

$9 \div 0.01 = 900$. (move decimal point

2 places to right)

Ask: "What happens to the decimal point in the quotient when you divide by 0.1? [It moves 1 place to the right.]

"What happens to the decimal point in the quotient when you divide by 0.01?" [It moves 2 places to the right.]

Make a decimal rule.

USING THE BOOK

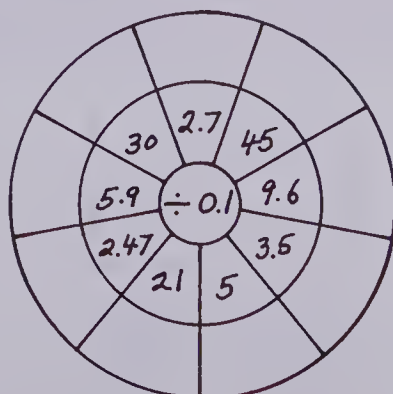
Go over the pattern boards at the top of the page. You may wish to write them on the board and use the "movable" decimal points made out of coloured paper to reinforce the short-cut method.

Do Exercises 1-6 orally with the students. Have the students write answers only for the rest of the questions.

ACTIVITIES

1. Make 2 decks of cards. One deck is made up of whole numbers and decimal numbers, e.g., 78, 93.2. The other deck is made up of 0.1 and 0.01 (several of each). Both decks are shuffled. Participating students draw a card from each deck. They divide the two numbers. If they are correct, they get a number of points equal to the quotient. The one with the most points after a predetermined number of rounds wins.

2. Clock drill:



3. Play "Football" as outlined in the Activity Reservoir. Use question cards which test division by 0.1 and 0.01.

OBJECTIVE

To provide further practice dividing by 0.1

PACING

- Level A All
- Level B All
- Level C All

SUGGESTIONS

Initial Activity Review with students the notion that division can be expressed as $10 \overline{)30}$ or $\frac{30}{10}$ (as on page 93) and that, in the example cited here, the “answer” in both instances is 3.

Review also the procedure used to create equivalent fractions.

Example

$\frac{1}{4} \rightarrow \frac{1 \times 2}{4 \times 2} \rightarrow \frac{2}{8}$, etc.



Point out the equivalent nature expressed here. Amounts haven’t changed, just numbers used to represent the amounts have changed.

USING THE BOOK

Have the students read the problem in the display at the top of the page. Go over the steps developed. Relate the short-cut method to the work they have done on the previous page (moving the decimal point to the right).

Do the first few examples of Exercise 1 with all students. With less able students you may wish to do the first two examples of Exercises 2 to 5 on the board.

ACTIVITIES

1. Make a stencil with questions similar to the ones in the exercises. At the bottom write the answers in mixed up order. Students match the questions and the answers.

2. “Snap”. Make 2 sets of cards with questions and corresponding answers. Two players play. Cards are shuffled together and each player is dealt half the deck. Players take turns playing one card at a time. When 2 corresponding cards turn up (i.e., question and answer), the first player to call “Snap” picks up the pile. The player with the most cards at the end

Ribbon Awards

Miss Carter made awards from ribbon for field day.
Each award measured 0.1 m.
How many awards did Miss Carter make from 3 m of ribbon?

$$0.1 \overline{)3} = \frac{3}{0.1}$$

Multiply the numerator and denominator by 10.

$$\frac{3 \times 10}{0.1 \times 10} = \frac{30}{1} = 30$$

Short cut:

$$0.1 \overline{)30} \rightarrow 1 \overline{)30}$$

She made 30 award ribbons.

Exercises

1. Copy and complete.

$(a) \ 0.1 \overline{)8} \rightarrow 1 \overline{)80}$

$(b) \ 0.1 \overline{)70} \rightarrow 1 \overline{)70}$

$(c) \ 0.1 \overline{)4} \rightarrow 1 \overline{)40}$

$(d) \ 0.1 \overline{)2} \rightarrow 1 \overline{)20}$

$(e) \ 0.1 \overline{)6} \rightarrow 1 \overline{)60}$

2. Divide.

$(a) \ 0.1 \overline{)30}$

$(b) \ 0.1 \overline{)90}$

$(c) \ 0.1 \overline{)50}$

$(d) \ 0.1 \overline{)10}$

3. Divide.

$0.1 \overline{)52} \rightarrow 1 \overline{)520}$

$0.1 \overline{)76} \rightarrow 1 \overline{)760}$

$(c) \ 0.1 \overline{)85} \rightarrow 1 \overline{)850}$

$(d) \ 0.1 \overline{)37} \rightarrow 1 \overline{)370}$

4. Divide.

$0.1 \overline{)726} \rightarrow 1 \overline{)7260}$

$(b) \ 0.1 \overline{)857} \rightarrow 1 \overline{)8570}$

$(c) \ 0.1 \overline{)407} \rightarrow 1 \overline{)4070}$

$(d) \ 0.1 \overline{)663} \rightarrow 1 \overline{)6630}$

5. Divide.

$(a) \ 0.1 \overline{)23} \rightarrow 1 \overline{)230}$

$(b) \ 0.1 \overline{)6852} \rightarrow 1 \overline{)68520}$

$(c) \ 0.1 \overline{)447} \rightarrow 1 \overline{)4470}$

$(d) \ 0.1 \overline{)750} \rightarrow 1 \overline{)7500}$

6. Greta cut 4 m of ribbon for awards.
Each award measured 0.1 m.
How many awards can she make? **40**

of the allotted time wins.

3. See “Rummy” as described in the Activity Reservoir. Use card sets such as:

$23 \div 1 = 23$

$23 \div 0.1 = 230$

$23 \div 0.01 = 2300$

Bazaar

Mrs. Jones bought 18 m of material to make aprons for the bazaar. She used 0.9 m of material for each apron. How many aprons did she make?

$$0.9 \overline{)18} = \frac{18}{0.9}$$

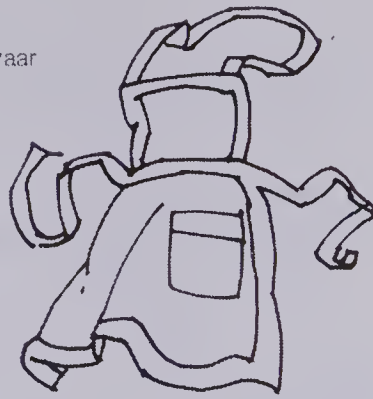
Multiply the numerator and denominator by 10

$$\frac{18 \times 10}{0.9 \times 10} = \frac{180}{9} = 20$$

Short cut:

$$0.9 \overline{)18.0} \rightarrow 9 \overline{)180}^{20}$$

She made 20 aprons.



Exercises

1. Copy and complete.

(a) $0.2 \overline{)12} \rightarrow 2 \overline{)120}$

(b) $0.4 \overline{)16} \rightarrow 4 \overline{)160}$

(c) $0.3 \overline{)24} \rightarrow 3 \overline{)240}$

(d) $0.7 \overline{)49} \rightarrow 7 \overline{)490}$

(e) $0.3 \overline{)18} \rightarrow 3 \overline{)180}$

(f) $0.6 \overline{)54} \rightarrow 6 \overline{)540}$

Divide.

2. (a) $0.4 \overline{)32}$

(b) $0.9 \overline{)63}$

(c) $0.7 \overline{)56}$

(d) $0.2 \overline{)14}$

(e) $0.8 \overline{)208}$

(f) $0.3 \overline{)135}$

(g) $0.5 \overline{)365}$

(h) $0.6 \overline{)336}$

3. (a) $0.8 \overline{)2136}$

(b) $0.5 \overline{)4465}$

(c) $0.2 \overline{)1326}$

(d) $0.7 \overline{)4011}$

(e) $0.4 \overline{)1068}$

(f) $0.9 \overline{)2259}$

(g) $0.8 \overline{)4192}$

(h) $0.6 \overline{)5352}$

4. Mr. Park used 24 m of cloth to make barbecue aprons.

He used 0.8 m for each one.

How many aprons did he make? 30

Division by 0.2 to 0.9 129

OBJECTIVE

To divide whole numbers by divisors 0.2 to 0.9

PACING

Level A All

Level B All

Level C All

SUGGESTIONS

Initial Activity Review the procedure for dividing by 0.1 developed in the previous lesson. Explain to students that one objective was to make the divisor into a whole number. This was achieved by multiplying numerator and denominator by 10. Thus in a question such as $9 \div 0.1$ to make the 0.1 into a whole number multiplication by 10 was required.

Write the question $0.2 \overline{)24}$. Rewrite in this form: $\frac{24}{0.2}$. Ask: "What must be

done to the denominator to make it a whole number?" [Multiply by 10.]

"What must be done to the numerator?" [Multiplication by 10.]

The question now is written as $\frac{240}{2}$ or $2 \overline{)240}$ and can then be solved in the established manner.

USING THE BOOK

Using the problem in the display, go over the steps developed to reinforce the work done in the Initial Activity. In the short cut, the students should take note that in order to make the divisor a whole number, the decimal point was moved one place to the right. The decimal in the dividend must also be moved the same number of places to the right to keep the relationship equal.

For the first two exercises, you may wish to have the student copy each question and rewrite it in its new form similar to the examples in Exercise 1. In Exercise 3 they can be told to "plan ahead" and write each question with the decimal point relocated.

ACTIVITIES

1. Play "Eraser" using cards which have questions similar to the ones in the exercises. An explanation of the game is in the Activity Reservoir.

2. See the "500 Grand" idea in the Activity Reservoir. Adjust the title and the grid to maintain division skills presented to date. As a source for grid items, use (a) some of the exercises from this and previous pages, (b) the extra practice exercises on pages 352 and 353, and (c) basic division facts.

Example

	etc.		
3	$0.8 \overline{)208}$	$\frac{24}{0.1}$	26×1
2	$5 \overline{)60}$	$0.6 \overline{)336}$	$0.01 \overline{)}$ etc.
1	$26 \div 0.1$	$\frac{100}{10}$	$28 \overline{)15}$
	1	2	3

3. To provide further practice recognizing and predicting patterns (page 127), provide some activity cards such as:

What comes next?

(a) 1, 0.1, 0.01, 0.001, ____.

(b) 5, 50, 500, 5000, ____.

(c) 2.851, 28.51, 285.1, 2851.0, ____.

(d) ↓, ↑↑, ↓↓↓, ____.

(e) $0.1, \frac{2}{10}, 0.3, \frac{4}{10},$ ____.

etc.

OBJECTIVE

To divide by a 2-digit divisor which includes 1 decimal place

PACING

Level A 1-4
Level B 1-5
Level C 2-6

SUGGESTIONS

Initial Activity Do a few questions similar to the ones in the previous lesson to review division by a decimal. Highlight the necessity to convert the denominator or divisor into a whole number. Remind students of the short cut and how this short cut can help them “plan ahead” and write the question in its final form before proceeding with the division.

USING THE BOOK

Go over the problem in the display to reinforce the procedure. You may wish to divide your class into 2 groups. Group A does the odd questions in each Exercise (i.e., (a), (c), (e), etc.), and group B does the even questions in each Exercise (i.e., (b), (d), (f), etc.).

Math Time

Danny answered 15 math questions in 2.5 min.

What was the average number of questions he answered in 1 min?

$$2.5 \overline{)15} = \frac{15}{2.5}$$

Write an equivalent fraction with the denominator a whole number:

$$\frac{15 \times 10}{2.5 \times 10} = \frac{150}{25} = 6$$

Short cut:

$$2.5 \overline{)15} \rightarrow 25 \overline{)150}$$

Danny answered an average of 6 questions each minute.

Exercises

1. Divide.

(a) $2.1 \overline{)63}$

(b) $4.7 \overline{)94}$

(c) $2.4 \overline{)96}$

(d) $3.4 \overline{)68}$

(e) $1.3 \overline{)65}$

(f) $3.2 \overline{)96}$

(g) $2.8 \overline{)84}$

(h) $4.2 \overline{)84}$

(i) $1.5 \overline{)60}$

(j) $2.7 \overline{)81}$

(k) $3.7 \overline{)10}$

(l) $2.3 \overline{)92}$

2. Find the quotient

(a) $4.8 \overline{)432}$

(b) $2.6 \overline{)156}$

(c) $6.1 \overline{)427}$

(d) $3.5 \overline{)175}$

(e) $2.9 \overline{)203}$

(f) $5.6 \overline{)448}$

(g) $4.3 \overline{)387}$

(h) $8.6 \overline{)602}$

(i) $9.8 \overline{)588}$

(j) $7.3 \overline{)365}$

(k) $8.2 \overline{)328}$

(l) $2.7 \overline{)216}$

3. Divide.

(a) $5.5 \overline{)3465}$

(b) $5.9 \overline{)3835}$

(c) $9.6 \overline{)7968}$

(d) $9.4 \overline{)7520}$

(e) $2.9 \overline{)2001}$

(f) $6.8 \overline{)3196}$

(g) $5.4 \overline{)5130}$

(h) $3.9 \overline{)1521}$

(i) $2.3 \overline{)1288}$

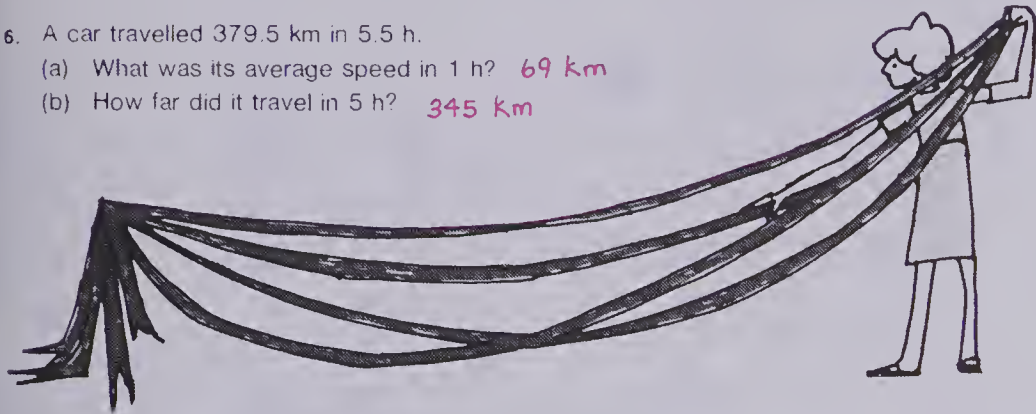
(j) $6.7 \overline{)1742}$

(k) $9.5 \overline{)4085}$

(l) $4.3 \overline{)1204}$

Solve these problems that Danny answered in his math class.

4. (a) How many small spice packets with a mass of 2.5 g each can be made from a large spice packet with a mass of 215 g? **86**
- (b) Each small packet sells for \$1.15.
What is the total selling price of all the small packets? **\$ 98.90**
5. (a) Susan made streamers from coloured ribbon.
How many streamers that measure 1.5 m each can she make from a roll of ribbon 36 m long? **24**
- (b) She needed 50 streamers.
How many more streamers did she have to make? **26**
- ★ 6. A car travelled 379.5 km in 5.5 h.
- (a) What was its average speed in 1 h? **69 km**
- (b) How far did it travel in 5 h? **345 km**



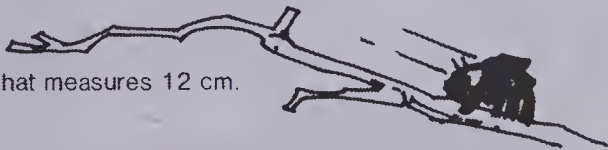
BRAINTICKLER

A small insect is creeping up a twig that measures 12 cm.

It creeps 3 cm in the daytime.

Each night it slips back 2 cm.

How many days does it take to creep to the top of the twig? **10 d**



ACTIVITIES

1. Play “Bingo” using division facts and answers. An explanation of the game is in the Activity Reservoir.

2. If you have not already done so, see Activity 2, page 102. Use division blanks of a sort similar to exercises on this page.

3. Have students compute the amount of their “math time” in a day, a week, a month, a year.

OBJECTIVE

To divide with decimals in the divisor and the dividend

PACING

Level A All
Level B All
Level C All

RELATED AIDS

HMS — DM31.
BFA COMP LAB II — 116.

SUGGESTIONS

Initial Activity Write the following questions on the chalkboard.

(a) $\frac{9}{0.3}$ (b) $\frac{15}{1.5}$ (c) $\frac{5.5}{0.5}$

As a review, develop questions (a) and (b) on the board making equivalent fractions by multiplying the numerator and denominator by the number that will make the denominator a whole number. [10] Progress to (c) and ask students to first identify the difference between (c) and the first two questions. [decimal in the numerator] Ask: "What must be done to the denominator to make it a whole number?" [Multiply by 10.] "What then must be done to the numerator?" [It too must be multiplied by 10.] Write the new form of the question: $\frac{55}{5} \rightarrow 5 \overline{)55} = 11$.

As reinforcement to convince students that the procedure is valid, ask "How many groups of $\frac{5}{10}$'s are in $5\frac{5}{10}$?" They should readily be able to answer 11. If not, show in diagram form.

USING THE BOOK

Go over the question in the display at the top of the page, developing it as outlined. Draw attention to the short cut where moving the decimal point to the right produces the same results.

For the first few exercises, you may wish to have students write the original question and then rewrite it in its new form. Having done this a few times, you may wish to then suggest that they "plan ahead" and write the question only once with the decimal moved to its appropriate place. Before asking students to do Exercise 2, draw attention to the "different" location of the decimal in the dividend, and do parts (a), (b), and if necessary, (c) on

Berry Picking

Bill picked 17.5 baskets of berries in 3.5 h.

What was the average number of baskets he picked in 1 h?

$$3.5 \overline{)17.5} = \frac{17.5}{3.5}$$

Write an equivalent fraction with the denominator a whole number:

$$\frac{17.5 \times 10}{3.5 \times 10} = \frac{175}{35} = 5$$

Short cut:

$$3.5 \overline{)17.5} \rightarrow 35 \overline{)175}$$

Bill picked an average of 5 baskets each hour.



Exercises

Find the quotient.

1. (a) $3.1 \overline{)27.9} \rightarrow 31 \overline{)279}$ (b) $4.3 \overline{)34.4} \rightarrow 43 \overline{)344}$ (c) $2.7 \overline{)16.2} \rightarrow 27 \overline{)162}$ (d) $4.6 \overline{)36.8} \rightarrow 46 \overline{)368}$
(e) $6.4 \overline{)44.8} \rightarrow 64 \overline{)448}$ (f) $7.8 \overline{)23.4} \rightarrow 78 \overline{)234}$ (g) $8.7 \overline{)52.2} \rightarrow 87 \overline{)522}$ (h) $9.2 \overline{)64.4} \rightarrow 92 \overline{)644}$
(i) $8.9 \overline{)80.1} \rightarrow 89 \overline{)801}$ (j) $5.3 \overline{)42.4} \rightarrow 53 \overline{)424}$ (k) $4.8 \overline{)14.4} \rightarrow 48 \overline{)144}$ (l) $7.9 \overline{)63.2} \rightarrow 79 \overline{)632}$
(m) $4.4 \overline{)30.8} \rightarrow 44 \overline{)308}$ (n) $9.6 \overline{)57.6} \rightarrow 96 \overline{)576}$ (o) $7.8 \overline{)70.2} \rightarrow 78 \overline{)702}$ (p) $6.3 \overline{)50.4} \rightarrow 63 \overline{)504}$

2. (a) $5.3 \overline{)12.72} \rightarrow 53 \overline{)127.2}$ (b) $6.2 \overline{)34.72} \rightarrow 62 \overline{)347.2}$ (c) $4.9 \overline{)42.14} \rightarrow 49 \overline{)421.4}$
(d) $8.1 \overline{)55.08} \rightarrow 81 \overline{)550.8}$ (e) $7.3 \overline{)50.37} \rightarrow 73 \overline{)503.7}$
(f) $9.5 \overline{)61.75} \rightarrow 95 \overline{)617.5}$ (g) $8.4 \overline{)40.32} \rightarrow 84 \overline{)403.2}$ (h) $2.9 \overline{)28.42} \rightarrow 29 \overline{)284.2}$ (i) $8.2 \overline{)37.72} \rightarrow 82 \overline{)377.2}$
(j) $48.84 \div 6.6$ (k) $39.33 \div 5.7$ (l) $57.62 \div 8.6$ (m) $43.61 \div 4.9$

3. A car uses 8.1 L of gasoline to travel 89.1 km.
How far does the car travel on 1 L of gasoline? **11 km**

132 Decimals in the divisor and the dividend

the board, drawing attention to its place in the dividend and its appearance in the quotient.

Discuss the rationale in solving Exercise 3, relating it to the solution of the problem at the top of the page.

ACTIVITIES

1. Divide the class into 2 teams. A division fact or question is posed to

the first two members on each team. The first to answer correctly scores a number of points equal to the quotient. At the end, add up the various points. The team with the most points wins.

2. See "Input-Output" as described in the Activity Reservoir.

3. See "Itza Fact!" as described in the Activity Reservoir.

Stacking Paper

Packages of paper are stacked against a wall.
One package is 0.01 m thick.
The height of a stack is 2 m.
How many packages of paper are in the stack?

$$0.01 \overline{)2} = \frac{2}{0.01}$$

Write an equivalent fraction with a whole number as denominator:

$$\frac{2 \times 100}{0.01 \times 100} = \frac{200}{1} = 200$$

Short cut.

$$0.01 \overline{)2.00} \rightarrow 1 \overline{)200}$$

There are 200 packages in the stack.



Exercises

Divide.

1. $0.01 \overline{)9} \rightarrow 1 \overline{)900}$ (Handwritten: 900)
2. $0.01 \overline{)200}$ (Handwritten: 20000, 900)
3. $0.01 \overline{)3}$ (Handwritten: 300)
4. $0.01 \overline{)18}$ (Handwritten: 1800)
5. $0.01 \overline{)0.74}$ (Handwritten: 74)
6. $0.01 \overline{)0.83}$ (Handwritten: 83)
7. $0.01 \overline{)0.57}$ (Handwritten: 57)
8. $0.01 \overline{)0.42}$ (Handwritten: 42)
9. $0.01 \overline{)0.68}$ (Handwritten: 68)
10. $0.01 \overline{)0.91}$ (Handwritten: 91)
11. $0.01 \overline{)0.25}$ (Handwritten: 25)
12. $0.01 \overline{)0.64}$ (Handwritten: 64)

Find the quotient.

13. $0.01 \overline{)4.83}$ (Handwritten: 483)
14. $0.01 \overline{)9.72}$ (Handwritten: 972)
15. $0.01 \overline{)16.39}$ (Handwritten: 1639)
16. $0.01 \overline{)10.24}$ (Handwritten: 1024)
17. $0.01 \overline{)6.17}$ (Handwritten: 617)
18. $0.01 \overline{)88.72}$ (Handwritten: 8872)
19. $0.01 \overline{)147.28}$ (Handwritten: 14728)
20. $0.01 \overline{)500.45}$ (Handwritten: 50045)
21. $0.01 \overline{)6.748}$ (Handwritten: 674.8)
22. $0.01 \overline{)9.463}$ (Handwritten: 946.3)
23. $0.01 \overline{)46.005}$ (Handwritten: 4600.5)
24. $0.01 \overline{)500.2}$ (Handwritten: 50020)
25. $0.01 \overline{)620.771}$ (Handwritten: 62077.1)
26. $0.01 \overline{)718.428}$ (Handwritten: 71842.8)
27. $0.01 \overline{)38.444}$ (Handwritten: 3844.4)
28. $0.01 \overline{)947.3}$ (Handwritten: 94730)

29. Each notebook is 0.01 m thick.
The stack of notebooks is 2.54 m high.
How many notebooks in the stack? (Handwritten: 254)



OBJECTIVE

To divide by 0.01

PACING

Level A 1-28
Level B 1-28
Level C 19-33

BACKGROUND

The main difference between the questions in this lesson and those in the lessons preceding this page is that in order to make the denominator or divisor a whole number, it is necessary to multiply by 100 instead of by 10.

SUGGESTIONS

Initial Activity Review a few of the exercises done in the previous lesson.

USING THE BOOK

Use the display to point out the similarities in the procedure used when dividing by a divisor with 1 decimal place and a divisor with 2 decimal places.

You may wish to do several examples with the students before assigning the page.

ACTIVITIES

1. Play "Bingo" as described in the Activity Reservoir.

2. Have students compile a list of situations where decimals are used in everyday life.

3. To review place value, see "The P.V. Game" as described in the Activity Reservoir.

OBJECTIVE

To divide by 0.02 to 0.99

PACING

Level A 1-36, 41

Level B 1-36, 39-41

Level C 17-41

RELATED AIDS

BFA COMP LAB II — 117.

CALC. ACTIVITY MASTERS — 81, 83.

SUGGESTIONS

Initial Activity In previous lessons, students progressed from questions such as $0.1 \overline{)26} \rightarrow 0.4 \overline{)2.8}$. Remind them of this and bring out the similarity of process. They have just completed division by 0.01 and are now going to progress to division by 0.02 to 0.99. Point out that the procedure is essentially the same as the division by tenths.

USING THE BOOK

Use the display at the top of the page to reinforce the procedure. You may wish to do several examples on the board before assigning Exercise 1.

Before assigning Exercise 25, you should draw attention to the necessity to insert a zero as a placeholder in the new dividend. You may wish to do several examples on the board before assigning the rest of the exercises.

ACTIVITIES

1. Play "Concentration" using division questions and answers. An explanation of the game is outlined in the Activity Reservoir.

2. If you have not already done so, see Activity 2, page 102. Use question blanks from this page.

3. Have students make up challenging questions similar to the starred questions in the book. They can challenge each other to solve them and use a calculator to check answers.

Rods and Wires

The diameter of a wire is 0.05 cm.

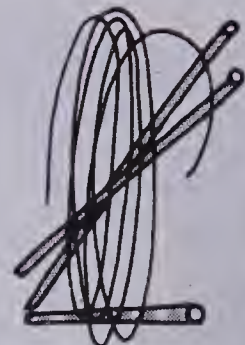
The diameter of a metal rod is 2.75 cm.

How many times thicker is the rod than the wire?

$$0.05 \overline{)2.75} = \frac{2.75}{0.05} \rightarrow \frac{2.75 \times 100}{0.05 \times 100} = \frac{275}{5} \rightarrow 5 \overline{)275}$$

$$\begin{array}{r} 55 \\ 5 \overline{)275} \\ \underline{25} \\ 25 \\ \underline{25} \\ 0 \end{array}$$

The rod is 55 times as thick as the wire.



Exercises

Divide.

1. $0.05 \overline{)6.25}$ $\frac{125}{63}$

5. $0.09 \overline{)5.67}$ $\frac{63}{810}$

2. $25.8 - 0.06$ $\frac{430}{810}$

13. $32.4 \div 0.04$ $\frac{9}{810}$

4. $0.26 \overline{)2.34}$ $\frac{9}{5}$

21. $0.73 \overline{)3.65}$ $\frac{5}{6}$

6. $0.08 \overline{)5.76}$ $\frac{72}{96}$

6. $0.02 \overline{)1.92}$ $\frac{96}{990}$

14. $79.2 \div 0.08$ $\frac{7}{990}$

15. $17.1 - 0.03$ $\frac{2}{570}$

19. $0.15 \overline{)1.05}$ $\frac{7}{6}$

22. $0.92 \overline{)5.52}$ $\frac{6}{6}$

3. $0.07 \overline{)1.61}$ $\frac{23}{98}$

7. $0.03 \overline{)2.94}$ $\frac{98}{470}$

11. $42.3 \div 0.09$ $\frac{470}{570}$

15. $17.1 - 0.03$ $\frac{2}{570}$

19. $0.72 \overline{)1.44}$ $\frac{2}{6}$

23. $0.67 \overline{)4.02}$ $\frac{6}{9}$

4. $0.04 \overline{)1.48}$ $\frac{37}{65}$

8. $0.05 \overline{)3.25}$ $\frac{65}{730}$

12. $36.5 \div 0.05$ $\frac{730}{870}$

16. $78.3 \div 0.09$ $\frac{870}{8}$

20. $0.68 \overline{)5.44}$ $\frac{8}{9}$

24. $0.28 \overline{)2.52}$ $\frac{9}{9}$

Find the quotient.

25. $0.14 \overline{)29.4}$ $\frac{210}{70}$

29. $0.51 \overline{)35.7}$ $\frac{70}{45}$

33. $0.23 \overline{)10.35}$ $\frac{45}{321}$

★37. $0.68 \overline{)21.828}$ $\frac{321}{64.3}$

26. $0.25 \overline{)12.5}$ $\frac{50}{80}$

30. $0.48 \overline{)38.4}$ $\frac{80}{94}$

34. $0.76 \overline{)71.44}$ $\frac{94}{85}$

★38. $0.45 \overline{)28.935}$ $\frac{64.3}{94.6}$

27. $0.47 \overline{)23.5}$ $\frac{50}{30}$

31. $0.95 \overline{)28.5}$ $\frac{30}{85}$

35. $0.61 \overline{)51.85}$ $\frac{85}{94.6}$

39. $0.12 \overline{)11.352}$ $\frac{94.6}{90.9}$

28. $0.56 \overline{)19.6}$ $\frac{35}{90}$

32. $0.31 \overline{)27.9}$ $\frac{90}{58}$

36. $0.79 \overline{)45.82}$ $\frac{58}{90.9}$

40. $0.38 \overline{)34.542}$ $\frac{90.9}{16.5}$

41. A metal beam is 0.54 cm thick.

A second beam is 8.91 cm thick.

How many times thicker is the second beam than the first?

16.5



Measuring

Using a trundle wheel, Peter measured 1 km.

Andrew measured 0.001 km with a ruler.

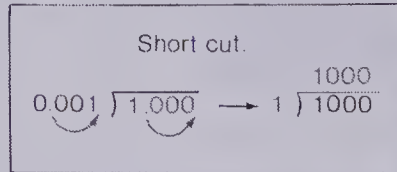
How many times greater is Peter's measurement than Andrew's?

$$0.001 \overline{)1} = \frac{1}{0.001}$$

Write an equivalent fraction with a whole number as a denominator:

$$\frac{1 \times 1000}{0.001 \times 1000} = \frac{1000}{1} = 1000$$

Peter's measurement is 1000 times greater than Andrew's.



Exercises

- Divide.
- | | | | |
|--|--|--|--|
| 1. $0.001 \overline{)7}$ 7000 | 2. $0.001 \overline{)9}$ 9000 | 3. $0.001 \overline{)16}$ 16000 | 4. $0.001 \overline{)25}$ 25000 |
| 5. $0.001 \overline{)634}$ 634000 | 6. $0.001 \overline{)483}$ 483000 | 7. $0.001 \overline{)500}$ 500000 | 8. $0.001 \overline{)6438}$ 6438000 |
| 9. $0.771 \div 0.001$ 771 | 10. $0.892 \div 0.001$ 892 | 11. $0.406 \div 0.001$ 406 | |
| 12. $0.517 \div 0.001$ 517 | 13. $0.602 \div 0.001$ 602 | 14. $0.953 \div 0.001$ 953 | |
| 15. $0.001 \overline{)8.27}$ 8270 | 16. $0.001 \overline{)3.14}$ 3140 | 17. $0.001 \overline{)19.28}$ 19280 | |
| 18. $0.001 \overline{)38.45}$ 38450 | 19. $0.001 \overline{)634.74}$ 634740 | 20. $0.001 \overline{)900.17}$ 900170 | |

Find the quotient.

- ★ 21. $24.5 \div 0.007$ **3500** ★ 22. $36.4 \div 0.008$ **4550** ★ 23. $3.075 \div 0.123$ **25**
- ★ 24. $1.587 \div 0.345$ **4.6** ★ 25. $7.290 \div 0.675$ **10.8** ★ 26. $608.235 \div 17.63$ **34.5**
27. Sergio's paper plane flew 0.615 m.
Nancy's plane flew 9.225 m.
How many times farther did Nancy's plane fly than Sergio's? **15**

Dividing by thousandths 135

OBJECTIVE

To divide by 0.001

PACING

Level A 1-20
Level B 1-20
Level C 12-27

RELATED AIDS

HMS — DM32.

CALC. ACTIVITY MASTERS — 80.

USING THE BOOK

Use the display to show that the procedure for dividing by 0.001 is essentially the same as dividing by 0.1 and 0.01. In this case, multiplication by 1000 is necessary in order to write an equivalent fraction with a whole number as denominator. It may be helpful to do some examples of division by 0.1 and 0.01 to lead up to the division by 0.001. When using the short-cut method, draw attention to the necessity to write zeros as placeholders before placing the decimal point in the dividend.

You may wish to do several examples on the board before assigning the exercises.

ACTIVITIES

1. Make 2 decks of cards. One deck contains any whole and decimal numbers. The other deck is made up of cards with 0.1, 0.01, and 0.001 written on them. Both decks are shuffled. Each player draws a card from each deck and he divides the first deck card by the second deck card. For every correct answer a point is awarded. The player with the most points wins.

2. Play the game "Square It" as outlined in the Activity Reservoir. The player who wins is the one who has the greatest number of points after he or she has divided the point total by 0.001.

3. Make up a stencil with examples similar to the ones on the page. Have the students play "Beat the Clock". They have to try to answer as many questions as they can within a designated time.

OBJECTIVE

To provide practice in writing and solving equations

PACING

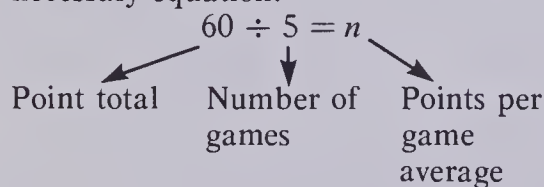
Level A 1-5
Level B 1-7
Level C 6-11

RELATED AIDS

BFA PROB. SOLVING LAB II — 230, 234, 236.

SUGGESTIONS

Initial Activity Review equations and writing equations for word problems as presented on pages 18, 19, and 86. Present a problem such as: Susan had scored a total of 60 points after 5 basketball games. What was her points per game average? Solve it, showing the origin of the necessary equation:



$$12 = n$$

She scored an average 12 points per game.

USING THE BOOK

Use the display at the top of the page to review the meaning of the term equations.

Have students read over silently all the assigned questions. They should solve the questions in any order they wish. Encourage them to do first the ones they find the easiest.

Equations

A very long game of monopoly lasted 120 h.
How many days is that?

$$120 \div 24 = n$$
$$5 = n$$

A number sentence with an equals sign is called an **equation**.

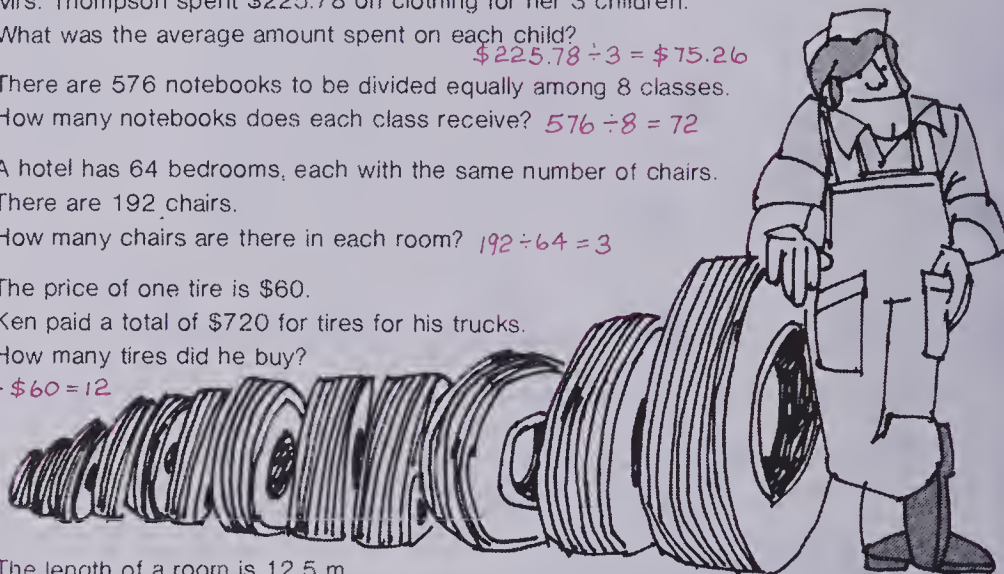
The game lasted 5 d.



Exercises

Write equations for the following, and solve.

1. Mrs. Thompson spent \$225.78 on clothing for her 3 children.
What was the average amount spent on each child?
 $\$225.78 \div 3 = \75.26
2. There are 576 notebooks to be divided equally among 8 classes.
How many notebooks does each class receive? $576 \div 8 = 72$
3. A hotel has 64 bedrooms, each with the same number of chairs.
There are 192 chairs.
How many chairs are there in each room? $192 \div 64 = 3$
4. The price of one tire is \$60.
Ken paid a total of \$720 for tires for his trucks.
How many tires did he buy?
 $\$720 \div \$60 = 12$
5. The length of a room is 12.5 m.
The width of the room is 0.7 times the length.
What is the width? $12.5 \times 0.7 = 8.75 \text{ (m)}$



6. Mrs. Miller paid \$2.88 for 2 dozen cookies.
What is the cost of 1 cookie? $\$2.88 \div 24 = \0.12

7. John is reading an adventure story.
The book has 243 pages.
He is $\frac{1}{3}$ of the way through.
How many pages has he read? $243 \div 3 = 81$

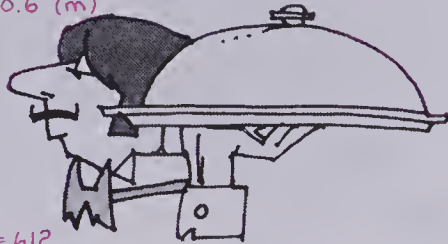


- ★ 8. The perimeter of a square is 20.92 cm.
What is the length of each side? $20.92 \div 4 = 5.23 \text{ (cm)}$

- ★ 9. If 27 ribbons have a total length of 48.6 m, what is the length of 1 ribbon? $48.6 \div 27 = 1.8 \text{ (m)}$
What is the length of 17 ribbons? $1.8 \times 17 = 30.6 \text{ (m)}$

- ★ 10. A box of cereal contains 432 g.
It costs 72¢.
How many cents does 1 g cost? $72 \div 432 = 0.17 \text{ ¢}$

11. A caterer charges \$6.75 per meal.
His bill for his customer was \$4131.
How many meals did he serve? $\$4131 \div \$6.75 = 612$



ACTIVITIES

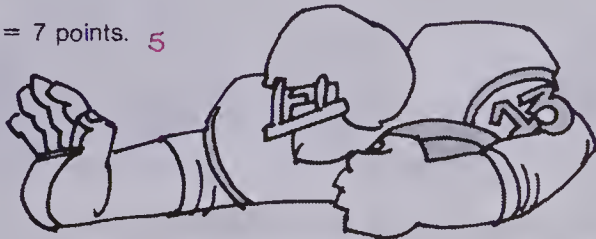
1. See Activity 1, page 86.
2. Make up a coded riddle following the steps outlined in the Activity Reservoir.
3. See Activity 1, page 22.
4. If you have not already done so, see the game "Number Sentence" as described in the Activity Reservoir.

BRAINTICKLER

The Rexford Roughriders football team scored a total of 53 points.
The number of converted touchdowns was one less than the number of field goals.
How many converted touchdowns were scored?

Hint: 1 field goal = 3 points.

1 converted touchdown = 7 points. **5**



OBJECTIVE

To provide practice in estimating

PACING

Level A 1-5

Level B 1-9

Level C 5-10

VOCABULARY

turtleneck, corduroy

SUGGESTIONS

Initial Activity Review the rounding and estimating skills necessary to complete the word problems on this page (see pages 12 to 15). Present a sample problem such as:

The gas tank in Oscar's car holds 56 L. Gasoline costs 39.1¢ for one litre.

How much will it cost?

Estimate:

56 L \longrightarrow 60 L

39.1¢ \longrightarrow $\times 40$ ¢

\$24.00

Calculate:

39.1

$\times 56$

\$21.90

Compare: \$21.90 is close to \$24.00.

Yes, \$21.90 for 56 L of gasoline at 39.1¢/L is reasonable.

USING THE BOOK

Go over the display at the top of the page to review the steps taken when finding an estimate.

Orally with the students, round the cost of the articles in Exercises 1 and 2 to ensure that they understand the procedure.

You may wish to briefly review the concept and computation of average as it applies to Exercises 4, 5, and 10.

Point out that, in Exercise 3, "140 km to work each day" means a *round trip* of 140 km, not 140 km there and 140 km back.

Be sure to clarify an acceptable answer format for the exercises on these two pages.

Estimating

Estimating helps us know if our answer is reasonable.

One barbecued chicken costs \$2.99.

How much will 19 chickens cost?

Step 1 Round \$2.99 to the nearest dollar amount \longrightarrow \$3.00.

Step 2 Round 19 chickens to the nearest 10 \longrightarrow 20

Step 3 Estimate \longrightarrow $\$3.00 \times 20 = \60.00 .
The chickens will cost about \$60.00.

Step 4 Find the exact amount: $\$2.99 \times 19 = \56.81

Step 5 Compare: \$56.81 is close to \$60.00.
Yes, \$56.81 for 19 chickens is reasonable.

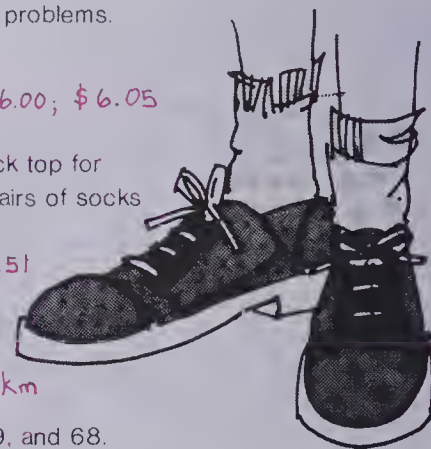
$$\begin{array}{r} \$2.99 \\ \times 19 \\ \hline 2691 \\ 2990 \\ \hline \$56.81 \end{array}$$



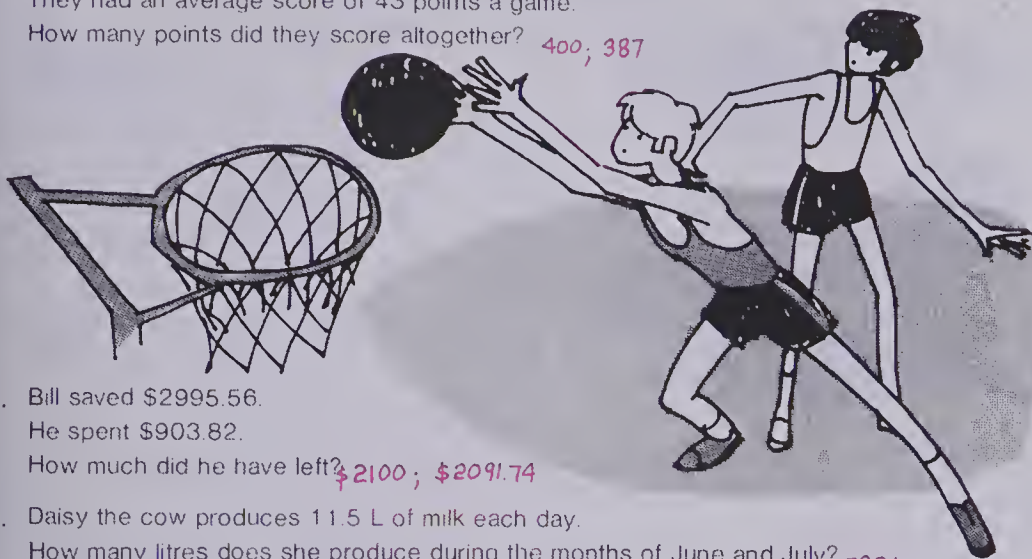
Exercises

Estimate by rounding, and then solve the following word problems.

- Bob's new shoes cost \$33.95.
How much change did he receive from \$40.00? *\$6.00; \$6.05*
- Mrs. West went shopping. She bought one turtleneck top for \$14.99, one pair of corduroy pants for \$17.87, 2 pairs of socks for \$3.88, and a winter jacket for \$34.77.
How much did she pay altogether? *\$72.00; \$71.51*
- John drives his car 140 km to work each day.
How far does he travel in 29 d. *4200 km; 4060 km*
- Mary's math test results last month were 84, 79, 89, and 68.
What was her average mathematics mark? *80; 80*

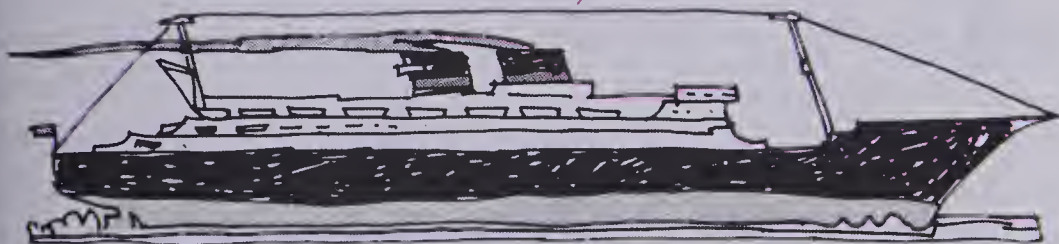


5. The school basketball team played 9 games.
They had an average score of 43 points a game.
How many points did they score altogether? *400; 387*



6. Bill saved \$2995.56.
He spent \$903.82.
How much did he have left? *\$2100; \$2091.74*
7. Daisy the cow produces 11.5 L of milk each day.
How many litres does she produce during the months of June and July? *720 L; 701.5 L*
8. A building has 19 stories. Each story is 3.9 m high.
What is the total height of the 19 stories? *80 m; 74.1 m*

9. 54 people paid a total of \$99 900.00 to go on a cruise.
How much did each person pay? *\$2000.00; \$1850.00*



- ★ 10. During a snowstorm, 100.8 cm of snow fell between 04.30 and 15.00 on the same day.
On the average, how many centimetres of snow fell each hour? *10 cm; 9.6 cm*

ACTIVITIES

1. See the Activities listed for pages 12 to 15 for rounding and estimating ideas which could be used here.

2. Play "Concentration" as described in the Activity Reservoir.
Use card pairs such as:

\$33.95 →
nearest dollar?

\$34

160 →
nearest hundred

200

etc.

3. See "Road Rally" as described in the Activity Reservoir.

OBJECTIVE

To provide practice in dividing with and without decimals

PACING

- Level A 1-5, parts (a), (b), and (c); 6; 7
 Level B All
 Level C 2-7, parts (b), (c), and (d); 6; 7

RELATED AIDS

CALC. ACTIVITY MASTERS — 27.

USING THE BOOK

Go over the display to review the division steps and the estimating procedure as presented up to page 134 and on pages 138 and 139.

You may wish to do on the board the first example of each question as a further review before assigning the page. Emphasize the consistency of procedure. The answers for the (a) part of Exercises 1 to 5 may be found in the back of the text, for those who wish to check their progress.

ACTIVITIES

1. Prepare a series of envelopes, each labelled in series: 100, 200, 300, ..., 1000; or 1000, 2000, ..., 10 000; or 10 000, 20 000, ..., 100 000, etc. Have the students generate numbers which when rounded, would yield amounts that match the envelope labels. Students should choose three numbers per envelope (i.e., for envelope 100, write, for example, 56, 95, 140) and write each number on its own card. When a sufficient number of number cards have been assembled, have them shuffled and include the following instruction card:

By rounding each number card to the nearest (e.g., "hundred") sort 20 cards into their correct envelopes.

2. Prepare a prototype pattern such as:

$$\begin{array}{l} 12 \div 3 = 4 \\ 120 \div 3 = 40 \\ 1200 \div 3 = \underline{\quad} \\ 12\,000 \div 3 = \underline{\quad} \end{array}$$

Have students write some of their own for exchange with classmates.

Sea Aquarium

On Thanksgiving Monday, the Sea Aquarium ticket office collected \$11 737.00.

The price of 1 ticket was \$2.75

How many people bought tickets?



4268 people bought tickets.

$$\begin{array}{r} 4\,268 \\ \$11\,737.00 - \$2.75 = 275 \overline{)11\,737.00} \\ \underline{1\,100} \\ 73\,700 \\ \underline{55\,000} \\ 18\,700 \\ \underline{16\,500} \\ 2\,200 \\ \underline{2\,200} \\ 0 \end{array}$$

Check by estimating:
 Round — then divide
 $\$11\,737.00 \div \2.75
 \downarrow
 $12\,000 \div 3 = 4000$
 Compare to actual answer
 $\$4268$ is close to $\$4000$.
 It checks!

Exercises

Copy and complete.

- | | | | |
|---|--|--|---|
| 1. (a) $7 \overline{)37\,436}$
$\underline{1242\,R24}$ | (b) $4 \overline{)30\,120}$
$\underline{206\,R13}$ | (c) $9 \overline{)48\,976}$
$\underline{620\,R48}$ | (d) $5 \overline{)26\,804}$
$\underline{4981\,R87}$ |
| 2. (a) $30 \overline{)37\,284}$
$\underline{11\,211\,R22}$ | (b) $80 \overline{)16\,493}$
$\underline{32\,124\,R14}$ | (c) $60 \overline{)37\,248}$
$\underline{7373\,R8}$ | (d) $90 \overline{)448\,377}$
$\underline{709\,R77}$ |
| 3. (a) $36 \overline{)403\,618}$
$\underline{6\,989}$ | (b) $21 \overline{)674\,618}$
$\underline{2\,853}$ | (c) $92 \overline{)678\,324}$
$\underline{0\,6646}$ | (d) $83 \overline{)58\,924}$
$\underline{0\,0176}$ |
| 4. (a) $5 \overline{)34\,945}$
$\underline{0\,125}$ | (b) $4 \overline{)11\,412}$
$\underline{5\,68}$ | (c) $9 \overline{)5\,9814}$
$\underline{2\,871}$ | (d) $2 \overline{)0\,0352}$
$\underline{6\,591}$ |
| 5. (a) $3.7 \overline{)0.4625}$ | (b) $0.83 \overline{)4.7144}$ | (c) $7.8 \overline{)22.3938}$ | (d) $6.2 \overline{)40.8642}$ |

Find the quotient. Check by estimating.

6. (a) $81.1195 \div 3.43$ 23.65 (b) $289.3026 \div 6.78$ 42.67 (c) $11.4276 \div 0.321$ 35.6
 (d) $20.0583 \div 4.37$ 4.59 (e) $48.8130 \div 7.95$ 6.14 (f) $53.9679 \div 6.51$ 8.29
7. On Tuesday, the Sea Aquarium ticket office collected \$6960.25.
 The price of 1 ticket was \$2.75.
 How many people bought tickets? 2531

Community Picnic

4 children tied for first place in a race.
The first-place prize was \$3.
The prize was divided equally among the winners.
How much did each child receive?



Divide the whole number.

$$\begin{array}{r} 0 \\ 4 \overline{)3} \end{array}$$

Place decimal point.
Annex a zero.

$$\begin{array}{r} 0 \\ 4 \overline{)30} \end{array}$$

Divide the tenths.

$$\begin{array}{r} 0.7 \\ 4 \overline{)3.0} \\ \underline{28} \\ 2 \end{array}$$

Annex another zero.

$$\begin{array}{r} 0.7 \\ 4 \overline{)3.00} \\ \underline{28} \\ 2 \end{array}$$

Divide the hundredths.

$$\begin{array}{r} 0.75 \\ 4 \overline{)3.00} \\ \underline{28} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

Each winner received \$0.75 or 75¢.

Exercises

Divide until you have a remainder of 0.

You may need to annex 3 zeros in some questions.

$$\bullet 4 \overline{)1}^{0.25}$$

$$\bullet 8 \overline{)6}^{0.75}$$

$$3. 8 \overline{)2}^{0.25}$$

$$4. 12 \overline{)9}^{0.75}$$

$$5. 12 \overline{)3}^{0.25}$$

$$6. 16 \overline{)5.28}^{0.33}$$

$$7. 14 \overline{)3.5}^{0.25}$$

$$8. 14 \overline{)3.36}^{0.24}$$

$$9. 36 \overline{)12.6}^{0.35}$$

$$10. 96 \overline{)24}^{0.25}$$

$$11. 8 \overline{)1}^{0.125}$$

$$12. 8 \overline{)3}^{0.375}$$

$$13. 16 \overline{)2}^{0.125}$$

$$14. 16 \overline{)6}^{0.375}$$

$$15. 24 \overline{)3}^{0.125}$$

$$16. 32 \overline{)4}^{0.125}$$

$$17. 32 \overline{)12}^{0.375}$$

$$18. 32 \overline{)28}^{0.875}$$

$$19. 96 \overline{)12}^{0.125}$$

$$20. 128 \overline{)80}^{0.625}$$

21. Four runners ran a relay.
Each ran the same distance.
The total distance was 1 km.
How far did each run?

0.25 km



Division: annexing zeros 141

OBJECTIVE

To divide when the divisor is greater than the dividend

PACING

Level A All
Level B All
Level C All

VOCABULARY

annex

MATERIALS

artificial or real money

RELATED AIDS

HMS — DM33.

BFA COMP LAB II — 118.

SUGGESTIONS

Initial Activity Review the names of the place-value locations to the right of the decimal place (see pages 2 and 3). Using two \$2 bills and one \$1 bill ask students to suggest a way to divide the money equally between 2 people. Elicit that each person should be given a \$2 bill and the single dollar is converted into 100 cents. 50 cents is given to each of the 2 people. Use the chalkboard to demonstrate how this is accomplished using numbers:

$$\begin{array}{r} 2 \\ 2 \overline{)5} \\ \underline{4} \\ 1 \end{array} \quad \begin{array}{r} 2.5 \\ 2 \overline{)5.0} \\ \underline{4} \downarrow \\ 10 \end{array} \quad \begin{array}{r} 2.50 \\ 2 \overline{)5.00} \\ \underline{4} \downarrow \\ 10 \\ \underline{10} \downarrow \\ 00 \end{array}$$

Each person receives \$2.50.

Repeat this for \$3 divided between 2 people and then \$2 divided among 4 people.

USING THE BOOK

Go over the display showing the procedure for dividing when a divisor is larger than a dividend. Emphasize that, when a zero is annexed, the value remains the same.

Example

$3 = 3.0 = 3.00 = 3.000$, etc.

Complete Exercises 1 and 2 orally before assigning the balance of the page.

ACTIVITIES

1. Play "Triple Concentration" as described in the Activity Reservoir. Use card sets such as:

14	14.0	14.00
----	------	-------

6	6.0	6.00
---	-----	------

2. See "The P.V. Game" as described in the Activity Reservoir.
3. See the "Coded Riddles" idea in the Activity Reservoir.

OBJECTIVE

To provide a computational skills activity

PACING

Level A All

Level B All

Level C All

MATERIALS

graph paper, Bristol board, pennies

USING THE BOOK

Read through the instructions, making sure that everyone is familiar with the rules of play. You may wish to provide materials so that the players can fabricate their own game boards, though the actual text page 143 can be used if the book is supported to keep the page flat. Also, you may wish (now or later) to change the grid numbers to increase or decrease the level of computational difficulty.

Flip-a-Penny Math

1. Make up the game board on graph paper.
2. Four people can play.
3. The first player declares the math operation for the first round (addition, subtraction, multiplication, or division).
4. The first player flips 2 pennies, one after the other.
5. If the operation is addition, the first player adds his two numbers.
6. The second player does the same, and so on, until all players have had a chance. The player with the highest number wins a point.
7. The winner starts the next game.

NOTE: If the penny touches more than 1 square, move it to the square with the greatest number value.

8. All players should do the operations as a check for each other.
9. Keep a total of all points to find the overall winner.



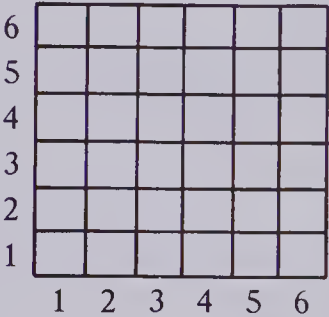
0	96	79	10	53	61
86	99	45	83	27	44
28	7	72	18	60	15
51	62	37	50	1	65
87	29	5	56	33	14
17	70	97	21	66	32

Activity 143

ACTIVITIES

1. To practise column addition, use the game board as shown on page 143 and 3 or more coin flips per player. The winner is the player with the greatest total after 5 turns.

2. Provide 2 number cubes (regular dice) and number the playing board on page 143 so that it has 2 axes:



Players roll both dice twice, thereby identifying 2 numbers on the grid (i.e., roll 4, then 2, → (4, 2) → 56). Players can perform any one of the 4 operations with the 2 numbers scoring a number of points equal to their answer. Player with the greatest point total after a predetermined number of rounds wins.

3. Use the method for choosing numbers from the grid outlined in either the book or in Activities 1 and 2. Have the children calculate the average of the numbers identified.

Example

Flip a penny 5 times to identify:
72, 53, 87, 18, 33.

$$\begin{aligned}
 \text{Average} &= \frac{72 + 53 + 87 + 18 + 33}{5} \\
 &= \frac{263}{5} \\
 &= 52.6
 \end{aligned}$$

Score 53 points (rounded to nearest whole). Players take turns. The player with the greatest (or least or closest to a predesignated number such as 250) total after 5 turns is the winner.

OBJECTIVE

To use the relationship: 1 L of water has a mass of 1 kg and occupies 1000 cm³

PACING

Level A Activity; 1-10

Level B All

Level C Activity; 1-3; 4-16 (even)

MATERIALS

litre containers, scales, construction materials, light plastic sheeting

RELATED AIDS

HMS — DM34.

BFA PROB. SOLVING LAB II — 94.

BACKGROUND

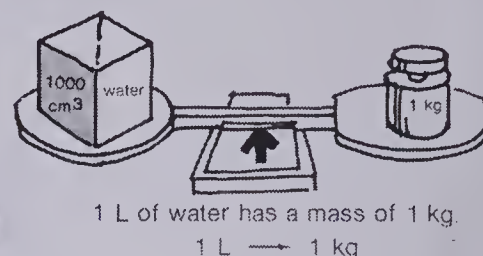
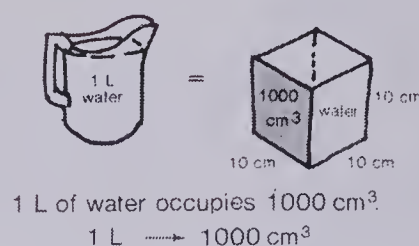
The quantity of liquid a container can hold is commonly referred to as its capacity. The amount of space an object or substance occupies is commonly called its volume. When we want to calculate the quantity of matter and can find or are given the dimensions of the space, we normally calculate the volume first. If the matter is a liquid or the space is to be filled with a liquid, we convert the measure of volume to a measure of capacity. Capacity is measured in litres and volume is measured in cubic units.

Do not formally differentiate between the two terms, capacity and volume, but rather use the two terms as indicated.

SUGGESTIONS

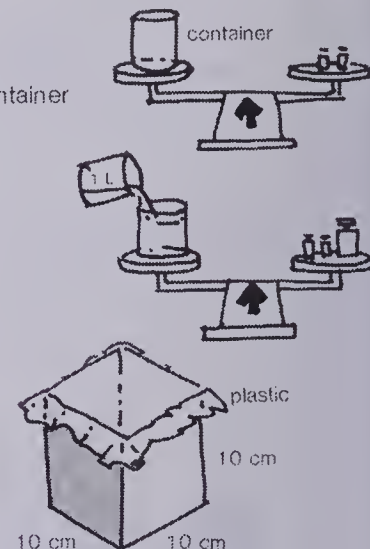
Initial Activity Review briefly litres and cubic centimetres as presented on page 115. The Activity on pupil page 144 has the students performing an experiment to confirm what some students may already know. Review that $0.25 = \frac{1}{4}$, $0.5 = \frac{1}{2}$, and $0.75 = \frac{3}{4}$. Also review the ratios: 1 pie costs \$4.00; $\frac{1}{2}$ pie costs \$____; $\frac{1}{4}$ pie costs \$____; $\frac{3}{4}$ pie costs \$____.

Capacity - Mass - Volume



Activity

- Use a container that will hold a litre of water. Place it on the balance and find the mass of the container.
 - Pour into the container exactly 1 L of water. Balance to find the mass of the container with the water in it.
 - Subtract to find the mass of 1 L of water.
1 L = ■ kg
- Make a container 10 cm × 10 cm × 10 cm.
 - Place plastic inside it to make it waterproof.
 - Pour in exactly 1 L of water.
 - Copy and complete:
1 L = ■ cm³



Exercises

Copy and complete.

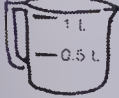
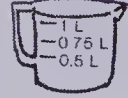
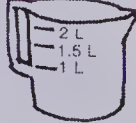
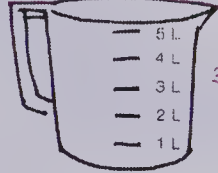
- The mass of 1 L of water is ■ kg.
- 1000 cm³ of water has a mass of ■ kg.
- 1 L of water fills ■ cm³




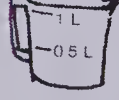

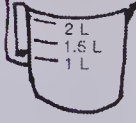
144 Activity: relationship between mass and volume of water

4. What is the mass of each quantity of water?
 (a) 1000 cm^3 (b) 500 cm^3 (c) 2 L (d) 0.5 L (e) 3.5 L
 1 kg 0.5 kg 2 kg 0.5 kg 3.5 kg
5. What is the measure in cubic centimetres of each quantity of water?
 (a) 1 kg (b) 3 kg (c) 0.5 kg (d) 2.5 kg (e) 1000 g
 1000 cm^3 3000 cm^3 500 cm^3 2500 cm^3 1000 cm^3
6. What is the measure in litres of each quantity of water?
 (a) 2 kg (b) 0.5 kg (c) 2000 cm^3 (d) 500 cm^3 (e) 1500 cm^3
 2 L 0.5 L 2 L 0.5 L 1.5 L

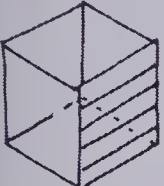
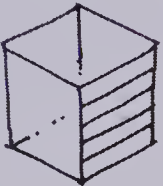
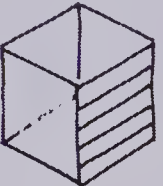
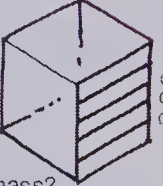
What is the mass of water in each container?

7.  0.5 kg
8.  0.75 kg
9.  1.5 kg
10.  3 kg

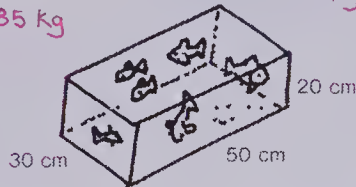
Find the total mass.

11.  Container only 750 g
-  Full of water Total mass? 1.75 kg
12.  Container only 1.2 kg
-  Full of water Total mass? 3.2 kg

What is the total mass of each container filled with water to the level shown?

13.  Container only 600 g
14.  Total mass? 1.1 kg
15.  Total mass? 0.85 kg
16.  Total mass? 1.6 kg

16. Calculate the (a) volume, (b) mass, and (c) number of litres of water in this aquarium.
 $(a) 30\,000 \text{ cm}^3$ $(b) 30 \text{ kg}$ $(c) 30 \text{ L}$



Relationship: mass, volume and capacity 145

USING THE BOOK

Arrange the students in groups so that each group does the Activity. You may arrange two sets of activities: half the group does Activity 1 while the other half does Activity 2. They then exchange stations.

There are basically two conclusions to be drawn:

- 1 L is equivalent to 1000 cm^3 .
- 1 L of water has a mass of 1 kg.

Following a thorough discussion of the Activity and the conclusions, assign the exercises. You may wish to put one example of each type on the chalkboard. Since there are three distinct skills here, some students need direction and assistance to get them started correctly.

ACTIVITIES

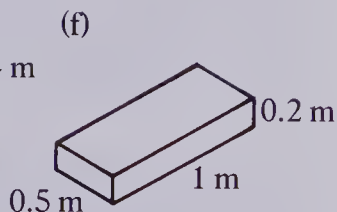
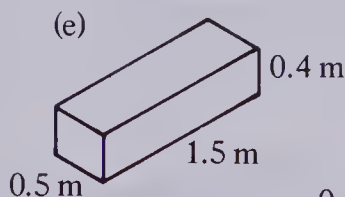
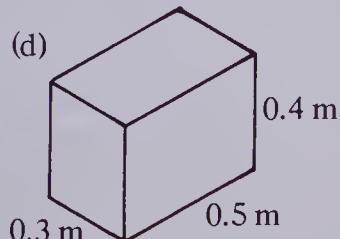
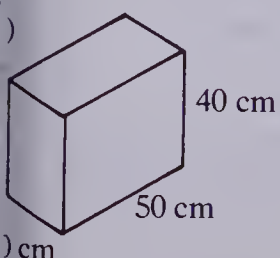
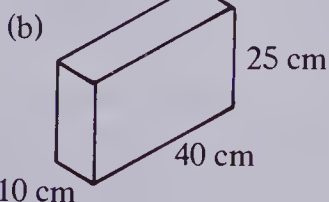
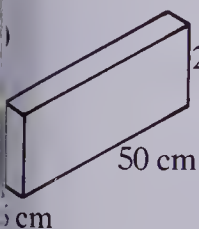
1. Provide the students with several unusual-shaped containers such as shampoo containers. Ask the students to calculate the capacity of each (remove or cover the labelled capacity) by filling the container with water and then pouring the water into a graduated litre container. Then have them check their findings against the labelled capacity or volume. (There may be a discrepancy since the labelled capacity usually does not mean "filled" to the cap.)

2. Have students calculate the capacity of any rectangular aquariums in the school.

3. Have students use different liquids and compare the masses of given quantities (e.g., 0.5 L) of the liquids to equal volume of water. Use oil, milk, and salt water.

EXTRA PRACTICE

What is the volume of each?
 What is the capacity of each?
 What is the mass of water each can hold?



OBJECTIVES

To calculate the number of hours and minutes between two times
To write the new time, given a time and a time-interval

PACING

Level A All
Level B All
Level C All

MATERIALS

a demonstration clock — twenty-four hour clock

RELATED AIDS

BFA PROB. SOLVING LAB II — 94.

BACKGROUND

Using the twenty-four hour clock system has the advantage that one does not need to specify a.m. or p.m.

SUGGESTIONS

Initial Activity Discuss the twenty-four hour clock: 01:00 is one o'clock in the morning, 06:00 is six o'clock in the morning, 12:00 is twelve noon, 18:00 is six o'clock in the evening and 24:00 or 00:00 is midnight. If necessary, conduct an oral drill session on this clock.

USING THE BOOK

Demonstrate the steps in the first part of the display at the top of the page. Then assign Exercises 1 to 4. Correct. Demonstrate the steps in the second part of the display. Assign the balance of the exercises. Any time students show mastery of a type of exercise, they should be encouraged to go on to the next type. They should not be forced to do an excessive number of questions when mastery has been demonstrated.

An alternate method you may wish to introduce with some students involves addition for Exercise 5(h).

22 : 45
+ 3 h 40 min
25 : 85 —→ 26:25 (60 min in 1 h)
—→ 02:25 (2 h 25 min after midnight)

ACTIVITIES

1. Arrange for a pair of students to work with the demonstration clock. One student writes two times on the chalkboard. The other student sets the

Time After Time

How many hours and minutes between



and



?

02:35 to 03:35 → 1 h

03:35 to 04:35 → 1 h

04:35 to 04:55 → 20 min

02:35 to 04:55 is 2 h 20 min

Write the time 3 h and 20 min after



22:10

Time is:

01:30

the next day.

22:10 23:10 24:10 01:10 01:30
1 h 1 h 1 h 20 min

Exercises

1. Write the time shown on the clock 08:25 or 20:25

Can you tell whether it is morning or evening? Why? No



2. Tell whether each time is before noon or after noon.

(a) 04:23 before (b) 16:23 after (c) 11:07 before (d) 20:48 after

3. How many hours from A to B?

3 h 8 h 7 h 11 h

A	04:25	08:20	20:00	15:15
B	07:25	16:20	03:00	02:15

4. How many hours and minutes from A to B?

1 h 15 min 4 h 5 min

A	02:30	11:20	10:35	02:48
B	03:45	12:30	14:40	05:13

1 h 10 min 2 h 25 min

5. Write the time.

(a) 3 h after 08:03 11:03

(c) 7 h after 16:05 23:05

(e) 4 h after 20:25 24:25 or 00:25

(g) 1 h 20 min after 08:10 09:30

(b) 5 h after 14:55 19:55

(d) 2 h after 23:15 01:15

(f) 3 h after 19:00 22:00

(h) 3 h 40 min after 22:45 02:25

6. Mark's family left home by car at 10:10.

They arrived at Aunt Millie's 6 h 20 min later.

What time did they arrive? 16:30

★ 7. Father drove for 8 h 20 min.

He arrived home at 06:30.

When did he leave? 22:10

demonstration clock at the first time, then counts as indicated in the display to the next time. The second student records the number of hours and minutes. The students then reverse roles.

2. Practice for the second objective can be had in a similar way: the first student writes the starting time and the hours and minutes later on the chalkboard. The second student sets the clock at the starting time and the final time.

3. Research the twenty-four hour clock and the use of a.m. and p.m. (an obsolete meaning of meridian is "noon" hence anti meridian means before noon and post meridian means afternoon). Who first used the twenty-four hour clock?

EXTRA PRACTICE

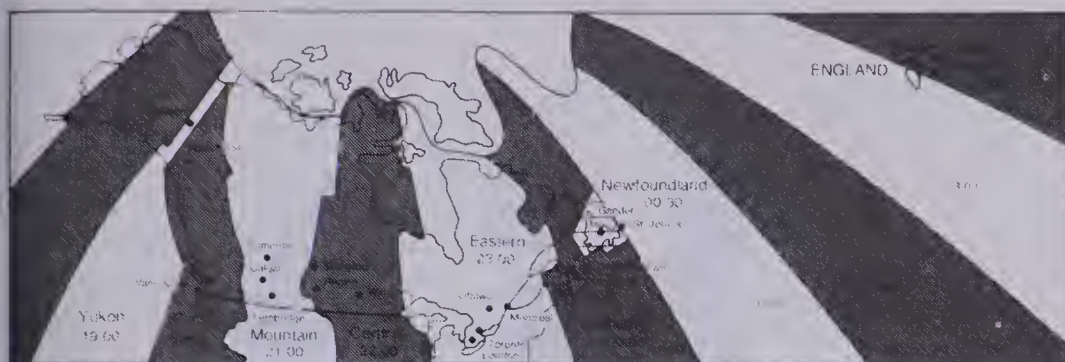
Have the students practise adding time.

1. 3 h 30 min + 1 h 25 min [4 h 55 min]	2. 4 h 25 min + 2 h 45 min [7 h 10 min]
3. 3 h 23 min + 2 h 53 min [6 h 16 min]	4. 4 h 33 min + 1 h 47 min [6 h 20 min]

Calculate the time.

- 3 h before 08:15 [05:15]
- 2 h before 13:15 [11:15]
- 6 h before 04:30 [22:30]
- 6 h before 02:15 [20:15]
- 4 h before 23:19 [19:19]
- 10 h before 09:30 [23:30]
- 6 h 30 min before 04:30 [22:00]
- 8 h 15 min before 22:40 [14:25]
- 12 h 30 min before 10:00 [21:30]

Time Zones



Exercises

Count the time zones to help you do these.

1. Tell how many hours difference in time between the following:

- (a) Edmonton and London, England **7h** (b) Toronto and London, England **5h**
 (c) Dawson and Charlottetown **5h** (d) Whitehorse and St. John's **4.5h**
 (e) Regina and London, England **6h** (f) Vancouver and London, England **8h**

2. A time is given in each city. Write the time in the second city.

- (a) 21:00 in Edmonton; Toronto **23:00** (b) 09:00 in Whitehorse; Gander, Newfoundland **13:30**
 (c) 12:00 in London, England; London, Ontario **07:00** (d) 12:00 in St. John's; Winnipeg **09:30**
 ★ (e) 05:15 in Calgary; Ottawa **07:15** (f) 08:35 in Halifax; London, England **12:35**
 ★ (g) 08:50 in Winnipeg; Vancouver **06:50** ★ (h) 10:20 in Saskatoon; Moncton **12:20**
 ★ (i) 22:40 in Lethbridge; London, England **05:40** (j) 18:35 in Vancouver; London, England **02:35**

- ★ 3. Jill left Vancouver at 08:00 on an Air Canada flight.

Flying time to Toronto is 5 h.

What is the time in Toronto when she arrives? **16:00**

- ★ 4. Martin left Montreal at 09:30 on a Canadian Pacific flight.

Flying time to Winnipeg is 4 h.

What is the time in Winnipeg when Martin arrives? **12:30**



Time zones 147

OBJECTIVE

To use a map of standard time zones of Canada

PACING

Level A 1-2

Level B 1-2

Level C All

VOCABULARY

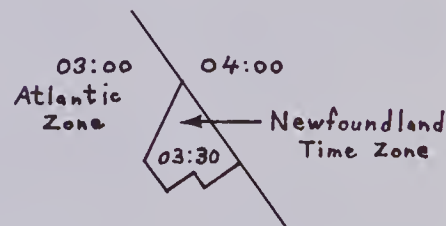
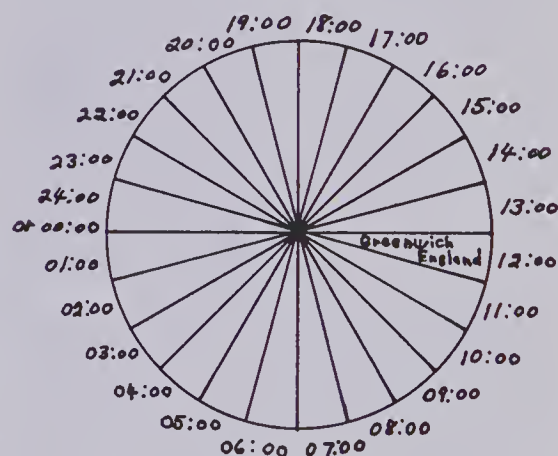
Greenwich (see various city names)

MATERIALS

globe

BACKGROUND

There are 24 time zones around the earth. The base time zone is centred in Greenwich, England.



Each zone as one goes west is an hour earlier than the previous zone. Hence Zone 5, which is the Eastern Time Zone is 5 h earlier than Greenwich Time; Zone 7, Mountain Time Zone, is 7 h earlier than Greenwich. Theoretically, the sun is at its zenith over the middle of a time zone at 12:00 noon. It is to be emphasized that the Newfoundland Time Zone is not an extra half-hour zone but rather just a partial step to the next time zone east of the Atlantic zone.

SUGGESTIONS

Initial Activity Use a globe to show where Greenwich is. Mark on the time zone lines on Canada — using coloured chalk — and mark 4 zones between Greenwich Zone and Atlantic Zone. Some students may need this globe to do some of the exercises.

USING THE BOOK

Discuss with the class how "live"

television coverage of such events as football games, sessions of parliament, etc. highlight the different time zones.

A game in Toronto beginning at 18:00 (Eastern time) will be seen in

Vancouver starting at 15:00 and in

Halifax at 19:00. Then discuss the

display and do Exercise 1, parts (a)

and (d), and Exercise 2, parts (a) and

(b) orally. Assign.

ACTIVITIES

Use four 4's and then any of the

operations +, -, ×, ÷ to make as

many of the numbers from 1 on as

possible.

Example

$$= \frac{4}{4} \times \frac{4}{4}$$

$$= 4 - \frac{4+4}{4}$$

2. Bring some airline schedules to class for discussion. Consider the differences in time of departures and arrivals between Toronto to Vancouver and Vancouver to Toronto. Have students plan trips with departures, arrivals, and time spent flying calculated.

3. Encourage students to read *Around the World in Eighty Days*. Then ask students to explain the surprise ending with reference to the International Date Line.

4. Research the history of Time Zones; Prime Meridians; and Greenwich, England. How long have time zones been in effect; who devised the system; etc.

OBJECTIVE

To use a map of daylight saving time

PACING

Level A All

Level B All

Level C All

VOCABULARY

daylight saving time, standard time

RELATED AIDS

HMS — DM35.

BACKGROUND

In order that the maximum use can be made of the daylight hours, many provincial and federal governments have declared that daylight saving time be implemented. Saskatchewan is the only major area of Canada that does not employ daylight saving time. Some countries are on daylight saving time all year round. In daylight saving time, the clocks are set ahead one hour in the spring and set back to standard time in the fall. Hence in the summer a clock reads 08:00 actually at what would have been 07:00.

Daylight saving time is utilized to take maximum advantage of the daylight hours. By setting the clocks ahead for the summer months, people can enjoy (and use) the extra hour of daylight at the end of the usual work day, yet have daylight in the morning also. In the winter months the return to standard time tends to equalize the normal work day over the daylight portion of the day.

USING THE BOOK

Discuss why daylight saving time is used. Assign the problems.

ACTIVITIES

1. Research: What is the purpose of daylight saving time.
2. Research: When does daylight saving time begin and when does it end.
3. Research: History of Measuring Time.

EXTRA PRACTICE

(These may be used with page 147 and page 148.) Given the time in the first city, what is the time in the second city?

Daylight Saving Time



In order to take maximum advantage of spring and summer sunlight hours, many areas go from standard time to daylight saving time. These areas use this rule:

In the spring: move clocks ahead 1 h. (spring ahead)

In the fall: move clocks back 1 h. (fall back)

The exact dates of the changes vary each year.

Exercises

1. Marcey lives in Sault Ste. Marie. In the spring she decided to set her watch for daylight saving time. Her watch showed 20:15. What should she set her watch to? **21:15**
2. Luke lives in Kingston, Ontario. In the fall he decided to set his watch to go off daylight saving time. His watch showed 07:30. What should he set his watch to? **06:30**
3. One of the provinces does not go on daylight saving time. Which province is it? **Saskatchewan**
4. The change in time is usually made at 02:00. Do you gain or lose an hour of sleep in (a) the spring? **lose** (b) the fall? **gain**
5. Use daylight saving time. When it is 22:10 in Regina, what time is it in: (a) Calgary? (b) Winnipeg? (c) St. John's? (d) Toronto? (e) Whitehorse?

148 Daylight saving time

ANSWERS:

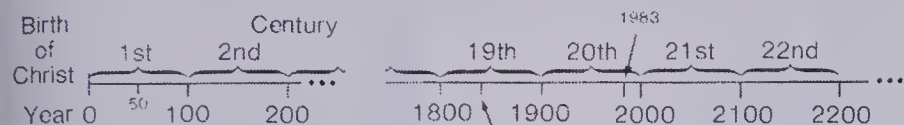
5. (a) 21:10 (b) 22:10 (c) 24:40 or 00:40 (d) 23:10 (e) 20:10

1. 02:00 Prince George, B.C.; Flin Flon, Man.
2. 20:00 Peace River, Alta.; Grand Falls, Nfld.
3. 09:10 Swift Current, Sask.; Georgetown, P.E.I.
4. 13:20 Brandon, Man.; (a) Trois-Rivières, Que. (b) Kamloops, B.C.
5. 18:50 Sault Ste. Marie, Ont.; (a) Pictou, N.S. (b) Milk River, Alta.
6. 22:15 Corner Brook, Nfld.; (a) Chatham, N.B. (b) Minnedosa, Man.
7. 12:00 Biggar, Sask.; Woodstock, Ont.
8. 07:00 Port aux Basques, Nfld.; (a) North Sydney, N.S. (b) Campbell River, B.C.

Centuries

A century is 100 years

We number the centuries as shown.



The year 140 is in the 2nd century.

The year 276 is in the 3rd century.

The year 1840 is in the 19th century.

The year 1983 is in the 20th century.

Exercises

1. Name the century each year is in.

76 (a) 103 (b) 267 (c) 1867 (d) 1985 (e) 2208 (f)

1st 2nd 3rd 19th 20th 21st 23rd

2. Write the interval of years for each century.

(a) 1st → 0 - 99 (b) 2nd → 100 - 199 (c) 3rd → 200 - 299

(d) 19th → 1800 - 1899 (e) 20th → 1900 - 1999 (f) 21st → 2000 - 2099

3. Vertically, on your page, draw a segment 23 cm long.

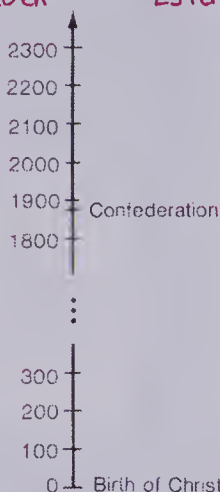
At the bottom mark 0 (for the birth of Christ), and let each centimetre represent 100 years.

Label each 100 years and name the centuries.

Mark and label each event.

- (a) 75 — Heron, Greek Mathematician
(b) 150 — Ptolemy, an astronomer
(c) 410 — Hypatia, first woman of mathematics
(d) 1066 — Norman Conquest
(e) 1492 — Columbus discovered America
(f) 1349 — Black Death
(g) 1431 — Joan of Arc burned at the stake
(h) 1500 — Leonardo da Vinci
(i) 1675 — Greenwich Observatory founded
(j) 1743 — La Verendrye sighted the Rockies
(k) This year

- (l) 1608 — Telescope invented
(m) 1867 — Confederation
(n) 1975 — A. Y. Jackson died
(o) 100 years from this year



Centuries 149

OBJECTIVE

To relate a specific year to its century

PACING

Level A All

Level B All

Level C All

VOCABULARY

century, see Exercise 3 for proper names

RELATED AIDS

BFA PROB. SOLVING LAB II — 38, 147.

BACKGROUND

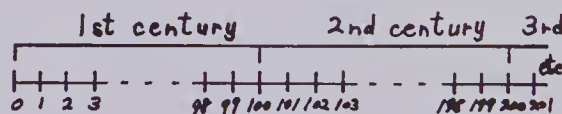
Centuries are numbered from the Birth of Christ. The first century covers those years 0 to 99 inclusive and the second from 100 to 199 inclusive, etc. The dates after the Birth of Christ are labelled A.D. (Anno Domini — meaning in the year of our Lord) and the years before labelled B.C. (before Christ).

SUGGESTIONS

Initial Activity Discuss the meaning of the word century derived from centum which means “one hundred”. Then tie this in with centi- (metric prefix), and such words as cent ($\frac{1}{100}$ of a dollar), centennial, percent, etc.

USING THE BOOK

To emphasize the first century, draw a time line on the chalkboard and number the years as shown and bracket the centuries. Explain that we start at 0.



Then ask which century each year is in: 56; 156; 256; 356; etc.

Do Exercises 1 and 2 orally.

ACTIVITIES

1. Extend the time line to include dates before Christ (B.C.). Some dates are:

1700 — possible beginning of Babylon calendar

1228 — probable date of Egyptian calendar

753 — Rome founded

600 — Thales

540 — Pythagoras

380 — Plato

340 — Aristotle

300 — Euclid

2. Research on the history of calendars. When was our current calendar adopted? What is the “World Calendar” movement?

3. Puzzle: “Rodney, a student of ancient history, returned from a trip to Greece. He brought back a coin marked 386 B.C. All the students were excited until one student, Gale, said ‘It’s a fake.’ How did Gale know it was a fake coin?”

4. Have students use the time-line concept to set up schedules for themselves: homework schedule, weekly schedule, sports schedule, etc.

5. Have students report on and use numeric dating:

1982 04 15 12:30 (year, month, date, time).

OBJECTIVE

To use temperature readings less than 0°C

PACING

Level A 1-15

Level B 1-15

Level C 2-16 (even)

BACKGROUND

Theoretically, a Celsius thermometer is made by enclosing mercury in a tube with a bulb at the lower end. When the temperature goes up the mercury expands and rises in the tube. Similarly, when the temperature drops the mercury contracts and falls in the tube. The scale is graduated by first placing the bulb in ice water and the resulting level of the mercury is marked 0. Then the bulb is submerged in boiling water (at sea level) and the resulting level of the mercury marked 100. The tube between 0 and 100 is marked in 100 equal segments, each being one degree Celsius. This thermometer is named after the Swedish astronomer Anders Celsius (1704-1744) who devised the scale.

USING THE BOOK

Do Exercises 1, 2, and 6 orally. Some students will want to use the thermometer shown in the text to do those exercises involving minus readings. Some students will have no need for the thermometer. The more able students might be encouraged to skip some of the easier questions, proceeding to the harder ones.

ACTIVITIES

1. Use the library to learn about Anders Celsius. When did he live; where did he live; for what is he noted; why did he choose 0° and 100° as two important points on his thermometer scale?

2. Have the students bring in newspapers for a week, showing the high and low temperatures for various places around the world. They can

- graph the data for certain cities
- find the average of the high and low for each city
- calculate the differences between

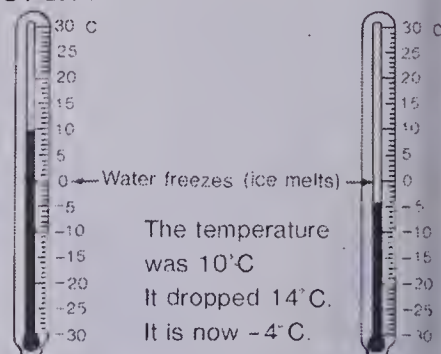
- highs and lows of each city
- highs (or lows) of different cities.

(This is a good geography lesson also!)

Measuring Temperatures

Temperatures colder than 0°C are identified by a **minus** sign in front of the numbers: -10°C , -23°C .

A temperature reading of -4°C means the temperature is colder than 0°C , or colder than the temperature at which water freezes.



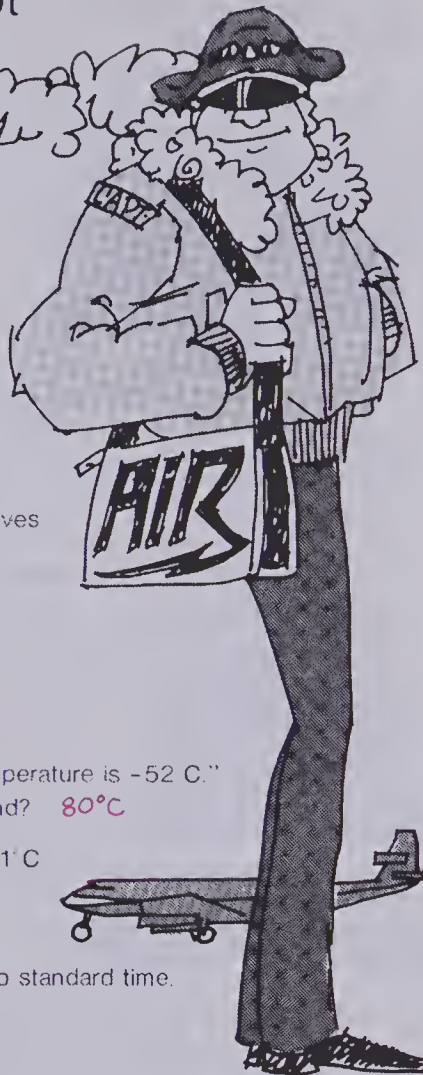
Exercises

1. What does the minus (-) sign in -28°C mean? *28°C below freezing*

Use the thermometer scale. Record the new temperatures.

2. It was 15°C ; temperature dropped 10°C . *5°C*
3. It was 8°C ; temperature dropped 10°C . *-2°C*
4. It was -12°C ; temperature dropped 12°C . *-24°C*
5. It was 4°C ; temperature rose 8°C . *12°C*
6. It was -5°C ; temperature rose 3°C . *-2°C*
7. It was -19°C ; temperature rose 35°C . *16°C*
8. It was 12°C ; temperature dropped 12°C . *0°C*
9. It was 0°C ; temperature dropped 15°C . *-15°C*
10. It was -22°C ; temperature dropped 8°C . *-30°C*
11. It was 0°C ; temperature rose 11°C . *11°C*
12. It was -11°C ; temperature rose 29°C . *18°C*
13. It was -28°C ; temperature rose 19°C . *-9°C*
14. The temperature rose 18°C . It is now 23°C . What was the original temperature? *5°C*
15. The temperature rose 12°C . It is now 7°C . What was it before it rose? *-5°C*
- ★ 16. In Canada, the coldest temperature recorded is -63°C (Yukon). The hottest temperature recorded is 46°C (B.C.). What is the difference between the hottest and coldest temperatures? *109°C*

Airplane Pilot



Use the time-zone maps on pages 147 and 148

1. A pilot is on a flight leaving Toronto for Victoria.
It departs at 10:15 Toronto time.
It arrives at 12:45 Victoria time.
What is the total flight time? **5 h 30 min**
2. An air crew leaves Edmonton at 08:10.
They arrive in Halifax at 18:45.
What is the total time of the trip? **7 h 35 min**
3. During daylight saving time of the year, an air crew leaves Winnipeg at 12:30.
 - (a) The flight to Regina is 1 h long.
What time do they arrive in Regina? **13:30**
 - (b) The flight to Calgary from Winnipeg is 2 h long.
What time do they arrive in Calgary? **13:30**
4. The temperature in Toronto at take-off time is 28°C.
At 11 000 m the captain announces "The outside temperature is -52°C."
How much lower is this temperature than on the ground? **80°C**
5. The temperature inside the airplane is a comfortable 21°C.
Outside, the temperature is 44°C lower.
What is the outside temperature? **-23°C**
6. A flight leaves Toronto for Vancouver at 01:00 Toronto standard time.
The flight takes 5 h 20 min.
The plane lands in Vancouver on daylight saving time.
What time is it when it lands?

Problems: time and temperature 151

OBJECTIVE

To solve problems involving time zones and minus temperatures

PACING

Level A All
Level B All
Level C All

SUGGESTIONS

Initial Activity Discuss the role of pilots. See the Career Awareness notes in the Chapter Overview (page 124).

USING THE BOOK

Assign the exercises to the B and C level students. With Level A students you may have students take turns reading the questions and then discussing them. Ask such questions as: "What do we need to find?" "What do we know that's related to the problem?" "How are we going to calculate the answer?" "Do we have enough information?" "Do we have more information than we need?" "Can you approximate the answer, that is, roughly guess what the answer should be?" "Is our answer reasonable?"

ANSWERS:

If it's spring - 04:20. If it's fall - 02:20.

ACTIVITIES

1. Have students prepare a report on the different kinds of airplanes used in history and that are in current use by commercial airlines and the armed forces.
2. Ask students to report on aspects of being a pilot such as navigating, radio operator, actual piloting of the plane, etc.
3. Provide 2 challenge cards such as:

A flight leaves Toronto for Vancouver at 01:00 standard time.
The flight time is 5 h 20 min.
It lands in Vancouver on daylight saving time.
What time is it when the flight arrives?

Flight 902 takes off from Montreal at 15:30 where the ground air temperature is 22°C. One hour and ten minutes later at an altitude of 12 000 m, the temperature is -72°C. The plane lands in St. John's where the temperature is 18°C.

- (a) What is the temperature difference between Montreal and the air temperature at 12 000 m?
- (b) What is the time in St. John's when the plane took off from Montreal?

OBJECTIVES

To identify extraneous information
in a problem

To identify missing information
in a problem

PACING

Level A All

Level B All

Level C All

RELATED AIDS

BFA PROB. SOLVING LAB II—23,
75.

BACKGROUND

In true problem situations, seldom is the exact information only provided—often there is either more than enough information or there is insufficient information. If students truly understand the problem, they will be able to identify what information is superfluous, if any, or what additional information is needed if there is insufficient information.

USING THE BOOK

Each student should be challenged to do a question. Then the question should be discussed in a group situation with each student making a contribution. The variety of answers that evolve will indicate the diversity of thinking of the group members.

For Exercise 2, one student may want to know the width of the room and calculate on the basis of a wall-to-wall carpet. Another may calculate on the basis of an area rug.

ACTIVITIES

1. Provide suitable additional information for those questions with insufficient information. Then direct the students to solve the problems.

Examples

Exercise 2: The width is 3 m.

Exercise 3: Mt. Logan is 3954 m tall.

Exercise 5: The width of Mr. Snow's field is 70 hm.

2. Have each student select 3 problems from some page earlier in the text. They are to rewrite each leaving out one piece of information from one, adding some additional information to another, and leaving the third as it is. Students then exchange sets of questions and challenge their friends to tell which questions are which.

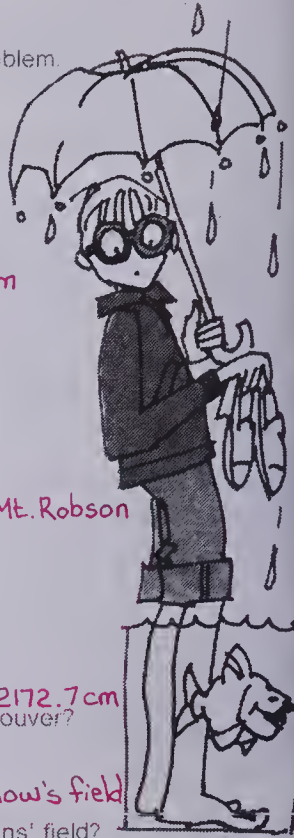
Problems

For each problem, *either*:

- (a) identify what information is *not* required
- or (b) identify what additional information is needed to solve the problem.

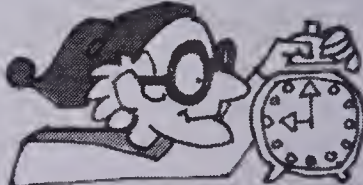
Solve any problem that has sufficient information.

- Rainfall is recorded in millimetres.
Snowfall is recorded in centimetres.
1 cm of snow provides the same moisture as 1 mm of rain.
It rained 14 mm on June 3, 1978. *Not needed*
How many millimetres of rain is the same as 9 cm of snow? *9 mm*
- The length of a room is 4 m.
Mark paid \$16.95 for each square metre of rug.
How much did the rug cost for the room? *Need width*
- The tallest mountain in Canada is Mt. Logan in the Yukon.
It is 6050 m tall.
How much taller is Mt. Logan than Mt. Robson? *Need height of Mt. Robson*
Mt. Robson is in B.C. *Not needed*
- The greatest recorded snowfall in Canada was at Kemano in B.C.
It snowed 2235 cm.
The average snowfall in Vancouver is 62.3 cm.
The average rainfall is 1432 mm. *Not needed*
How much more snow fell in Kemano than is the average in Vancouver? *2172.7 cm*
- The area of Mrs. Burns' field is 730 ha.
The length of Mr. Snow's field is 80 hm. *Need width of Mr. Snow's field*
How many more hectares are in Mr. Snow's field than in Mrs. Burns' field?



BRAINTICKLER

Professor Q was very tired. He went to bed at 9:00 p.m.
"I'll sleep till noon tomorrow!" he said, and he set his alarm.
How many hours did he sleep? (15 h is incorrect!) *3 h*



152 Problems: missing and extraneous information

Alternately, each question can be put on a card and a special Problem Box be established. In which case, the solutions should be written on the back of each card.

Ask students to solve the problems that can be solved and to provide suitable additional information to those problems with insufficient information, then solve.

Have students prepare a display of questions they should ask themselves when solving problems. They might also include things they look for in problems in order to solve it.

3. See Activity 1 on page 22 for an idea to help maintain the important skill—choosing the correct operation.

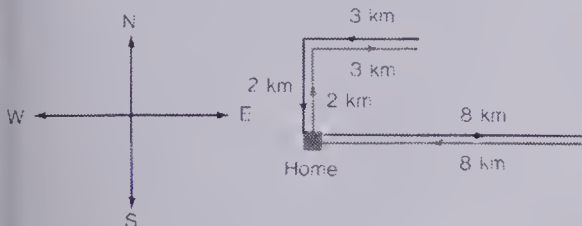
Drawing Pictures to Help Solve Word Problems

Mark rode his bicycle 8 km east.

He returned home.

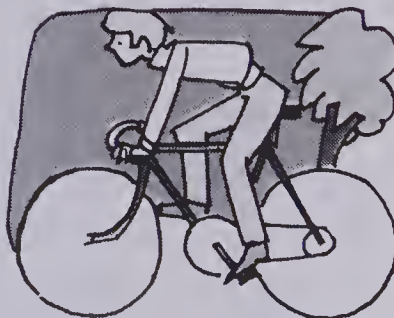
Then he rode 2 km north, then 3 km east, and he returned the same way

How far did he ride?



He rode: $8 \text{ km} + 8 \text{ km} + 2 \text{ km} + 3 \text{ km} + 3 \text{ km} + 2 \text{ km}$.

He rode 26 km.



Exercises

Draw a picture for each problem. Solve.

- Mark rode his bicycle 2.3 km west, then 4.5 km south. He returned the same way. Then he rode 3.4 km north, 2.3 km east, and back the same way. How far did he ride? **25 km**
- A garden is rectangular. It is $100 \text{ m} \times 500 \text{ m}$. A square is marked out of one corner for a lawn. This square is $50 \text{ m} \times 50 \text{ m}$. How much fencing is needed to fence the garden? (No fence is required where the lawn is.) **1100 m**
- A spacecraft blasts off for Mars. It circles Mars 3 times, then returns to Earth. The distance to Mars is 209 215 000 km. One orbit of Mars is 14 000 km. How far did the spacecraft travel? **418 472 000 km**
- A taxi costs \$0.80 for any trip up to 1 km, then \$0.50 for each kilometre after. How much does a trip of 4 km cost? **\$2.30**



Problems drawing diagrams 153

OBJECTIVE

To draw pictures to help solve problems

PACING

Level A All
Level B All
Level C All

RELATED AIDS

HMS — DM36.

BACKGROUND

Translating a word problem into a diagram often provides the students with a better grasp of what is wanted and how to solve it. However, drawing the diagram to represent the word problem is no small task — unless the student has had practice in doing it. This exercise is presented here and the teacher should have students make diagrams for problems wherever suitable hereafter.

USING THE BOOK

Direct the students to put their hands over the diagram in the display. Then read the question. Then direct the students to look at the diagram and read the question again. Ask: "Do you understand the problem better with the diagram?" (Answers will generally be "Yes".) Solve the question in the display. Then assign Exercise 1 providing individual assistance where needed. Each question requires a different type of diagram, therefore, individual assistance will likely be required.

ACTIVITIES

1. Make up 10 problems consisting of 5 different types (2 of each type) but using different numbers. Put these on numbered cards. Draw, on other cards, labelled diagrams to go with each problem. Label the cards A, B, C, etc. Challenge the students to match the problems and diagrams.

2. Have the students make up or select a problem from previous pages. Then have the students select a problem and draw a picture to illustrate the problem. Make sets of 5 cards and matching pictures. Students are to match the problems and pictures. Alternately, the students can be given the diagrams and asked to reconstruct the problem.

3. Have the students solve the problems in Activities 1 and 2.

EXTRA PRACTICE

Solve the problems. Draw a picture to

help solve each.

- Millie Wing had 5 m of carpet. She used 2.8 m on the steps and 1.5 m in the hallway. How much carpet did she have left? [0.7 m]
- Mr. Henry made 5 pies. He kept 2.5 pies for the family picnic. He and his wife kept 0.5 pie. He gave the rest to a neighbour. How much did he give the neighbour? [2 pies]
- A garden is rectangular and is 30 m long and 24 m wide. The owner fences the two short sides and one long side. How much fencing does he need? [78 m]
- A snail climbs a flower stalk. It climbs 4 cm each night and slides back 2 cm each day. How high is it after 4 nights? [10 cm]

OBJECTIVE

To evaluate achievement of the chapter objectives

PACING

- Level A All
- Level B All
- Level C All

RELATED AIDS

HMS — DM37.

USING THE BOOK

Each student should do this test independently under supervision. Assistance should be given only when the instructions are not understood. After the work has been corrected, you should provide appropriate remedial work. You may wish to reteach if a large number of students had difficulty with a particular topic or concept.

The following chart will help in this regard. The specific objectives are listed in the Chapter Overview (see page 124).

An alternate Chapter Test can be found in the Holt Mathematics System Duplicating Masters available for use with this grade level.

Test Item	Objective	Text Page Number
1(a), 1(b), 3	A	127, 128, 133
1(c), 1(d)	B	129
2, 4, 5,	C	130-132, 134
6(c), 6(d)		
6(a), 6(b)	D	141
9, 10	E	144
11, 12, 15	F	146-149
13, 14	G	150
7, 8	H	136-139

Chapter Test

- Divide.
- 14790

0.1

1479
- 7324

0.1

7324
- 6530

0.8

5224
- 29.4

0.6

1764
- 80

3.4

272
- 60

6.8

408
- 370

4.7

1739
- 640

9.6

6144
- 17

17

0.01
- 16.347

16.347

0.01
- 7658

7658

0.001
- 0.583

0.583

0.001
- 0.27

0.27

918
- 0.61

0.61

28.67
- 3.8

3.8

19.76
- 6.4

6.4

54.464
- 15.3552

15.3552

3.36
- 286.728

286.728

5.2
- 10.1232

10.1232

1.48
- 4

4

3
- 50

50

45
- 0.681

0.681

50.394
- 26.9

26.9

63.484
- Write an equation for the following, and then solve.

There are 100 file folders in 1 box.

How many boxes are needed for 35 000 file folders?

35000

100

=

350
- Estimate by rounding, and then solve.

The cost per student for a class trip was \$4.88.

The total amount collected was \$180.56.

How many students were in the class?

36

37
- The mass of 1 L of water is $\frac{1}{1000}$ kg.
- The volume of 1 L of water is $\frac{1}{1000}$ cm³.
- How much time from 08:35 to 14:50?

6 h

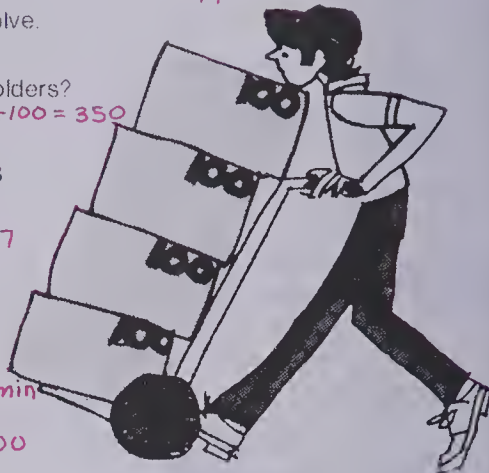
15 min
- What is the time 3 h 10 min after 12:50?

16:00
- The temperature was -3°C. It is now 17°C. How many degrees warmer is it now?

20°C
- The temperature was -23°C. It rose 32°C. What is the temperature now?

9°C
- In which century is the year 1983?

20th



Cumulative Review

1. Add.

$$\begin{array}{r} 9734 \\ 768 \\ 2973 \\ + 501 \\ \hline 13976 \end{array} \quad \begin{array}{r} 9476 \\ 3501 \\ 224 \\ + 7198 \\ \hline 20399 \end{array} \quad \begin{array}{r} \$347.95 \\ 24.06 \\ 685.19 \\ + 44.74 \\ \hline \$1101.94 \end{array} \quad \begin{array}{r} 0.625 \\ 0.761 \\ 0.448 \\ + 0.006 \\ \hline 1.840 \end{array}$$

2. Subtract

$$\begin{array}{r} 85465 \\ - 21627 \\ \hline 63838 \end{array} \quad \begin{array}{r} \$5965.42 \\ - 610.76 \\ \hline \$5354.66 \end{array} \quad \begin{array}{r} 80000 \\ - 4276 \\ \hline 75724 \end{array} \quad \begin{array}{r} 0.488 \\ - 0.109 \\ \hline 0.379 \end{array}$$

3. Multiply.

$$\begin{array}{r} 3.9 \times 0.7 \\ \hline 2.73 \end{array} \quad \begin{array}{r} 438 \times 765 \\ \hline 335070 \end{array} \quad \begin{array}{r} 458 \times \$8.61 \\ \hline \$39.43 \end{array} \quad \begin{array}{r} 90 \times 4732 \\ \hline 425880 \end{array}$$

4. Divide

$$\begin{array}{r} 8.41 \\ 0.1 \overline{)0.841} \end{array} \quad \begin{array}{r} 370 \\ 47 \overline{)1739} \end{array} \quad \begin{array}{r} 264 \\ 0.17 \overline{)44.88} \end{array} \quad \begin{array}{r} 7.920 \\ 5.1 \overline{)40.392} \end{array}$$

5. Round according to instructions.

$$\begin{array}{l} 9128 \text{ (nearest ten)} \\ 9130 \end{array} \quad \begin{array}{l} 68543 \text{ (nearest thousand)} \\ 69000 \end{array} \quad \begin{array}{l} 751245 \text{ (nearest hundred)} \\ 751200 \end{array}$$

6. Write in words.

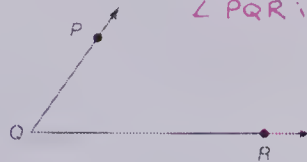
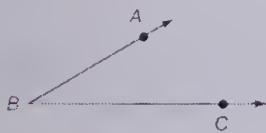
$$6427 \quad 468452 \quad 3649067$$

7. Copy and complete.

$$68 \times 42 = n \times 68 \quad 7 \times n = 70 \quad 1000 \times 63 = n$$

8. Use your protractor. Are $\angle ABC$ and $\angle PQR$ congruent? Explain

No $\angle ABC$ is 30° and $\angle PQR$ is 55° .

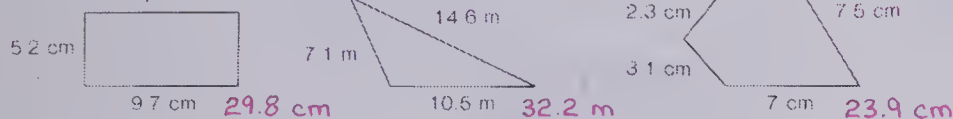


9. Draw a straight angle and label it $\angle RST$.



10. Draw an obtuse angle and label it $\angle PQR$. Measure $\angle PQR$.

11. Calculate the perimeter.



Chapters 1-5 cumulative review 155

OBJECTIVE

To review and test selected concepts and skills previously covered

PACING

Level A All
Level B All
Level C All

USING THE BOOK

This page may be used for diagnostic and remedial as well as review purposes. Students should check their work, correct any errors, and review the pages that contain any problems of the type they missed. Some students can do this on their own while others may need help. If a large number of students have a particular problem incorrect, you may want to reteach that topic to the groups, then assign a duplicated worksheet to reinforce that topic or, refer to an appropriate skill card in the BFA Computational Skills Kit II or BFA Problem Solving Kit II.

Test Item	Text Page Number
1	4, 6, 7
2	5, 8, 9
3	66, 67, 75, 77
4	130, 132
5	12
6	10
7	62, 86
8-10	36, 37
11	105

ANSWERS:

Six thousand, four hundred twenty-seven; four hundred sixty-eight thousand, four hundred fifty-two; three million, six hundred forty-nine thousand, sixty-seven

CHAPTER 6 OVERVIEW

This chapter involves number sense and number theory. Prime and composite numbers are studied; factor sets, factor trees, and divisibility rules are determined.

Factors, greatest common factors, multiples, and least common multiples provide a basis in whole numbers which may be used in fraction work.

Powers, exponents, squares and cubes, and expanded notation are alternate equivalent numerical formats for whole numbers.

OBJECTIVES

- A To identify factors of whole numbers, divisibility, concepts and rules for divisibility, and the use of *zero* and *one* in both multiplication and division
- B To investigate and determine which whole numbers are prime and which are composite
- C To determine prime factors of whole numbers and write factor trees and product of prime factors (i.e., Prime Factorization)
- D To write the greatest common factor (GCF) for a given pair of whole numbers
- E To list sets of multiples and determine the least common multiple (LCM) for a given pair of whole numbers
- F To express whole numbers as powers and in expanded notation using powers of ten
- G To calculate squares and cubes of whole numbers, products of powers; to solve problems

BACKGROUND

1. Divisibility rules save a student much time doing mechanical divisions and rule out some frivolous trial divisions when determining factor sets.

Example

For finding the set of all factors of 45:

There is no use dividing by 2 since the number is odd.

45 should divide by 3 since the sum of the digits is divisible by 3. It should also divide by 5 since the unit's digit is 5.

$$1 \times 45 = 45$$

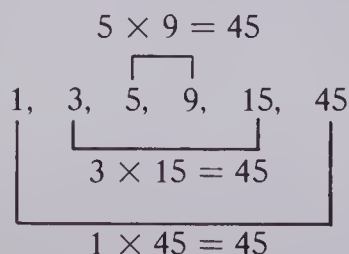
$$3 \times 15 = 45$$

$$5 \times 9 = 45$$

The set of factors of 45 is 1, 3, 5, 9, 15, 45.

2. Factor sets can be checked by multiplying extreme pairs.

Example



If the factors are written in numerical order, these products should all equal 45.

3. The number one is neither prime nor composite because 1 is its only factor.

4. Composite numbers with an odd number of factors are perfect squares.

Example

The factors of 25 are 1, 5, and 25.

25 is a perfect square.

5. Step division is a simple algorithm for determining the prime factorization of a whole number.

Example

$$\begin{array}{r} 7 \\ 2 \overline{)14} \\ 2 \overline{)28} \\ 2 \overline{)56} \end{array} \quad \begin{array}{l} 56 = 2 \times 2 \times 2 \times 7 \\ \text{or } 2^3 \times 7 \end{array}$$

6. By convention, prime factors in prime factorization are arranged in ascending numerical order.

Example

$$56 = 2 \times 2 \times 2 \times 7 \leftarrow \text{Yes}$$

$$56 = 2 \times 2 \times 7 \times 2 \leftarrow \text{No}$$

MATERIALS

2 cm or 1 cm cubes

graph paper

blank cubes for making dice

4-function calculator

bingo chips or round plastic chips

standard playing cards

place-value chart

CAREER AWARENESS

Lab Technician [175]

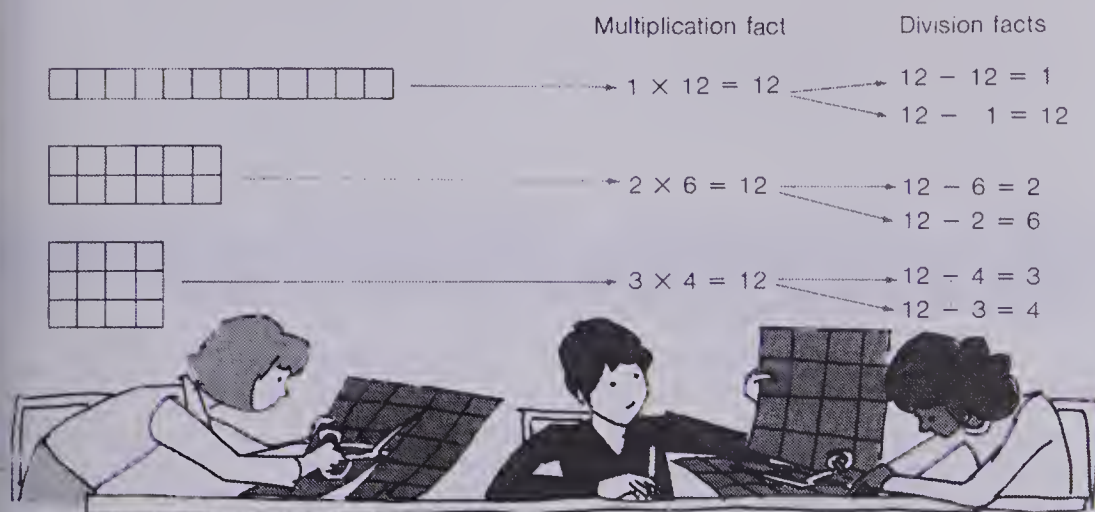
Many large companies (such as Bell, IBM, forestry, oil and pharmaceutical companies) provide laboratories and hire chemists, biologists, physicists, and other scientists and engineers to help in the testing and development of the company's products. A laboratory technician works in tandem with them doing tests or performing experiments. The results of these tests or experiments would be shared with other departments within the company to help them make decisions regarding product change and new developments.

The laboratory provides information regarding future directions, checks on present functions, and records the discoveries of the past.

The lab technician is a person who is interested in science, has completed secondary school, and has pursued further education at a community college or a university.

Making Rectangles

These rectangles can be made by using 12 squares



Exercises

- Draw all the possible rectangles using 20 squares. Write the multiplication fact for each rectangle.
 - Write 2 related division facts for each multiplication statement.
- Write all the possible multiplication facts for rectangles made from these sets of squares.

(a) 24 squares	(b) 30 squares	(c) 32 squares
(d) 36 squares	(e) 50 squares	(f) 56 squares
(g) 72 squares	(h) 96 squares	(i) 100 squares
- Write two related division facts for each multiplication fact in Exercise 2.

ANSWERS: continued on page 158.

ACTIVITIES

- To provide a quick review of multiplication facts, have the children complete a blank multiplication table.

×	1	2	3	4	5	6	7	8	9	10	11	12
1												
2												
3												
4												
5												
6												
7												
8												
9												
10												
11												
12												

- Use a deck of cards which show multiplication facts and a game board (use a cribbage board or one as shown for "Road Rally" in the Activity Reservoir). Players (from 2 to 4) take turns: (a) taking a multiplication fact card from the shuffled deck; (b) turning the card over and calling out the first related

division fact that comes to mind; (c) moving a counter on the game board a number of spaces equal to the quotient of the stated division fact.

Example

Draw $3 \times 5 = 15$

Call either (a) $15 \div 5 = 3$, move ahead 3 spaces or (b) $15 \div 3 = 5$, move ahead 5 spaces. First player to reach the end of the game board wins.

Variations:

- You may wish to confine the related division fact to the one which uses the multiplication fact product as the dividend and the second factor as the divisor (i.e., $6 \times 3 = 18 \rightarrow 18 \div 3 = 6$ NOT $18 \div 6 = 3$).
- Instead of a deck of cards showing multiplication facts, use a multiplication table (see Activity 1) and a penny or bottle cap. Play shuffleboard style.
- To make the game more of a mental process, have the multiplication facts (which have been generated by either cards or

OBJECTIVE

To review multiplication and division patterns

PACING

Level A All
Level B All
Level C All

MATERIALS

2 cm cubes, centimetre cubes or 2 cm \times 2 cm cardboard squares, graph paper, overhead projector

BACKGROUND

Using many rectangular regions which have equal areas, multiplication and division can be shown.

SUGGESTIONS

Initial Activity Using the overhead projector and 3 or 4 sets of 12 cubes or squares, show the rectangular arrays which can be formed. Discuss the length and width of these rectangles and determine the products which result. By questioning, have students determine 2 related division statements for each.

USING THE BOOK

Have students refer to the display on page 157 before beginning the exercises.

Provide graph paper or grid paper so that the students can construct rectangles having an area of 20 square units.

Encourage the students to organize their multiplication statements and pairs of related division statements as shown in the display.

table) called out to the players.

3. See "Itza Fact!" in the Activity Reservoir.

OBJECTIVE

To use rectangular arrays to explore divisibility

PACING

Level A 1, 2(a)-(e)
Level B All
Level C All

MATERIALS

2 cm cubes, centimetre cubes or
2 cm \times 2 cm cardboard squares,
graph paper, overhead projector

SUGGESTIONS

Initial Activity Using the overhead projector show the rectangles which can be made using 15 squares.



Record the related division statements.

$$15 \div 1 = 15 \quad 15 \div 3 = 5$$

$$15 \div 5 = 3 \quad 15 \div 15 = 1$$

Therefore 15 is divisible (without remainder) by 1, 3, 5, and 15.

USING THE BOOK

The above can be shown using 2 related divisions for each rectangular array, as shown in the display.

Encourage the students to draw the rectangles on graph paper for all solutions. It may take longer but it enhances their "number sense".

ACTIVITIES

1. Continue to review multiplication facts using drill, flash cards, or multiplication matrices, etc.

2. If you have not already done so, see the closely related Activities listed on page 157.

3. Play "Concentration" as described in the Activity Reservoir. Use cards such as:

3×7 $21 \div 7$ 5×8 $40 \div 8$ etc.

Rectangles and Divisibility

These rectangles can be made by using 15 squares.



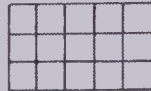
Multiplication fact

$$1 \times 15 = 15$$

Division facts

$$15 \div 1 = 15$$

$$15 \div 15 = 1$$



$$3 \times 5 = 15$$

$$15 \div 5 = 3$$

$$15 \div 3 = 5$$

15 is divisible by 1, 3, 5, and 15.

Exercises

- (a) Draw the possible rectangles by using 8 squares
(b) Write multiplication and division facts for each.
(c) Write a divisibility statement



$$1 \times 8 = 8$$

$$8 \div 1 = 8$$

$$8 \div 8 = 1$$



$$2 \times 4 = 8$$

$$8 \div 4 = 2$$

$$8 \div 2 = 4$$

8 is divisible by 1, 2, 4, and 8

- Repeat Parts (a), (b), and (c) in Exercise 1 for:

- | | |
|----------------|-----------------|
| (a) 10 squares | (b) 18 squares |
| (c) 25 squares | (d) 48 squares |
| (e) 64 squares | (f) 84 squares |
| (g) 96 squares | (h) 120 squares |

Note: A square is a special rectangle with all 4 sides equal

158 Divisibility

ANSWERS: Continued from page 157.

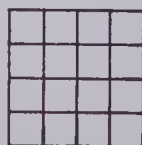
1. (b) $1 \times 20 = 20$ $2 \times 10 = 20$ $4 \times 5 = 20$ (c) $20 \div 20 = 1$ $20 \div 10 = 2$ $20 \div 4 = 5$ $20 \div 1 = 20$ $20 \div 2 = 10$ $20 \div 5 = 4$
2. (a) $1 \times 24 = 24$; $2 \times 12 = 24$; $3 \times 8 = 24$; $4 \times 6 = 24$ (b) $1 \times 30 = 30$; $2 \times 15 = 30$; $3 \times 10 = 30$; $5 \times 6 = 30$
(c) $1 \times 32 = 32$; $2 \times 16 = 32$; $4 \times 8 = 32$ (d) $1 \times 36 = 36$; $2 \times 18 = 36$; $3 \times 12 = 36$; $4 \times 9 = 36$; $6 \times 6 = 36$
(e) $1 \times 50 = 50$; $2 \times 25 = 50$; $5 \times 10 = 50$ (f) $1 \times 56 = 56$; $2 \times 28 = 56$; $4 \times 14 = 56$; $7 \times 8 = 56$ (g) $1 \times 72 = 72$; $2 \times 36 = 72$; $3 \times 24 = 72$; $4 \times 18 = 72$; $6 \times 12 = 72$; $8 \times 9 = 72$ (h) $1 \times 96 = 96$; $2 \times 48 = 96$; $3 \times 32 = 96$; $4 \times 24 = 96$; $6 \times 16 = 96$; $8 \times 12 = 96$ (i) $1 \times 100 = 100$; $2 \times 50 = 100$; $4 \times 25 = 100$; $5 \times 20 = 100$; $10 \times 10 = 100$
3. (a) $24 \div 24 = 1$, $24 \div 1 = 24$; $24 \div 12 = 2$, $24 \div 2 = 12$; $24 \div 8 = 3$, $24 \div 3 = 8$; $24 \div 6 = 4$, $24 \div 4 = 6$
(b) $30 \div 30 = 1$, $30 \div 1 = 30$; $30 \div 15 = 2$, $30 \div 2 = 15$; $30 \div 10 = 3$, $30 \div 3 = 10$; $30 \div 6 = 5$, $30 \div 5 = 6$
(c) $32 \div 32 = 1$, $32 \div 1 = 32$; $32 \div 16 = 2$, $32 \div 2 = 16$; $32 \div 8 = 4$, $32 \div 4 = 8$ (d) $36 \div 36 = 1$, $36 \div 1 = 36$; $36 \div 18 = 2$, $36 \div 2 = 18$; $36 \div 12 = 3$, $36 \div 3 = 12$; $36 \div 9 = 4$, $36 \div 4 = 9$; $36 \div 6 = 6$ (e) $50 \div 50 = 1$, $50 \div 1 = 50$; $50 \div 25 = 2$, $50 \div 2 = 25$; $50 \div 10 = 5$, $50 \div 5 = 10$ (f) $56 \div 56 = 1$, $56 \div 1 = 56$; $56 \div 28 = 2$, $56 \div 2 = 28$; $56 \div 14 = 4$, $56 \div 4 = 14$; $56 \div 8 = 7$, $56 \div 7 = 8$ (g) $72 \div 72 = 1$, $72 \div 1 = 72$; $72 \div 36 = 2$, $72 \div 2 = 36$; $72 \div 24 = 3$, $72 \div 3 = 24$; $72 \div 18 = 4$, $72 \div 4 = 18$; $72 \div 12 = 6$, $72 \div 6 = 12$; $72 \div 9 = 8$, $72 \div 8 = 9$ (h) $96 \div 96 = 1$, $96 \div 1 = 96$; $96 \div 48 = 2$, $96 \div 2 = 48$; $96 \div 32 = 3$, $96 \div 3 = 32$; $96 \div 24 = 4$, $96 \div 4 = 24$; $96 \div 16 = 6$, $96 \div 6 = 16$; $96 \div 12 = 8$, $96 \div 8 = 12$; (i) $100 \div 100 = 1$, $100 \div 1 = 100$; $100 \div 50 = 2$, $100 \div 2 = 50$; $100 \div 25 = 4$, $100 \div 4 = 25$; $100 \div 20 = 5$, $100 \div 5 = 20$; $100 \div 10 = 10$

ANSWERS:

2. (a) Divisible by 1, 2, 5, 10. (b) Divisible by 1, 2, 3, 6, 9, 18 (c) Divisible by 1, 5, 25.
(d) Divisible by 1, 2, 3, 4, 6, 8, 12, 16, 24, 48. (e) Divisible by 1, 2, 4, 8, 16, 32, 64 (f) Divisible by 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 84. (g) Divisible by 1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 96.
(h) Divisible by 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, 120.

4. Divide a 4×4 square into 2 congruent parts, in as many ways as you can.

Example



EXTRA PRACTICE

- Which of the solutions in Exercise 2 in the text, page 158, have a "square" rectangle as one of the solutions?
- List 2 other sets of squares which would have a "square" rectangle as one of the solutions.

Divisibility

24 is divisible by 2, 3, 4, and 6 but not by 5

$$\begin{array}{r} 12 \\ 2 \overline{)24} \\ \underline{2} \\ 04 \\ \underline{4} \\ 0 \end{array}$$

$$\begin{array}{r} 8 \\ 3 \overline{)24} \\ \underline{24} \\ 0 \end{array}$$

$$\begin{array}{r} 6 \\ 4 \overline{)24} \\ \underline{24} \\ 0 \end{array}$$

$$\begin{array}{r} 4 \\ 6 \overline{)24} \\ \underline{24} \\ 0 \end{array}$$

$$\begin{array}{r} 4 \\ 5 \overline{)24} \\ \underline{20} \\ 4 \end{array}$$

24 is not divisible by 5

A whole number is divisible by a given whole number if it divides without remainder (the remainder is zero).

Exercises

1. Which of these numbers are divisible by 2?

14 17 20 30 35 62 77

2. How can you recognize numbers that are divisible by 2?

They end in an even number.

3. Which of these numbers are divisible by 5?

10 27 55 70 96 110

4. How can you recognize numbers that are divisible by 5?

They end in 5 or 0.

5. Which of these numbers are divisible by 10?

20 75 140 207 330 950

6. How can you recognize numbers that are divisible by 10?

They end in 0.

7. Which of these numbers are divisible by 8?

24 54 78 100 136 232

8. Which of these numbers are divisible by 9?

63 82 135 171 216 321

- ★ 9. A whole number that is divisible by 9 has a digit sum of 9 or 18 or 27 or

$72 \rightarrow 7 + 2 = 9$ 72 is divisible by 9.

$585 \rightarrow 5 + 8 + 5 = 18$ 585 is divisible by 9.

Write 6 three-digit numbers that are divisible by 9. Check by dividing. *Many possible answers.*



OBJECTIVE

To introduce divisibility rules

PACING

Level A 1-8

Level B All

Level C All

MATERIALS

squares or centimetre cubes

RELATED AIDS

HMS — DM38.

BACKGROUND

The purpose of this exercise is to encourage the students to state their own divisibility rules.

SUGGESTIONS

Initial Activity Show how 15 is divided by 2, 3, 4, 5, and 6.

$$\begin{array}{r} 7 \\ 2 \overline{)15} \\ \underline{14} \\ 1 \end{array} \quad \begin{array}{r} 5 \\ 3 \overline{)15} \\ \underline{15} \\ 0 \end{array} \quad \begin{array}{r} 3 \\ 4 \overline{)15} \\ \underline{12} \\ 3 \end{array} \quad \begin{array}{r} 3 \\ 5 \overline{)15} \\ \underline{15} \\ 0 \end{array} \quad \begin{array}{r} 2 \\ 6 \overline{)15} \\ \underline{12} \\ 3 \end{array}$$

State the definition: "A number is divisible by a given whole number if it divides without remainder (i.e., the remainder is zero)."

Ask: "What is 15 divisible by?"

Elicit an answer such as: "3 and 5 from the above because the remainder is zero each time. 15 is *not* divisible by 2, 4, or 6 because there is a remainder."

USING THE BOOK

Show that 24 is divisible by 2, 3, 4, and 6. Use the overhead projector to show the rectangles which represent each.



$$24 \div 2 = 12$$

$$24 \div 3 = 8$$



$$24 \div 4 = 6$$



$$24 \div 6 = 4$$

(Continued on page 161)

ACTIVITIES

1. "Divisibility Sorting"

(a) Work in pairs.

(b) Write the numerals 2 through 60 on cards (playing-card size).

(c) Sort the cards under the following headings: — divisible by 2

- divisible by 3
- divisible by 4
- divisible by 5
- divisible by 6
- divisible by 7
- divisible by 8
- divisible by 9
- divisible by 10.

What did you notice as you were using this activity?

Share these observations with other students.

2. Play "Triple Concentration" (rules for "Concentration" are described in the Activity Reservoir)

using 3 card sets such as:

$$5 \times 4$$

$$20 \div 4$$

$$20 \div 5$$

$$8 \times 7$$

$$56 \div 7$$

$$56 \div 8$$

3. See "Square It" as described in the Activity Reservoir. When a number has been squared, have the players identify a number by which the squared number is divisible. Stipulate that they must do this correctly in order to score points.

EXTRA PRACTICE

For Exercises 1, 3, 5, 7, and 8 in the text, list 5 numbers more which are divisible by the given number.

OBJECTIVE
To use divisibility rules

PACING
Level A 1-40
Level B 1-50
Level C 1-55

MATERIALS
simple 4-function calculator, overhead projector, 2 dice

RELATED AIDS
CALC. ACTIVITY MASTERS — 28, 58, 59.

SUGGESTIONS
Initial Activity Write the divisibility rules on acetate for use on the overhead projector. Have students refer to the rules on page 160.
Briefly explain each rule using a couple of examples for each in the following order:
—divisible by: [10, 5] [2, 4, 8] [3, 6, 9]

Note: The chart is in numerical order for easier student reference only.

USING THE BOOK

While the students are working on the exercises allow them to refer to the divisibility rules.

Allow students to use the calculator on occasion to prove to themselves whether or not the very large numbers are in fact divisible based on the given rules.

Encourage students to remember these rules.

Note that there is no divisibility rule for seven provided in this text.

You may wish to remind the children that the answers to some exercises (see Exercises 1-3, 11-13, 21-23, 31-33, 41, 42, 51) can be checked in the back of the text.

Rules for Divisibility

360 is divisible by 2, 3, 4, 5, 6, 8, 9, and 10.

Table with 7 columns showing division of 360 by 2, 3, 4, 5, 6, 8, and 9. Each column shows the quotient and remainder.

Rules for Divisibility

Table with 3 columns: A Counting Number is Divisible, If, Check. Rows include rules for 2, 3, 4, 5, 6, 8, 9, and 10.

Exercises

Use the divisibility rules.

Tell which are divisible by 3, and which by 4.

1. 39 **3** 2. 48 **3 & 4** 3. 54 **3** 4. 116 **4** 5. 165 **3**
 6. 404 **4** 7. 639 **3** 8. 1272 **3 & 4** 9. 17 008 **4** 10. 25 476 **3 & 4**

Tell which are divisible by 3, and which by 6.

11. 42 **3 & 6** 12. 63 **3** 13. 84 **3 & 6** 14. 201 **3** 15. 210 **3 & 6**
 16. 723 **3** 17. 732 **3 & 6** 18. 5874 **3 & 6** 19. 25 482 **3 & 6** 20. 100 002 **3 & 6**

Tell which are divisible by 3, and which by 9.

21. 21 **3** 22. 45 **3 & 9** 23. 102 **3** 24. 549 **3 & 9** 25. 594 **3 & 9**
 26. 2823 **3** 27. 9720 **3 & 9** 28. 34 860 **3** 29. 300 003 **3** 30. 500 076 **3 & 9**

Tell which are divisible by 8, and which by 10.

31. 3410 **10** 32. 4168 **8** 33. 5234 **Neither** 34. 56 170 **10** 35. 30 256 **8**
 36. 173 564 **Neither** 37. 81 432 **8** 38. 72 345 **Neither** 39. 63 008 **8** 40. 7 000 000 **8 & 10**

State whether these numbers are divisible

by 2; by 3; by 4; by 5; by 6; by 7; by 9; or by 10.

41. 24 **2, 3, 4, 6** 42. 40 **2, 4, 5, 10** 43. 72 **2, 3, 4, 6, 9** 44. 100 **2, 4, 5, 10** 45. 108 **2, 3, 4, 6, 9**
 46. 136 **2, 4** 47. 288 **2, 3, 4, 6, 9** 48. 306 **2, 3, 6, 9** 49. 352 **2, 4** 50. 1000 **2, 4, 5, 10**

Tell whether these statements are true or false. Explain.

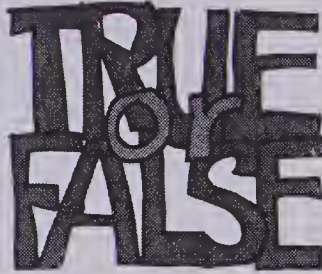
★ 51. "All numbers divisible by 8 are also divisible by 2 and 4."

★ 52. "All numbers divisible by 9 are also divisible by 3."

★ 53. "All numbers divisible by 3 are also divisible by 9."

★ 54. "All numbers divisible by 6 are also divisible by 2 and 3."

★ 55. "All numbers divisible by 5 are also divisible by 10."

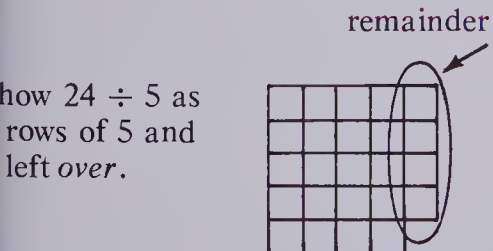


Divisibility 161

Answers:

1. True, 2 and 4 are factors of 8.
 2. True, 3 is a factor of 9.
 3. False, 9 is not a factor of 3.
 4. True, 2 and 3 are factors of 6.
 5. False, 10 is not a factor of 5.

Continued from page 159)



Point out: *not* divisible, then not rectangular — or conversely — not rectangular, then not divisible.

Assign the exercises, *encouraging* and *accepting* workable divisibility rules proposed by the students, (i.e., see Exercises 2, 4, and 6).

ACTIVITIES

1. Provide an activity card such as:

What do you get when you add:

- (a) an even number to an even number?
 (b) an odd number to an odd number?
 (c) an even number to an odd number?
 (d) an odd number to an even number?

What do you get when you multiply:

- (a) an even number by an even number?
 (b) an odd number by an odd number?
 (c) an even number by an odd number?
 (d) an odd number by an even number?

2. "Divisibility Sorting". Repeat the activity suggested on page 159 (Activity 1) with the following instructions.

(a) If a numeral belongs to more than one of the headings, write new cards so that each numeral is included in all appropriate sets.

Example

You will need 30 cards for each of the divisible by 2, 3, 5, 6, and 10 headings.

(b) Answer some of the following questions:

- (i) Which heading has the most cards? Explain why?
 (ii) Which has the least? Why?

3. "Toss Up". A game for 2 players.

(a) Throw a pair of dice. Multiply the 2 amounts.

(b) Each player lists all the numbers that are "divisible" into the product in 30 s.

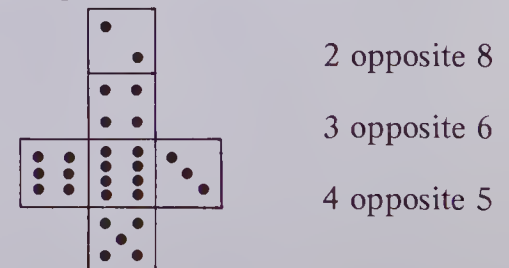
Example



15 is divisible by 1, 3, 5, and 15.

After 10 products have been used, the player with the most correct numbers which are *divisible* into the products is the winner.

Each die should have the following amounts on the faces.



4. See "Itza Fact!" as described in the Activity Reservoir.

OBJECTIVE

To list and chart factors

PACING

Level A All
Level B All
Level C All

MATERIALS

overhead projector, centimetre cubes or 2 cm × 2 cm cardboard square

BACKGROUND

Factors are the numbers a certain number is divisible by.

SUGGESTIONS

Initial Activity Use the centimetre cubes or squares and the overhead projector to show the rectangles possible using 20 squares.

List the factors of 20, the lengths of the sides of the rectangles.

Note that the *factors* are simply numbers which divide (without remainder) into 20.

Example

The factors of 20 are 1, 2, 4, 5, 10, and 20.

Repeat the above routine to show the set of factors of 24. Chart these factors as shown on page 163.

Ask the students if they see an easy way to check if all the factors are listed.

Example

The factors of 20 are 1, 2, 4, 5, 10, 20.

The factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24.

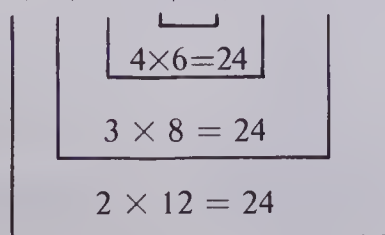
USING THE BOOK

Allow about 10 min for the students to work on the exercises on page 162; or use the exercises as oral exercises.

Encourage students to list the factors in order from smallest to largest so that they can be checked quickly.

The factors of 24 are

1, 2, 3, 4, 6, 8, 12, 24.



$1 \times 24 = 24$

Factors

These rectangles can be made by using 20 squares.



$$1 \times 20 = 20$$

1 and 20 are factors of 20.



$$2 \times 10 = 20$$

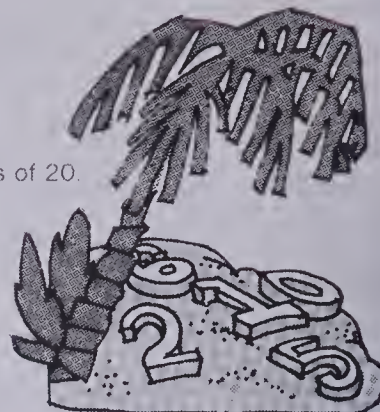
2 and 10 are factors of 20.



$$4 \times 5 = 20$$

4 and 5 are factors of 20.

The factors of 20 are 1, 2, 4, 5, 10, and 20.



Exercises

Copy and find the missing factors.

1. $1 \times \blacksquare = 18$

$\blacksquare \times 9 = 18$

$3 \times \blacksquare = 18$

The factors of 18 are 1, \blacksquare , 3, \blacksquare , 9, and \blacksquare .

2. $1 \times \blacksquare = 24$

$\blacksquare \times 12 = 24$

$\blacksquare \times 8 = 24$

$4 \times \blacksquare = 24$

The factors of 24 are 1, \blacksquare , \blacksquare , 4, \blacksquare , 8, 12, and \blacksquare .

3. $1 \times \blacksquare = 36$

$2 \times \blacksquare = 36$

$\blacksquare \times 12 = 36$

$4 \times \blacksquare = 36$

$6 \times \blacksquare = 36$

The factors of 36 are 1, 2, \blacksquare , 4, \blacksquare , 6, 9, 12, \blacksquare , and \blacksquare .

4. $1 \times \blacksquare = 42$

$2 \times \blacksquare = 42$

$3 \times \blacksquare = 42$

$6 \times \blacksquare = 42$

The factors of 42 are 1, 2, 3, 6, \blacksquare , \blacksquare , \blacksquare , and \blacksquare .

5. $1 \times \blacksquare = 50$

$2 \times \blacksquare = 50$

$5 \times \blacksquare = 50$

The factors of 50 are 1, 2, 5, \blacksquare , \blacksquare , and \blacksquare .

Write all the factors for each of the following numbers.

6. 10 $1, 2, 5, 10$

7. 21 $1, 3, 7, 21$

8. 30 $1, 2, 3, 5, 6, 10, 15, 30$

9. 33 $1, 3, 11, 33$

10. 40 $1, 2, 4, 5, 8, 10, 20, 40$

11. 45

12. 48

13. 56

14. 60

15. 100

162 Factors

ANSWERS:

11. 1, 3, 5, 9, 15, 45 12. 1, 2, 3, 4, 6, 8, 12, 16, 24, 48 13. 1, 2, 4, 7, 8, 14, 28, 56

14. 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60 15. 1, 2, 4, 5, 10, 20, 25, 50, 100



Charting Factors

What are the factors of 24?

$$\blacktriangle \times \blacksquare = 24$$

\blacktriangle	\blacksquare	
1	24	because $1 \times 24 = 24$
2	12	because $2 \times 12 = 24$
3	8	because $3 \times 8 = 24$
4	6	because $4 \times 6 = 24$

The factors of 24 are
1, 2, 3, 4, 6, 8, 12, and 24

What are the factors of 32?

$$\blacktriangle \times \blacksquare = 32$$

\blacktriangle	\blacksquare	
1	32	because $1 \times 32 = 32$
2	16	because $2 \times 16 = 32$
4	8	because $4 \times 8 = 32$

The factors of 32 are
1, 2, 4, 8, 16, and 32.

Exercises

Complete these factor charts.

List the factors of 40, 28, and 27.

$$\blacktriangle \times \blacksquare = 40$$

\blacktriangle	\blacksquare	
1	40	
2	20	
4	10	
5	8	

$$2. \quad \blacktriangle \times \blacksquare = 28$$

\blacktriangle	\blacksquare	
1	28	
2	14	
4	7	

$$3. \quad \blacktriangle \times \blacksquare = 27$$

\blacktriangle	\blacksquare	
1	27	
3	9	

Make factor charts for these.

$$1 \times 16, 2 \times 8, 4 \times 4$$

$$1 \times 20, 2 \times 10, 4 \times 5$$

$$1 \times 30, 2 \times 15, 3 \times 10, 5 \times 6$$

$$4. \quad \blacktriangle \times \blacksquare = 16$$

$$5. \quad \blacktriangle \times \blacksquare = 20$$

$$6. \quad \blacktriangle \times \blacksquare = 30$$

$$1 \times 36, 2 \times 18, 3 \times 12, 4 \times 9, 6 \times 6$$

$$1 \times 25, 5 \times 5$$

$$1 \times 64, 2 \times 32, 4 \times 16, 8 \times 8$$

$$7. \quad \blacktriangle \times \blacksquare = 36$$

$$8. \quad \blacktriangle \times \blacksquare = 25$$

$$9. \quad \blacktriangle \times \blacksquare = 64$$

$$1 \times 72, 2 \times 36, 3 \times 24, 4 \times 18, 6 \times 12, 1 \times 90, 2 \times 45, 3 \times 30, 5 \times 18, 1 \times 108, 2 \times 54, 3 \times 36, 4 \times 27, 6 \times 18,$$

$$10. \quad \blacktriangle \times \blacksquare = 72 \quad 8 \times 9$$

$$11. \quad \blacktriangle \times \blacksquare = 90 \quad 6 \times 15, 9 \times 10$$

$$12. \quad \blacktriangle \times \blacksquare = 108 \quad 9 \times 12$$

13. (a) In Exercises 4-12, write the set of factors for each product.

(b) Count the number of factors for each product.

Which numbers have an odd number of factors? **16, 36, 25, 64**

★ (c) Numbers that have an odd number of factors are special.

What are they called? **perfect squares**

Factor sets 163

Assign the exercises on page 163 to be recorded as factor charts.

Example

$$1. \quad \blacktriangle \times \blacksquare = 40$$

\blacktriangle	\blacksquare	
1	40	
2	20	
4	10	
5	8	

The factors of 40 are 1, 2, 4, 5, 8, 10, 20, 40.

ACTIVITIES

1. "Factors Toss Up". Using the pair of dice from Activity 2 for pages 160 and 161, list the factors of the products rolled.

The player with the largest number of correct factors wins.

2. "Giant Factors Toss Up". Similar to above, but using 3 normal dice.

3. Use normal playing cards to play "Royal Factor". Use these 28 cards only: 4 aces (count as one); from 2 to 6 from each of the four suits; 4 kings (or queens). Shuffle and place the cards face down on a table. The players (2-4) take turns turning over 1 card and saying the total of the cards turned at that time. The player who turns up a king must list as many factors for the total as possible. That player scores 1 point for each correct factor. Cards are all turned back over, remixed up, and play begins again. Player with the most points after a predetermined amount of time (or turns) wins.

Example

Turn up: 2, 6, 5, 3, king. Total was 16. The player finding the king lists 1, 16, 4. Score 3 points.

Variation:

(1) If a player correctly lists all factors, have that player score a number of points equal to the total (i.e., in the example above, score 16, not 3).

(2) Allow other players to identify (and score points for) any factors that have been missed.

ANSWERS:

3. (a) **16** - 1, 2, 4, 8, 16 (5 factors) **20** - 1, 2, 4, 5, 10, 20 (6 factors)
10 - 1, 2, 3, 5, 6, 10, 15, 30 (8 factors) **36** - 1, 2, 3, 4, 6, 9, 12, 18, 36 (9 factors)
25 - 1, 5, 25 (3 factors) **64** - 1, 2, 4, 8, 16, 32, 64 (7 factors)
72 - 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72 (12 factors) **90** - 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, 90 (12 factors) **108** - 1, 2, 3, 4, 6, 9, 12, 18, 27, 36, 54, 108 (12 factors)

OBJECTIVE

To identify elements, zeros and ones

PACING

Level A 1-15

Level B 1-20

Level C All

All students should do the Tune Up.

MATERIALS

bingo chips, overhead projector

RELATED AIDS

CALC. ACTIVITY MASTERS — 68.

BACKGROUND

When working with zero make sure that it is the first element.

Example

$$0 \times 5 = \blacksquare$$

$$0 \div 2 = \blacksquare$$

SUGGESTIONS

Initial Activity Using the bingo chips and the overhead projector, demonstrate these four situations:

1. $6 \times 1 = 6$ as this is one group of 6.
 $6 \div 1 = 6$ as this is *how* many ones in 6? [6]

2. Place *no* chips on the overhead projector. Multiplying by any number *leads* to a zero answer.

“Nothing times any amount is still nothing.”

Example

$$0 \times 8 = 0 \quad 0 \times 4 = 0$$

$$0 \times 10 = 0 \quad 0 \times 1 \text{ million} = 0$$

3. To divide zero, again place *no* chips on the overhead projector.

Demonstrate

$$0 \div 2 = 0$$

$$0 \div 3 = 0$$

$$0 \div 4 = 0$$

If there is *nothing* altogether, there is nothing to share.

4. Place 5 bingo chips on the overhead projector. Using these, how many groups can be formed containing *zero* chips? [Many! None! A whole bunch!] At this level the question is meaningless.

USING THE BOOK

Assign the exercises. Allow the students to refer to the display at the top of the page when they need to.

Use the Tune Up the following day to review and test the concepts.

Have students cover the display on the top of the page.

Zeros and Ones

Multiplication

$$6 \times 1 = 6$$

$$1 \times 6 = 6$$

A number multiplied by 1 is the number itself.

$$0 \times 4 = 0$$

A number multiplied by 0 is 0.

Division

$$6 \div 1 = 6$$

A number divided by 1 is the number itself.

$$6 \div 6 = 1$$

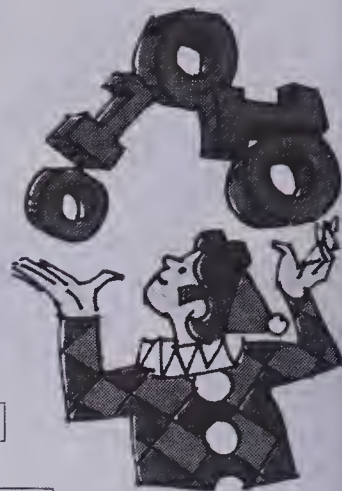
A number divided by itself is 1.

$$0 \div 4 = 0$$

0 divided by a number is 0

$5 \div 0$ is meaningless.

Division by zero is meaningless.



Exercises

Copy and complete where possible.

1. $7 \times 1 = 7$
2. $1 \times 8 = 8$
3. $12 \div 1 = 12$
4. $73 \times 1 = 73$
5. $28 \div 1 = 28$
6. $0 \times 9 = 0$
7. $17 \times 0 = 0$
8. $0 \div 2 = 0$
9. $0 \div 10 = 0$
10. $37 \times 0 = 0$
11. $14 \times 1 = 14$
12. $14 \div 0 = \text{meaningless}$
13. $14 \div 14 = 1$
14. $0 \div 14 = 0$
15. $8 \div 8 = 1$
16. $7 \times n = 7$
17. $7 \div n = 7$
18. $7 \div 7 = n$
19. $n \div 8 = 0$
20. $12 \div n = 12$
21. $50 \div n = 1$
22. $13 \times n = 0$
23. $n \times 8 = 0$
24. $n \div 2 = 0$
25. $15 \div n = 15$

Tune Up

Watch your operation signs.

1. $6 + 0 = 6$
2. $8 - 0 = 8$
3. $0 + 12 = 12$
4. $1 + 7 = 8$
5. $19 + 0 = 19$
6. $12 \times 1 = 12$
7. $18 - 1 = 18$
8. $0 \div 2 = 0$
9. $1 \times 10 = 10$
10. $0 \times 20 = 0$
11. $19 + 1 = 20$
12. $19 - 1 = 19$
13. $19 \times 1 = 19$
14. $19 \times 0 = 0$
15. $19 \div 0 = 19$

164 Zeros and ones in multiplication and division, practice

ACTIVITIES

1. See “Bingo” as described in the Activity Reservoir. Use numbers on the grids from 0 to 30 inclusive. Be sure to call out statements typical of this page.

2. See “Input-Output” as described in the Activity Reservoir.

3. To review and practise missing parts of equations, have the students make up Punch Cards such as:

punch holes

$15 \div \bullet = 15$ $0 \div 21 = \bullet$ $26 \times \bullet = 0$ $\bullet \times 17 = 17$ $13 \times 0 = \bullet$ $108 \div \bullet = 108$ $108 \div \bullet = 1$	$\bullet 1$ $\bullet 0$ $\bullet 0$ $\bullet 1$ $\bullet 0$ $\bullet 1$ $\bullet 108$
Front	Back

The cardboard strips should be about 4.5 cm wide to allow the hole punch easy access to the centre of the strip. These strips could test any basic facts you wish and could be exchanged with other classmates, groups, or even classes in the school.

Information — Too Little or Too Much!

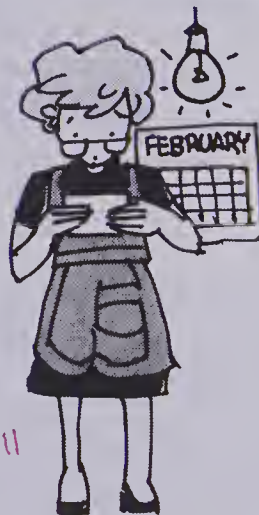
Some problems contain extra information, and some problems do not contain enough information.

If the problem does not contain enough information, write some additional information, and then solve.



Exercises

1. Jody has a part-time delivery job. She earns \$3.25/h. How much does she earn in a week? *Answers will vary.*
2. The hockey team purchased 5 dozen hockey sticks and 30 pucks. Sticks sell for \$8.50 each and pucks sell for \$1.80 each. What is the total cost of the sticks? *\$510.00*
3. Miss Bennett's class collected \$32.40 for charity. There are 24 students in the class — 10 boys and 14 girls. What was the average donation for each student? *\$1.35*
4. Mr. Kowalski's class collected \$51.20 for charity. What was the average donation for each student? *Answers will vary.*
5. One afternoon, Joan spent 2 h gardening. She spent 20 min trimming the hedge, a half hour mowing the lawn, 40 min raking leaves, and the rest of the time weeding the garden. How much time altogether did Joan spend mowing the lawn and raking leaves? *1 h 10 min*
6. The MacPhersons' electricity bill was \$62.40 for the month of February. What was the average cost for electricity per day? *\$2.23*



Extraneous and insufficient information 165

OBJECTIVE

To solve problems involving extraneous or insufficient information

PACING

Level A Any 4
Level B Any 5
Level C All

RELATED AIDS

BFA PROB. SOLVING LAB II — 23, 75.

BACKGROUND

If the problem requires more information, the student has to understand the context of the problem in order to add the required information. Once this is done the child has more *ownership* of the problems. Discuss these problems and the answers individually with the students.

SUGGESTIONS

Initial Activity Review the steps involved in solving word problems with the class using a sample problem constructed jointly by the class based on some local event, depending on the season, etc. (For a suggested problem-solving format, see pages 17 and 22 in Chapter 1.)

USING THE BOOK

Assign the exercises, based on the Pacing suggested above. You may wish to read and discuss the context of the problems with some of the students. Exercises 3, 4, and 6 involve computing averages. You may need to review with some groups the concept and technique for finding averages.

ACTIVITIES

1. For those who need practice choosing the correct operation in word problems, see Activity 1 on page 22.

2. The "Problem Box". Have each student write *one* problem and solve it in his/her notebook.

After the teacher has discussed the problem wording, accuracy, etc. have each student write the problem on a card, add some art, and sign it.

Place problems in the "Problem Box". Each student is to solve a minimum of 2 problems a week. (A child may choose not to do a problem and trade it for another.)

OBJECTIVE

To determine whether a whole number is prime or composite

PACING

Level A 1-5

Level B 1-5

Level C 1-6

MATERIALS

cubes for making dice

BACKGROUND

This is basically a sorting exercise.

SUGGESTIONS

Initial Activity Write the numbers 2, 3, 5, 7, 11, 13, and 17 on the front chalkboard and the numbers 4, 6, 8, 9, 10, 12, 14, 15, 16, and 18 on the side chalkboard.

Ask the students to write the numbers 19, 20, 21, 22, 23, 24, and 25 on the appropriate chalkboard. Do not yet explain why.

Ask questions to determine why the numerals are sorted as they are. Record all reasonable explanations.

USING THE BOOK

Define and show the prime and composite numbers.

Prime

Composite

Exactly 2 factors
1 and the number
itself.

More than 2
factors.

Emphasize that 1 is neither prime nor composite.

Have the students do the exercises and check all work with the class.

In Exercise 5, all of the even counting numbers except 2 are *composite* because 2 is a factor of all *even* numbers. The question may be answered in many ways but this is the essential reason.

ACTIVITIES

1. "Prime Time". Using 2 dice marked as shown calculate the products. List all of the factors for each product.

Prime Numbers and Composite Numbers

Prime numbers have exactly two different factors.

$$7 = 7 \times 1$$

$$11 = 11 \times 1$$

The two factors are 1 and the number itself.

7 and 11 are prime numbers.



Composite numbers have more than two different factors.

$$8 = 1 \times 8$$

$$8 = 2 \times 4$$

The factors of 8 are 1, 2, 4, and 8.

8 is a composite number.

The number 1 is neither prime nor composite as $1 = 1 \times 1$. It has only one factor, 1.

Exercises

- Copy and complete this chart, using the numbers 2, 3, 4, 5, 9, 10, 12, 13, 15, 18, 22, 23, 25, 27, 29, 31, 35, and 36.

Number	Products	Factors	Kind of Number
2	$2 \times 1 = 2$	1, 2	Prime
3	$3 \times 1 = 3$	1, 3	Prime
4	$4 \times 1 = 4$ $2 \times 2 = 4$	1, 2, 4	Composite
5	$5 \times 1 = 5$	1, 5	Prime

- List the composite numbers greater than 39 and less than 51.

40, 42, 44, 45, 46, 48, 49, 50

- List the prime numbers greater than 39 and less than 51.

41, 43, 47

- List the prime numbers between 50 and 70.

53, 59, 61, 67

- Most even numbers are composite numbers. Why is this true?

Because they are divisible by 2.

- Twin primes are pairs of primes that are two apart. For example, 3 and 5 are twin primes.

List all twin primes less than 75. 3, 5 5, 7 11, 13 17, 19 29, 31 41, 43 59, 61 71, 73



166 Primes and composites

ANSWERS:

1. 9- $9 \times 1 = 9$, $3 \times 3 = 9$; 1, 3, 9; Composite 10- $10 \times 1 = 10$, $5 \times 2 = 10$; 1, 2, 5, 10; Composite
12- $12 \times 1 = 12$, $6 \times 2 = 12$, $4 \times 3 = 12$; 1, 2, 3, 4, 6, 12; Composite
13- $13 \times 1 = 13$, 1, 13; Prime 15- $15 \times 1 = 15$, $5 \times 3 = 15$; 1, 3, 5, 15; Composite
18- $18 \times 1 = 18$, $9 \times 2 = 18$, $6 \times 3 = 18$; 1, 2, 3, 6, 9, 18; Composite
22- $22 \times 1 = 22$, $11 \times 2 = 22$; 1, 2, 11, 22; Composite 23- $23 \times 1 = 23$; 1, 23; Prime
25- $25 \times 1 = 25$, $5 \times 5 = 25$; 1, 5, 25; Composite 27- $27 \times 1 = 27$, $9 \times 3 = 27$; 1, 3, 9, 27; Composite
29- $29 \times 1 = 29$; 1, 29; Prime 31- $31 \times 1 = 31$; 1, 31; Prime
35- $35 \times 1 = 35$, $7 \times 5 = 35$; 1, 5, 7, 35; Composite
36- $36 \times 1 = 36$, $18 \times 2 = 36$, $12 \times 3 = 36$, $9 \times 4 = 36$, $6 \times 6 = 36$; 1, 2, 3, 4, 6, 9, 12, 18, 36; Composite

Prime Die

	2	
	3	
13	7	5
	11	

Normal Die

	1	
	2	
4	6	3
	5	

List the factors for 6 different products.

2. "Prime Circles". Have 3 or 4 circles of 6 to 8 players. Each team counts by primes in rotation until a total of 2 errors are made. The team counting to the highest amount wins. (Appoint a student to act as referee. He or she should have a list of primes to 200.)

3. See "Pro(bability) Golf" as described in the Activity Reservoir.

Special Numbers

An Investigation

All composite numbers have more than two different factors.

Let's investigate the number of factors that some composite numbers have.

- Copy and complete this chart using the numbers 4, 6, 8, 9, 14, 16, 21, 25, 27, 30, 36, 48, 49, 56, 64, 75, 81, 96, and 100.

Number	Factors	Number of Factors
4	1, 2, 4	Three
6	1, 2, 3, 6	Four
8	1, 2, 4, 8	Four
9	1, 3, 9	Three

- Which of the numbers from Exercise 1 have an *odd* number of factors? List them. *4, 9, 16, 25, 36, 49, 64, 81, 100*
- Which whole number has only one factor?
- Numbers with an *odd* number of factors are special. What are these numbers called? *perfect squares*

BRAINTICKLER

Add these odd numbers.

$$\begin{aligned}
 1 + 3 &= \blacksquare 4 & 1 + 3 + 5 + 7 + 9 + 11 + 13 &= \blacksquare 49 \\
 1 + 3 + 5 &= \blacksquare 9 & 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 &= \blacksquare 64 \\
 1 + 3 + 5 + 7 &= \blacksquare 16 & 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 &= \blacksquare 81 \\
 1 + 3 + 5 + 7 + 9 &= \blacksquare 25 & 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 &= \blacksquare 100 \\
 1 + 3 + 5 + 7 + 9 + 11 &= \blacksquare 36
 \end{aligned}$$

The sums are special numbers.

What are these numbers called? *perfect squares*

Composite numbers 167

ANSWERS:

14 - 1, 2, 7, 14; 4 factors 16 - 1, 2, 4, 8, 16; 5 factors 21 - 1, 3, 7, 21; 4 factors
 25 - 1, 5, 25; 3 factors 27 - 1, 3, 9, 27; 4 factors
 30 - 1, 2, 3, 5, 6, 10, 15, 30; 8 factors 36 - 1, 2, 3, 4, 6, 9, 12, 18, 36; 9 factors
 48 - 1, 2, 3, 4, 6, 8, 12, 16, 24, 48; 10 factors 49 - 1, 7, 49; 3 factors
 56 - 1, 2, 4, 7, 8, 14, 28, 56; 8 factors 64 - 1, 2, 4, 8, 16, 32, 64; 7 factors
 75 - 1, 3, 5, 15, 25, 75; 6 factors 81 - 1, 3, 9, 27, 81; 5 factors
 96 - 1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 96; 12 factors
 100 - 1, 2, 4, 5, 10, 20, 25, 50, 100; 9 factors

ACTIVITIES

To practise recognizing primes and composites, prepare a deck of 20 or so numeral cards showing various numbers from 1 to 50. Have the children sort these into envelopes, one labelled "PRIMES" and the other "COMPOSITES". Have the sort checked and reshuffled by a classmate.

2. See the "Royal Factor" idea (Activity 3, pages 162 to 163). Play it with this variation: when a king is turned up, have that player identify the total as "prime" or "composite".

If correct, that player scores a number of points equal to the total.

3. If you have not already done so, see the "Pro(bability) Golf" idea listed in the Activity Reservoir.

EXTRA PRACTICE

Predict the answer to:

- $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 = \blacksquare$
- $1 + 3 + 5 + 7 + \dots + 23 = \blacksquare$
- $1 + 3 + 5 + 7 + \dots + 25 = \blacksquare$
- $1 + 3 + 5 + 7 + \dots + 27 = \blacksquare$
- $1 + 3 + 5 + 7 + \dots + 29 = \blacksquare$

OBJECTIVE

To investigate composite numbers

PACING

Level A All
 Level B All
 Level C All

MATERIALS

centimetre cubes or blocks

BACKGROUND

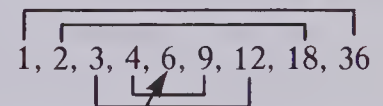
A composite number with an odd number of factors is a perfect square. This is what the students are to determine.

USING THE BOOK

Assign this exercise after a brief review of factors and sets of factors.

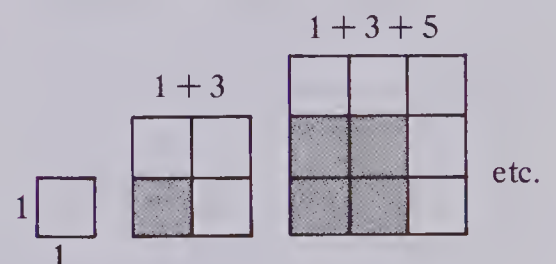
When students are finished they should try the Braintickler.

The sums in the Braintickler and the answer to Exercise 4 are related. Discussion: You may wish to show the students why the *odd* number of factors indicates a perfect square. The factors of 36 are

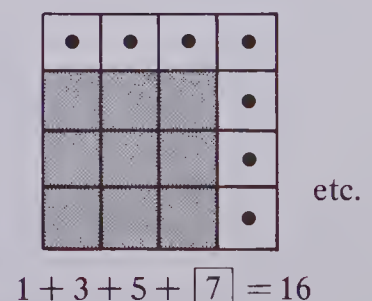


The middle factor has to be multiplied by itself to produce 36.

You can illustrate the Braintickler using squares on the overhead projector.



In each case an *odd* number of squares are added.



OBJECTIVE

To use division to find prime factors

PACING

Level A 1-3(e)
Level B 1-3(g)
Level C All

BACKGROUND

This page involves repeated short division using prime-number divisors. Students divide by primes until the quotient is also *prime*.

SUGGESTIONS

Initial Activity (a) Review some simple short division techniques involving divisors of 2, 3, 5, and 7. (b) Demonstrate a Step Division using 8.

Example

$$\begin{array}{l} 2 \longrightarrow 2 \\ 2 \overline{)4} \longrightarrow 4 \div 2 = 2 \\ 2 \overline{)8} \longrightarrow 8 \div 2 = 4 \end{array}$$

Ask students to divide by the smallest prime factors such as 2, 3, 5, 7, or 11.

USING THE BOOK

On the chalkboard, demonstrate the step divisions in the display shown at the top of the page.

Example

For 12. "What is the smallest prime number which is a factor of 12?" [2]
"Divide 12 by 2." [6]
"What is the smallest prime number which is a factor of 6?" [2]
"Divide 6 by 2." [3]
"Since 3 is also prime, then
 $12 = 2 \times 2 \times 3$.

$$\begin{array}{l} 3 \\ 2 \overline{)6} \longrightarrow 6 \div 2 = 3 \\ 2 \overline{)12} \longrightarrow 12 \div 2 = 6 \end{array}$$

Use the same questioning technique for the step divisions of 30. Assign the exercises based on the Pacing suggested above.

ACTIVITIES

1. Play "Concentration" as described in the Activity Reservoir. Use cards such as:

$$2 \times 2 \times 3 \quad 12$$

$$3 \times 3 \times 5 \quad 45$$

2. Provide a challenge card such as:

Step Division

We can find the prime factors of a number by using **step division**.

Step division is repeated short division.

Choose prime divisors

$$\begin{array}{r} 3 \overline{)15} \\ 2 \overline{)30} \end{array}$$

12 as a product is $2 \times 2 \times 3$.

This is a prime number, too.

$$\begin{array}{r} 3 \\ 2 \overline{)6} \\ 2 \overline{)12} \end{array}$$

30 as a product is $2 \times 3 \times 5$.

12 = $2 \times 2 \times 3$
30 = $2 \times 3 \times 5$

Exercises

- Copy and complete these step divisions. Write the missing factors.

(a)

$$\begin{array}{r} 3 \overline{) \square} \\ 2 \overline{) \square} \\ 2 \overline{)12} \end{array}$$

$12 = 2 \times 2 \times \square$

(b)

$$\begin{array}{r} 2 \overline{) \square} \\ 2 \overline{) \square} \\ 2 \overline{) \square} \\ 2 \overline{)16} \end{array}$$

$16 = 2 \times \square \times \square \times \square$

(c)

$$\begin{array}{r} 3 \overline{) \square} \\ 3 \overline{) \square} \\ 3 \overline{)27} \end{array}$$

$27 = 3 \times \square \times \square$
- Use step division. Write products for each number.

(a) $\square \overline{)20}$
 $20 = 2 \times 2 \times 5$

(b) $\square \overline{)27}$
 $27 = 3 \times 3 \times 3$

(c) $\square \overline{)36}$
 $36 = 2 \times 2 \times 3 \times 3$

(d) $\square \overline{)42}$
 $42 = 2 \times 3 \times 7$

(e) $\square \overline{)50}$
 $50 = 2 \times 5 \times 5$

(f) $\square \overline{)56}$
 $56 = 2 \times 2 \times 2 \times 7$

(g) $\square \overline{)65}$
 $65 = 5 \times 13$

(h) $\square \overline{)100}$
 $100 = 2 \times 2 \times 5 \times 5$
- Use step division. Write products for each.

(a) 96

(b) 104

(c) 110

(d) 116

(e) 140

(f) 168

(g) 200

★ (h) 225

★ (i) 320

★ (j) 576

168 Dividing to find factors

ANSWERS:

3. (a) $96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$ (b) $104 = 2 \times 2 \times 2 \times 13$ (c) $110 = 2 \times 5 \times 11$ (d) $116 = 2 \times 2 \times 29$
(e) $140 = 2 \times 2 \times 5 \times 7$ (f) $168 = 2 \times 2 \times 2 \times 3 \times 7$ (g) $200 = 2 \times 2 \times 2 \times 5 \times 5$
(h) $225 = 3 \times 3 \times 5 \times 5$ (i) $320 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5$ (j) $576 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3$

- Choose 3 numbers between 50 and 100.
- Use step division to find the prime factors of each of your 3 numbers, if possible.
- Were any of your choices prime numbers? How do you know? Explain.

3. Have the students prepare Punch Cards as described in Activity 3, page 164. On the front of the cards, have them show numbers such as 12, 15, 18, 30, 40, etc. with the instruction: "What are the products for each?" Answers should be listed for each on the back of the cards.

4. Prepare a challenge card such as:

A prime is a number divisible by 1 and itself only.
Every even number, except 2, can be expressed as the sum of two primes.

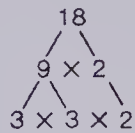
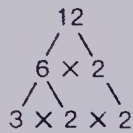


Make your own patterns.
Try these:



Factor Trees

All composite numbers can be expressed as a product of prime factors.



$$10 = 2 \times 5$$

$$12 = 2 \times 2 \times 3$$

$$18 = 2 \times 3 \times 3$$

$$15 = 3 \times 5$$

Keep building the factor tree until all the factors are prime numbers.

Exercises

Copy and finish these factor trees.

Write each number as the product of prime factors.

(a) $8 = 2 \times 2 \times 2$

(b) $15 = 3 \times 5$

(c) $20 = 2 \times 2 \times 5$

(d) $24 = 2 \times 2 \times 2 \times 3$

2. (a) $12 = 2 \times 2 \times 3$

(b) $28 = 2 \times 2 \times 7$

(c) $30 = 2 \times 3 \times 5$

3. Build a factor tree.
Write each number as a product of prime factors.

(a) 36 (b) 40 (c) 42 (d) 48 (e) 54
(f) 56 (g) 60 (h) 64 (i) 70 (j) 72

Products of primes 169

OBJECTIVE

Use factor-tree diagrams to find the product of prime factors

PACING

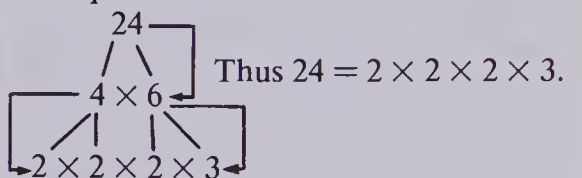
Level A All
Level B All
Level C All

SUGGESTIONS

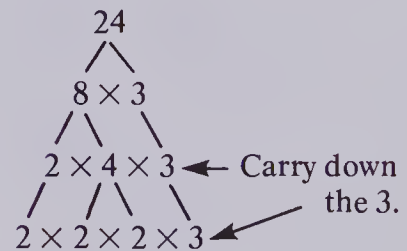
Initial Activity Review factor trees as a method of finding a product of prime factors for a given number.

The method is to write each number as a product of 2 smaller numbers. Repeat this procedure until all factors are prime.

Example



If the student chose



he/she must remember to carry down all primes to the base of the factor tree.

USING THE BOOK

Assign the exercises. You may wish to do some orally, before assigning the remainder of the exercises. Remind the students if necessary that 1 is neither prime nor composite.

ACTIVITIES

- Have the children:
 - Choose 3 numbers between 50 and 100 at random (different from the numbers chosen if you did the Activity from page 168);
 - Make factor trees for each of the three numbers, if possible;
 - Answer these questions:

“Were any of your choices prime numbers? How do you know? Explain.”
- See “Pro(bability) Golf” as described in the Activity Reservoir.
- See “Rummy” as described in the Activity Reservoir. In preparing the deck of cards, include a series of composite numbers and their prime factors.

OBJECTIVE

To determine the product of prime factors (prime factorization)

PACING

Level A 1-2
Level B 1-3(c)
Level C All

VOCABULARY

factorization

RELATED AIDS

CALC. ACTIVITY MASTERS — 35, 66.

BACKGROUND

This is a review of the previous 2 methods of finding prime factors.

SUGGESTIONS

Initial Activity (a) Review step division to find the prime factorization for 24.

(b) Show how the 3 factor trees in the display all result in the same prime factorization for 24, i.e., $24 = 2 \times 2 \times 2 \times 3$.

(c) Mention that for ease of checking, and by convention, the factors are written in order from least through to greatest.

Note: It also makes writing powers easier eventually.

Example

$$\begin{array}{l} \boxed{24 = 2 \times 2 \times 2 \times 3} \\ \rightarrow 24 = 2^3 \times 3 \end{array}$$

The identical factors are grouped.

USING THE BOOK

Assign the exercises. Allow the students to use any method they wish to find the prime factorization for each. Encourage them to record answers as:

$$2(a) 27 = 3 \times 3 \times 3$$

ACTIVITIES

1. Have the students write the first 15 prime numbers.

Answer

The first 15 primes are:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, and 47.

Some students may write 39 as a prime. $39 = 3 \times 13$

If a prime is missed, some may write 51 as a prime. $51 = 3 \times 17$

2. Use a 10×10 grid numbered from 1 to 100 to play "Choice Choice".

Prime Factorization

Factor trees help in finding the prime factorization.

The above factor trees look different, but result in the same set of factors.

The prime factorization of 24 = $2 \times 2 \times 2 \times 3$

smallest largest

Using step division
 $24 = 2 \times 2 \times 2 \times 3$.

When every factor is prime, we have prime factorization.

Exercises

- Find the missing prime factors

(a) $24 = 2 \times \blacksquare \times 2 \times 3$

(c) $32 = 2 \times \blacksquare \times 2 \times \blacksquare \times \blacksquare$

(e) $54 = \blacksquare \times \blacksquare \times \blacksquare \times 3$

(g) $52 = 2 \times 2 \times \blacksquare \times \blacksquare$

(i) $44 = \blacksquare \times \blacksquare \times 11$

(b) $30 = 2 \times \blacksquare \times 5$

(d) $48 = 2 \times \blacksquare \times \blacksquare \times \blacksquare \times \blacksquare$

(f) $81 = \blacksquare \times \blacksquare \times \blacksquare \times \blacksquare$

(h) $21 = \blacksquare \times \blacksquare$

(j) $92 = 2 \times \blacksquare \times \blacksquare$
- Write the following as a product of prime factors.

(a) 27

(b) 32

(c) 35

(d) 45

(e) 50

(f) 66

(g) 75

(h) 80

(i) 88

(j) 96
- ★ (a) 120 (b) 108 (c) 132 (d) 144
(e) 175 (f) 192 (g) 196 (h) 200

170 Prime factorization

ANSWERS:

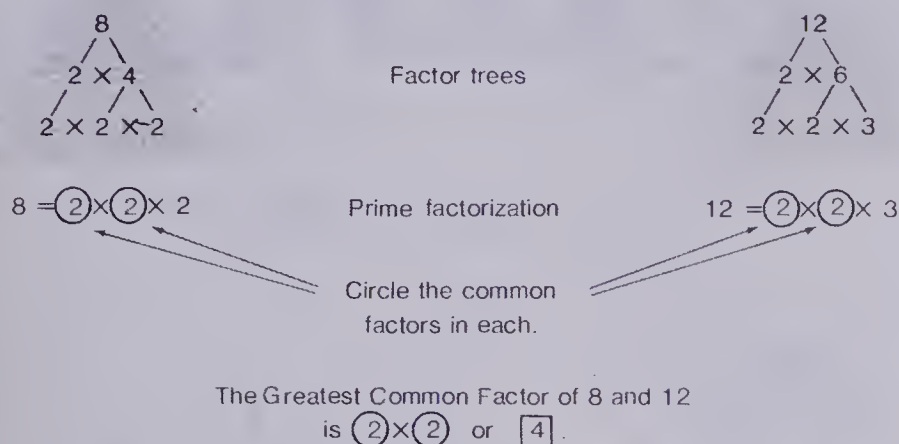
2. (a) $27 = 3 \times 3 \times 3$ (b) $32 = 2 \times 2 \times 2 \times 2 \times 2$ (c) $35 = 5 \times 7$ (d) $45 = 3 \times 3 \times 5$
 (e) $50 = 2 \times 5 \times 5$ (f) $66 = 2 \times 3 \times 11$ (g) $75 = 3 \times 5 \times 5$ (h) $80 = 2 \times 2 \times 2 \times 2 \times 5$
 (i) $88 = 2 \times 2 \times 2 \times 11$ (j) $96 = 2 \times 2 \times 2 \times 2 \times 3$
3. (a) $120 = 2 \times 2 \times 2 \times 3 \times 5$ (b) $108 = 2 \times 2 \times 3 \times 3 \times 3$ (c) $132 = 2 \times 2 \times 3 \times 11$
 (d) $144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$ (e) $175 = 5 \times 5 \times 7$ (f) $192 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$
 (g) $196 = 2 \times 2 \times 7 \times 7$ (h) $200 = 2 \times 2 \times 2 \times 5 \times 5$

Players (2) flip a coin to determine who goes first. First player chooses a number (e.g., 12), crosses it off the grid, and takes it as his/her score. Second player gets all of the factors of the first person's choice that have not been crossed off the grid (i.e., 1, 2, 3, 4, 6) and takes that total as a score (i.e., 16). First player chooses again (e.g., 17, total now $12 + 17 = 29$). Second player has only 1 factor to

choose (i.e., 1) but it has already been crossed off the grid. Score remains 16. Play continues in this fashion till all numbers have been crossed off the grid. Player with the greatest point total wins. (**Note:** you may eventually wish to help players discover a strategy: first player should always choose primes!)

3. See "Itza Fact!" as described in the Activity Reservoir.

Greatest Common Factor (GCF)



Exercises

What is the greatest common factor for each pair of numbers?

1. 9, 12 3 15, 35 5 8, 10 2 (d) 16, 24 8 (e) 10, 25 5

(f) 12, 30 6 (g) 14, 28 14 (h) 6, 11 1 (i) 16, 36 4 (j) 20, 32 4
2. 12, 15 3 18, 24 6 (c) 10, 14 2 (d) 6, 16 2 (e) 8, 24 8

(f) 18, 30 6 (g) 20, 30 10 (h) 21, 35 7 (i) 18, 45 9 (j) 22, 55 11

★3. Find the greatest common factor for these sets of three numbers.

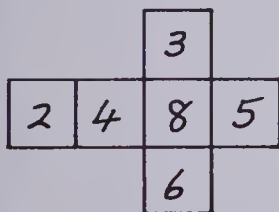
- 8, 10, 12 2 (b) 12, 18, 36 6 (c) 15, 25, 35 5
 (d) 12, 21, 27 3 (e) 24, 40, 56 8 (f) 28, 42, 70 14

Greatest common factors 171

ACTIVITIES

Have the students play "GCF Toss p".

1. Make 4 dice with the following numbers on the faces, to make 2 pairs of dice.



(b) Roll a pair of these dice. Record the product.

(c) Roll the second pair of dice. Record the second product.

(d) Find the GCF for these two products. Write a statement.

(e) Repeat the above Activity until you find a GCF larger than 8.

Variation: Use 36 playing cards, deuce through 10. Deal out pairs.

2. See "Choice Choice" as described in Activity 2, page 170.

3. See "Pro(bability) Golf" as described in the Activity Reservoir.

OBJECTIVE

To determine the greatest common factor

PACING

Level A 1-3(a)
Level B 1-3(any 2)
Level C All

RELATED AIDS

HMS — DM39.

BACKGROUND

The GCF is not a very large number in most instances. Some children hear the word "greatest" and look for a large answer. Most students should now know how to find the prime factorization. All they need now do is circle all the common factors to determine the greatest common factor. Be sure to define "common" for the students.

USING THE BOOK

Read through the display at the top of the pupil page together. Review the prime factorization using 8 and 12. Identify "2" as the common (i.e., a factor of *both* products) factor and "4" as the greatest common factor.

While doing the exercise students must recognize common factors instantly. Encourage them to check to see if double or triple that common factor is also a factor.

Example

Exercise 1(f):

12 and 30 have 3 as a common factor. Is 2×3 or 6 a common factor? [Yes]
Is 3×3 or 9 a common factor? [No]
Is 4×3 or 12 a common factor? [No]
The GCF of 12 and 30 is $[6]$.

OBJECTIVE

To review and provide practice with divisibility, factors, primes and composites, and factorization

PACING

Level A 1-9

Level B All

Level C All

USING THE BOOK

Use this as a review and/or diagnostic exercise. Have students mark their work and then help them improve any problem areas.

ACTIVITIES

There are many games and activities which could be used with the topics on this page. Check the Activities section of the past 15 pages to see if there are any which would appeal to your group. See also the ideas suggested in the Activity Reservoir.

Practice



- List all the rectangles that can be drawn by using the following.
(a) 16 squares $1 \times 16 = 16$, $2 \times 8 = 16$, $4 \times 4 = 16$
(b) 21 squares $1 \times 21 = 21$, $3 \times 7 = 21$
(c) 30 squares $1 \times 30 = 30$, $2 \times 15 = 30$, $3 \times 10 = 30$, $5 \times 6 = 30$
- Which numbers are divisible by 3? by 5? by 9?
7, 12, 18, 25, 27, 39, 42, 50
 $3 - 12, 18, 27, 39, 42$
 $5 - 25, 50$
 $9 - 18, 27$
- Write the set of factors for each.
(a) 16 $1, 2, 4, 8, 16$
(b) 21 $1, 3, 7, 21$
(c) 32 $1, 2, 4, 8, 16, 32$
(d) 35 $1, 5, 7, 35$
(e) 40 $1, 2, 4, 5, 8, 10, 20, 40$
- Find the missing prime factors.
(a) $18 = 2 \times 3 \times \blacksquare^3$
(b) $22 = 2 \times \blacksquare^{11}$
(c) $24 = 2 \times 2 \times \blacksquare^2 \times \blacksquare^3$
(d) $36 = 2 \times 2 \times \blacksquare^3 \times \blacksquare^3$
(e) $42 = \blacksquare^2 \times 3 \times \blacksquare^7$
(f) $48 = \blacksquare^2 \times \blacksquare^2 \times \blacksquare^2 \times \blacksquare^2 \times \blacksquare^3$
- Which of these numbers are prime?
1, 2, 3, 4, 8, 15, 17, 21, 29, 31, 35, 39
- Write a pair of twin primes greater than 20 and less than 100. Possible answers: 29 and 31; 41 and 43; 59 and 61; 71 and 73.
- Use step division to find the prime factorization for each.
(a) 28 $28 = 2 \times 2 \times 7$
(b) 40 $40 = 2 \times 2 \times 2 \times 5$
(c) 52 $52 = 2 \times 2 \times 13$
(d) 96 $96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$
- Build factor trees and write the prime factorization for each.
(a) 21 $21 = 3 \times 7$
(b) 32 $32 = 2 \times 2 \times 2 \times 2 \times 2$
(c) 75 $75 = 3 \times 5 \times 5$
(d) 92 $92 = 2 \times 2 \times 23$
- Find the greatest common factor for each pair of numbers.
(a) 3, 12 3
(b) 8, 20 4
(c) 10, 25 5
(d) 18, 60 6
- ★10. There are more composite numbers than prime numbers from 1000 to 2000. Explain why this is true.

Multiples

The multiples of 2 are:

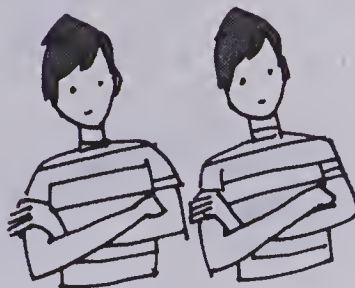
2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, ...

The multiples of 3 are:

3, 6, 9, 12, 15, 18, 21, 24, 27, ...

The common multiples of 2 and 3 are:

6, 12, 18, 24, ...



Exercises

Copy and write the next 10 multiples.

1. The multiples of 4 are: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48

2. The multiples of 6 are: 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72.

3. The multiples of 7 are: 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91

4. The multiples of 8 are: 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96.

Copy and find the missing multiples.

5. 3, 6, 9, 12; 15, 18, 21, 24, 27; 30, 33, 36, 39, 42, 45

6. 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75

7. 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99, 108

8. 10, 20, 30, 40, 50, 60, 80, 90, 100, 110, 120, 130, 140

9. 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 132, 144

Use your lists of multiples to find the following.

10. Three common multiples of:

(a) 9 and 12 36, 72, 108 (b) 6 and 8. 24, 48, 72

11. Four common multiples of:

(a) 3 and 6 6, 12, 18, 24 (b) 4 and 6. 12, 24, 36, 48

★ 12. Three common multiples of 2, 3, and 6. 6, 12, 18



OBJECTIVE

To list multiples

PACING

Level A All

Level B All

Level C All

RELATED AIDS

CALC. ACTIVITY MASTERS — 34.

BACKGROUND

Relate multiples to counting by 2's, 3's, etc.

SUGGESTIONS

Initial Activity Have the students count by: 2's to 30, 3's to 30, 4's to 48, 5's to 75. Explain that these are multiples of 2, 3, 4, or 5.

USING THE BOOK

Have students count to 60 by 2's. Write these on the board. Have students count to 60 by 3's. Write these on the board. Circle the common multiples of 2 and 3.

Assign the exercises. Discuss the answers to Exercises 10, 11, and 12 only.

The students can check Exercises 1 through 9 themselves in pairs.
Note: You may wish to use this page as an oral developmental exercise for page 174 — Least Common Multiple.

ACTIVITIES

1. Prepare and distribute "Complete-the-pattern" exercises such as:

(a) 4, 8, 12, , 20, , , 32.

(b) 1, 4, 7, 10, , , .

(c) 240, 120, 60, , .

(d) , , , , , , .

(e) , , , , , , , .

etc.

2. Play "Rummy" as described in the Activity Reservoir. Use a deck of cards which shows multiples of 2, 3, 4, 5, 6, and 7.

3. Have the children prepare and exchange their own "complete-the-pattern" challenge as described in Activity 1.

OBJECTIVE

To find the least common multiple

PACING

- Level A All
- Level B All
- Level C All

RELATED AIDS

HMS — DM40.

BACKGROUND

The least common multiple is usually larger than both. Students hear least and look for small numbers. The least common multiple is also the least common denominator when adding and subtracting unlike fractions.

USING THE BOOK

Using the examples in the display at the top of the page, circle the common multiples of 3 and 5. The smallest one is 15 so the least common multiple of 3 and 5 is 15.

In Exercises 6 through 20, encourage students to list enough multiples to determine the LCM.

ACTIVITIES

1. "LCM Rummy". Using 40 playing cards, ace through 10, deal pairs to 3 or 4 players face down. At a signal from the dealer all players pick up their pairs of cards, figure out the LCM, and state the LCM for their pair.
First correct answer — 4 points.
Second correct answer — 3 points.
Third correct answer — 2 points.
Fourth correct answer — 1 point.
Any incorrect answer — subtract 5 points.
Highest total after 10 deals wins. The teacher may wish to appoint a dealer/referee at each table.

- 2. If you have not already done so, see the Activities listed for page 173.
- 3. Have the children use a multiplication grid to help recognize and find least common multiples. Prepare a 9 × 9 multiplication grid (partial grid shown) as a playing board and use the deck of cards described in "Itza Fact!" in the Activity Reservoir.

Least Common Multiple (LCM)

The multiples of 3 are:
3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, ...
The multiples of 5 are:
5, 10, 15, 20, 25, 30, 35, 40, 45, 50, ...
The common multiples of 3 and 5 are:
15, 30, 45, ...
The Least Common Multiple of 3 and 5 is 15



Exercises

- 1. List ten multiples of 2.
List ten multiples of 5.
What is the LCM of 2 and 5? 10
- 2. List ten multiples of 3.
List ten multiples of 7.
What is the LCM of 3 and 7? 21
- 3. List ten multiples of 4.
List ten multiples of 3.
What is the LCM of 4 and 3? 12
- 4. List ten multiples of 5.
List ten multiples of 9.
What is the LCM of 5 and 9? 45
- 5. List twenty multiples of 2. 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40
List ten multiples of 3. 3, 6, 9, 12, 15, 18, 21, 24, 27, 30
List ten multiples of 5. 5, 10, 15, 20, 25, 30, 35, 40, 45, 50
What is the LCM of 2, 3, and 5? 30

List multiples to find the LCM for the following pairs of numbers.

- 6. 4, 5 20
- 7. 2, 7 14
- 8. 4, 6 12
- 9. 6, 9 18
- 10. 4, 10 20
- 11. 5, 8 40
- 12. 9, 12 36
- 13. 6, 10 30
- 14. 8, 6 24
- 15. 6, 15 30

Find the LCM for the following sets of three numbers.

- 16. 2, 4, 5 20
- 17. 3, 4, 6 12
- 18. 4, 5, 10 20
- 19. 6, 5, 10 30
- 20. 3, 6, 8 24

174 Least common multiples

ANSWERS:

- 1. 2, 4, 6, 8, 10, 12, 14, 16, 18, 20
5, 10, 15, 20, 25, 30, 35, 40, 45, 50
- 2. 3, 6, 9, 12, 15, 18, 21, 24, 27, 30
7, 14, 21, 28, 35, 42, 49, 56, 63, 70
- 3. 4, 8, 12, 16, 20, 24, 28, 32, 36, 40
3, 6, 9, 12, 15, 18, 21, 24, 27, 30
- 4. 5, 10, 15, 20, 25, 30, 35, 40, 45, 50
9, 18, 27, 36, 45, 54, 63, 72, 81, 90

×	1	2	3	4	5
1	1	2	3	4	5
2	2	4	6	8	10
3	3	6	9	12	15
4	4	8	12	16	20
5	5	10	15	20	25

Players (from 2 to 4) will need 4 of their own colour of counters. Players

take turns taking a card from the shuffled, face-down deck and using one of their counters to cover a LCM on the grid for the numbers shown on their card (i.e., draw card 4/3; cover any uncovered "12" on the grid). Winner is the first player to place his/her 4 markers in a horizontal, vertical, or diagonal row. Note, when all 4 markers have been placed (but not in a straight row), player may move one as he/she draws each subsequent card.

The Lab Technician

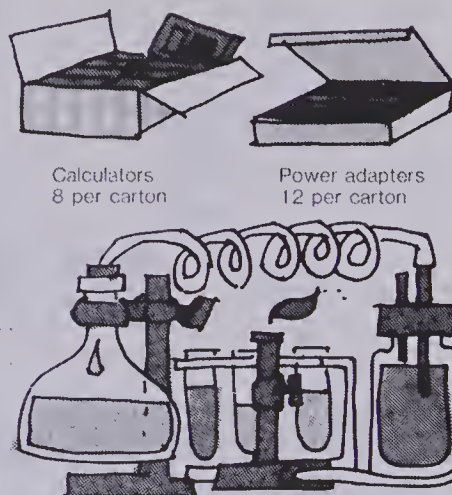
The High-Light Paint Company has a large chemical laboratory where chemists develop and test new paints.

Georgio is a lab technician and one of his jobs is to order supplies for the laboratory. Georgio wants to order calculators and power adapters, and the supplier ships only carton lots

What is the smallest order he could place so that he has an equal number of each?

Solution: The multiples of 8 are 8, 16, 24, 32, 40, ...
The multiples of 12 are 12, 24, 36, 48, ...
The LCM is 24.

He could order 24 of each:
3 cartons of calculators and 2 cartons of adapters.



Exercises

- Georgio wants to order an equal number of flasks and glass tubes. Flasks are shipped 12 per carton. Glass tubes are shipped 20 per carton. How many of each should he order? **60**
- Georgio is ordering Bunsen burners and gas lines. Gas lines are packaged 10 per carton. Bunsen burners are packaged 8 per carton. How many should he order so that he has a gas line for each Bunsen burner? **40**
- Electronic scales are shipped in cartons of 24. AC power packs are shipped in cartons of 30. How many of each must be ordered so that each electronic scale will have an AC power pack? **120**
- ★ Paint mixers are shipped in cartons of 24. Batteries are shipped in cartons of 40. Each paint mixer uses 2 batteries. How many of each should be ordered so that there are 2 batteries for each paint mixer? **120 paint mixers and 240 batteries**

Problems 175

OBJECTIVE

To solve problems involving multiples and least common multiples

PACING

Level A All (discuss)
Level B All
Level C All

VOCABULARY

chemical laboratory, chemists, technician, adapters, flasks, Bunsen burners, AC power packs

MATERIALS

interlocking centimetre cubes (in 2 colours, if available)

RELATED AIDS

HMS — DM41.

BACKGROUND

These problems are real-life problems as most retail stores must order in carton lots to obtain reasonable wholesale prices.

SUGGESTIONS

Initial Activity Present a problem such as:

"Here we have stacks of centimetre cubes.

Some of the stacks contain 6 cubes, some contain 8.

How many of each size stack must you take to have 2 equal quantities of cubes?"

Receive answers from the class, discussing the rationale of each.

Demonstrate on the chalkboard to be sure all can see the process:

Number of "6" stacks	Total	Number of "8" stacks	Total
1	6	1	8
2	12	2	16
3	18	3	24
4	24	4	32

Point out that, when four "6" stacks and three "8" stacks are selected, the total number of cubes is equal — 24.

Repeat using "4" stacks and "5" stacks.

USING THE BOOK

Discuss the problem in the display so that the context of the problem is clear. You may wish to sketch the various cartons on the chalkboard.

The number of calculators he can order is 8, 16, **24**, 32, 40, 48, ...

The number of adapters he can order is 12, **24**, 36, 48, 60, ...

He should order 24 of each — 3 cartons of calculators and 2 cartons of adapters.

Assign the problems.

ACTIVITIES

1. Discuss with the class, items which go together but can be shipped separately. List these pairs, (groups) of things.

2. Using some of the items listed from Activity 1 above, have the children make up some problems for the problem box or centre. You may wish to have the students use the problems on this page as models.

OBJECTIVE

To write powers

PACING

Level A 1-21

Level B 1-23

Level C 4-6, 9-12, 13-24

MATERIALS

wooden cubes for making dice,
playing cards (one through nine)

BACKGROUND

A use for powers of 10 is for expanded notation which follows on page 177.

USING THE BOOK

Using the display at the top of the page, discuss:

(a) What the exponent must mean in each case.

Answer — The exponent states the number of times the base number is multiplied.

Example

$$10^4 = \underset{\uparrow 1}{10} \times \underset{\uparrow 2}{10} \times \underset{\uparrow 3}{10} \times \underset{\uparrow 4}{10} = 10\,000$$

(b) What the parts of the power are.
 10^4 is the 4th power of 10.

10 is called the base of the power.

4 is the exponent.

(c) Have students read the powers and the numbers.

Example

" 10^3 means $10 \times 10 \times 10 = 1000$ "
reads as: "10 to the exponent 3 means 10 times 10 times 10 which equals one thousand."

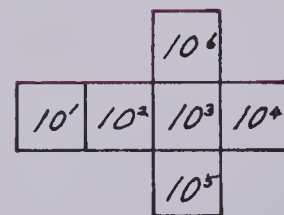
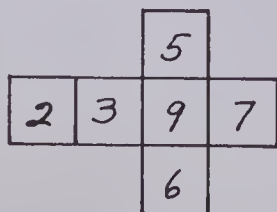
(d) Show how 7×10^2 is expressed as a single numeral.

$$\begin{aligned} 7 \times 10^2 &= 7 \times 10 \times 10 \rightarrow \text{Why? } [10^2 = 10 \times 10] \\ &= 7 \times 100 \rightarrow \text{Why? } [10 \times 10 = 100] \\ &= 700 \rightarrow \text{Why? } [7 \times 100 = 700] \end{aligned}$$

Assign the exercises.

ACTIVITIES

1. Provide pairs of dice with the following faces and have the students play "Powerful Dice".



(a) Roll a pair of dice.

(b) Record the product. $\boxed{7} \boxed{10^3}$
 7×10^3

(c) Express this product as a single numeral.

Example

$$\begin{aligned} 7 \times 10^3 &= 7 \times 1000 \\ &= 7000 \end{aligned}$$

Powerful Tens

Powers

10^1	means
10^2	means
10^3	means
10^4	means
10^5	means
10^6	means

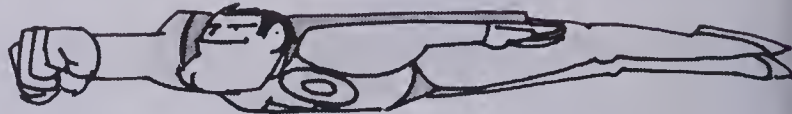
Products

10
10×10
$10 \times 10 \times 10$
$10 \times 10 \times 10 \times 10$
$10 \times 10 \times 10 \times 10 \times 10$
$10 \times 10 \times 10 \times 10 \times 10 \times 10$

Numbers

=	10
=	100
=	1000
=	10 000
=	100 000
=	1 000 000

10^4 is a power of ten.
The 10 is the base.
The 4 is the exponent.



Exercises

Write as a power.

1. $100 = 10^2$

2. $1000 = 10^3$

3. $100\,000 = 10^5$

4. $1\,000\,000 = 10^6$

5. $100\,000\,000 = 10^8$

6. $1\,000\,000\,000 = 10^9$

Write as products of 10.

7. $10^3 = 10 \times 10 \times 10$

8. $10^2 = 10 \times 10$

9. $10^5 = 10 \times 10 \times 10 \times 10 \times 10$

10. $10^6 = 10 \times 10 \times 10 \times 10 \times 10 \times 10$

11. $10^6 = 10 \times 10 \times 10 \times 10 \times 10 \times 10$

12. $10^9 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$

Write as a single number.

13. $10^3 = 10 \times 10 \times 10 = 1000$

14. $10^2 = 10 \times 10 = 100$

15. $10^5 =$

16. $10^8 =$

17. $10^9 =$

Find the products.

18. $3 \times 10^2 = 3 \times 10 \times 10 = 300$

19. $5 \times 10^1 = 5 \times 10 = 50$

20. $7 \times 10^3 = 7 \times 10 \times 10 \times 10 = 7000$

21. $9 \times 10^5 = 9 \times 10 \times 10 \times 10 \times 10 \times 10 = 900\,000$

Write each of these as a single digit times a power of 10.

22. $300 = 3 \times 100 = 3 \times 10^2$

23. $50 = 5 \times 10 = 5 \times 10^1$

24. $40\,000 = 4 \times 10\,000 = 4 \times 10^4$

176 Exponents

ANSWERS:

15. $10 \times 10 \times 10 \times 10 \times 10 = 100\,000$

16. $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 100\,000\,000$

17. $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1\,000\,000\,000$

(d) Complete as many as you can in the time allotted.

2. Have the students use 36 playing cards, ace through nine, and the power die described above.

(a) Select one card and roll the die.

(b) Proceed as in Activity 1.

3. Play "Concentration" as described in the Activity Reservoir. Use card pairs such as:

10^2	100	5×10^1	50
--------	-----	-----------------	----

Expanded Notation

$2 \times 1\,000\,000 = 2 \times 10^6$	$5 \times 100\,000 = 5 \times 10^5$	$3 \times 10\,000 = 3 \times 10^4$	$8 \times 1\,000 = 8 \times 10^3$	$7 \times 100 = 7 \times 10^2$	$6 \times 10 = 6 \times 10^1$	4×1
millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones
2	5	3	8	7	6	4

We say: "two million, five hundred thirty-eight thousand, seven hundred sixty-four."

In expanded notation 2 538 764 is

$$(2 \times 1\,000\,000) + (5 \times 100\,000) + (3 \times 10\,000) + (8 \times 1\,000) + (7 \times 100) + (6 \times 10) + (4 \times 1)$$

$$(2 \times 10^6) + (5 \times 10^5) + (3 \times 10^4) + (8 \times 10^3) + (7 \times 10^2) + (6 \times 10^1) + (4 \times 1)$$

Powers of ten show the place value of the digits.

Exercises

Find the missing numerals or powers of ten.

- $635 = (6 \times 10^{\underline{4}}) + (3 \times 10^{\underline{1}}) + (5 \times \underline{1})$
- $427 = (\underline{} \times 10^2) + (2 \times \underline{}) + (7 \times \underline{})$
- $1289 = (1 \times 10^3) + (\underline{} \times 10^2) + (\underline{} \times \underline{}) + (\underline{} \times 1)$
- $8375 = (\underline{} \times \underline{}) + (3 \times 10^2) + (\underline{} \times \underline{}) + (\underline{} \times \underline{})$
- $3042 = (\underline{} \times 10^3) + (0 \times 10^2) + (\underline{} \times 10^1) + (2 \times \underline{})$
or $(\underline{} \times 10^3) + (\underline{} \times 10^1) + (2 \times \underline{})$
- $208 = (2 \times \underline{}) + (\underline{} \times \underline{})$
- $9003 = (9 \times \underline{}) + (\underline{} \times \underline{})$
- $30\,702 = (3 \times \underline{}) + (7 \times \underline{}) + (2 \times \underline{})$

Write in expanded notation.

- 238
- 519
- 4239
- 7010
- 9005
- 85 027
- 300
- 7000
- 50 000
- 90
- 8 000 000
- 100 000



Expanded notation 177

ANSWERS:

$$238 = (2 \times 100) + (3 \times 10) + (8 \times 1) = (2 \times 10^2) + (3 \times 10^1) + (8 \times 1)$$

$$519 = (5 \times 100) + (1 \times 10) + (9 \times 1) = (5 \times 10^2) + (1 \times 10^1) + (9 \times 1)$$

$$4239 = (4 \times 1000) + (2 \times 100) + (3 \times 10) + (9 \times 1) = (4 \times 10^3) + (2 \times 10^2) + (3 \times 10^1) + (9 \times 1)$$

$$7010 = (7 \times 1000) + (1 \times 10) = (7 \times 10^3) + (1 \times 10^1)$$

$$9005 = (9 \times 1000) + (5 \times 1) = (9 \times 10^3) + (5 \times 1)$$

$$85\,027 = (8 \times 10\,000) + (5 \times 1000) + (2 \times 10) + (7 \times 1) = (8 \times 10^4) + (5 \times 10^3) + (2 \times 10^1) + (7 \times 1)$$

$$300 = (3 \times 100) = (3 \times 10^2)$$

$$7000 = (7 \times 1000) = (7 \times 10^3)$$

$$50\,000 = (5 \times 10\,000) = (5 \times 10^4)$$

$$90 = (9 \times 10) = (9 \times 10^1)$$

$$8\,000\,000 = (8 \times 1\,000\,000) = (8 \times 10^6)$$

$$100\,000 = (1 \times 100\,000) = (1 \times 10^5)$$

ACTIVITIES

1. To review place value with an expanded notation twist, prepare an exercise such as:

Write the value of each underlined digit using expanded notation. The first is done for you.

- 2716 $\rightarrow 7 \times 10^2 = 700$
- 3684 \rightarrow
- 2878 \rightarrow
- 99 226 \rightarrow
- 11 377 \rightarrow

2. Review place value using puzzle cards such as:



3. Play "Expanded Draw"—a card game for 4. Have blank place-value charts for *all* players (millions to ones). Use a regular deck of 52 cards. (Jacks, queens, and kings mean *zero*.)

Game #1

- Deal 4 cards to each player face down.
- Place the cards on the one's, ten's, hundred's, and thousand's places in order as each card is turned.
- Record your number.
- Write your number in expanded notation.

(e) When 5 deals have been completed, add up your 5 numbers. The largest total is the winner.

Game #2

Repeat dealing 6 cards to each player. Remember: Face cards (jacks, queens, kings mean *zero* for this game).

OBJECTIVE

To express numerals in expanded notation

PACING

Level A 1-10
Level B 1-15
Level C 1-20

MATERIALS

a place-value chart showing the powers of tens in the proper places

BACKGROUND

The purpose of this is to recognize the *place value* of each digit in a numeral. Allow students to refer to the place-value chart in the display.

SUGGESTIONS

Initial Activity Review the meanings of various digits using a place-value chart.

Example

7852 is 7 thousands
8 hundreds
5 tens
and 2 ones

or $(7 \times 1000) + (8 \times 100) + (5 \times 10) + (2 \times 1)$.

Replace the multiples of 10 by powers of 10 to get expanded notation:

$$(7 \times 10^3) + (8 \times 10^2) + (5 \times 10^1) + (2 \times 1)$$

Show how

$764 = (7 \times 10^2) + (6 \times 10^1) + (4 \times 1)$
and then how $8764 = (8 \times 10^3) + \dots$,
etc. by uncovering digits on the place-value chart as required.

Explain that zeros in a place means there are "none of these" and therefore that place is skipped over in expanded notation.

Example

30 072 is 3 ten thousands
7 tens
and 2 ones
or $30\,072 = (3 \times 10^4) + (7 \times 10^1) + (2 \times 1)$.

USING THE BOOK

Read through the display at the top of the page together. Emphasize that the number 2 538 764 is actually expressed 5 different ways there: in numerals; in a place-value chart; in words; in expanded notation; in expanded notation using powers.

Complete Exercises 1 and 2 orally before assigning the balance of the exercises to the class.

OBJECTIVE

To evaluate powers which are *squares* or *cubes*

PACING

- Level A All
- Level B All
- Level C All

MATERIALS

centimetre graph paper, centimetre cubes

RELATED AIDS

BFA COMP LAB II — 44.

BACKGROUND

We can show “squares” and “cubes” by physical models. However, for any power above 3, it is not reasonable to construct a model.

SUGGESTIONS

Initial Activity Have students draw “squares” and make “cubes”. In this way, they can get a physical construction for a power.

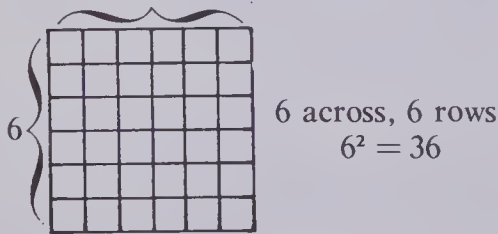
Have students draw all the squares from a 1 cm by 1 cm square to a 10 cm by 10 cm square on graph paper. Label the area of each square.

Have some students make a 2 cm cube, 3 cm cube, and a 4 cm cube, using centimetre interlocking cubes. Have students describe what they did and tell how many cubes they used for each.

USING THE BOOK

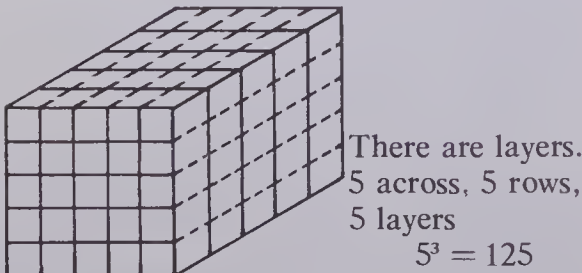
Discuss the squares and cubes shown in the display at the top of the page. Be sure to mention 2 dimensions of squares,

Example 6

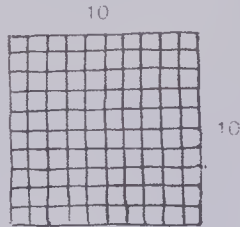


and the 3 dimensionality of cubes.

Example

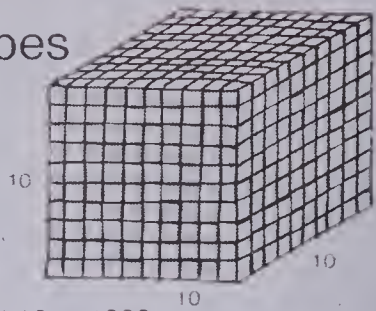
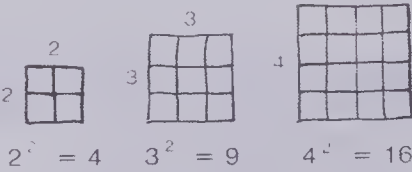


Squares and Cubes



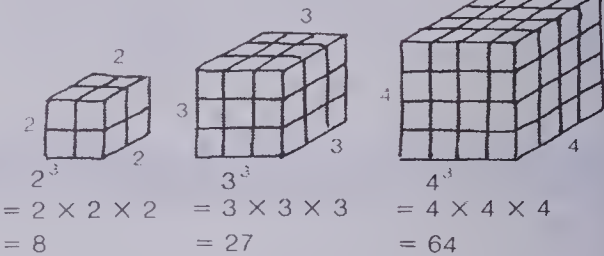
This is “10 squared” or 10^2 which equals 100.

Here are some other squares



This is “10 cubed” or 10^3 which equals $10 \times 10 \times 10$ or 1000

Here are some other cubes.



Exercises Copy and complete these charts.

1.	Power	Meaning	Standard Numeral
	2^2	2×2	4
	3^2	3×3	9
	4^2	4×4	16
	5^2	■ 5x5	■ 25
	6^2	■ 6x6	■ 36
	7^2	■ 7x7	■ 49
	8^2	■ 8x8	■ 64
	9^2	■ 9x9	■ 81
	10^2	■ 10x10	■ 100

2.	Power	Meaning	Standard Numeral
	2^3	$2 \times 2 \times 2$	8
	3^3	$3 \times 3 \times 3$	27
	4^3	$4 \times 4 \times 4$	64
	5^3	■ 5x5x5	■ 125
	6^3	■ 6x6x6	■ 216
	7^3	■ 7x7x7	■ 343
	8^3	■ 8x8x8	■ 512
	9^3	■ 9x9x9	■ 729
	10^3	■ 10x10x10	■ 1000

3. Find the products.
(a) 5×4^2 80 (b) 10×3^2 90 (c) 7×2^3 56 (d) $5^2 \times 2^3$ 200 (e) $3^2 \times 4^2$ 144 (f) $6^2 \times 3^2$ 324

Assign Exercise 1 using the squares drawn on graph paper. The rest of the questions can be solved by multiplying.

For Exercise 3(a):
 $5 \times 4^2 = 5 \times (4 \times 4)$
 $= 5 \times 16$
 $= 80.$

ACTIVITIES

1. Play “Concentration” as described in the Activity Reservoir using cards such as:



2. Have the children make up their own fill-in-the-blank charts as shown. Have them exchange these with classmates.

Power	Meaning	Standard Numeral
3^3		
		25
	4×4	
3×2^2		12
	$4 \times 6 \times 6$	

3. Play “Bingo” as described in the Activity Reservoir. Have the players randomly write numbers on their blank grids from this set: 1; 4; 8; 9; 10; 16; 27; 25; 36; 49; 64; 81; 100; 125; 200; 216; 300; 343; 400; 512; 729; 1000; 10 000; 100 000. Call out these corresponding powers: $1; 2^2; 2^3; 3^2; 10^1; 4^2; 3^3; 5^2; 6^2; 7^2; 8^2$ or $4^3; 9^2; 10^2; 5^3; 2 \times 10^2; 6^3; 3 \times 10^2; 7^3; 4 \times 10^2; 8^3; 9^3; 10^3; 10^4; 10^5$. Provide a chart showing these values (such as the ones made in Exercises 1 and 2 on this pupil page) or, to make the game more challenging, paper and pencil so that the players may perform the calculations.

Squares, Cubes, and Other Powers

5^3 is a **power**.

5 is the **base**.

$$5 \times 5 = 25$$

3 is the **exponent**.

$$25 \times 5 = 125$$

5^3 means $5 \times 5 \times 5$, which equals 125

Exercises

Write as a product of factors.

1. $5^4 = 5 \times 5 \times 5 \times 5$

3. $2^4 = 2 \times 2 \times 2 \times 2$

4. $6^3 = 6 \times 6 \times 6$

2. $7^3 = 7 \times 7 \times 7$

5. $10^3 = 10 \times 10 \times 10$

6. $8^5 = 8 \times 8 \times 8 \times 8 \times 8$

Write as a power.

7. $3 \times 3 \times 3 \times 3 \times 3 = 3^5$

9. $7 \times 7 \times 7 \times 7 \times 7 \times 7 = 7^6$

8. $5 \times 5 \times 5 = 5^3$

10. $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^8$

11. $9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 = 9^7$

12. $12 \times 12 \times 12 \times 12 = 12^4$

Write as a product of factors and solve.

13. $7^2 = 7 \times 7 = 49$

15. $3^3 = 3 \times 3 \times 3 = 27$

17. $2^2 \times 3^2 = 2 \times 2 \times 3 \times 3 = 36$

18. $2^3 \times 4^2 = 2 \times 2 \times 2 \times 4 \times 4 = 128$

19. $3^2 \times 5^2 = 3 \times 3 \times 5 \times 5 = 225$

20. $7^2 \times 3^2 = 7 \times 7 \times 3 \times 3 = 441$

21. $6^2 \times 8^2 = 6 \times 6 \times 8 \times 8 = 2304$

★ Which is greater?

22. 2^6 or 6^2

24. 2^8 or 8^2

23. 2^3 or 3^2

25. 3^3 or 5^3

Products of powers 179

OBJECTIVE

To evaluate powers and products of powers

PACING

Level A 1-18

Level B 1-21

Level C All

RELATED AIDS

HMS — DM42.

BFA COMP LAB II — 44.

CALC. ACTIVITY MASTERS — 75.

BACKGROUND

This is a review and diagnostic exercise for powers.

SUGGESTIONS

Initial Activity Review the meaning of powers, base, and exponents.

Demonstrate the evaluation of these:

(a) $4^3 = 4 \times 4 \times 4 = 64$

(b) $2^2 \times 4^2 = (2 \times 2) \times (4 \times 4) = 4 \times 16 = 64$

USING THE BOOK

Assign the exercises as a review and diagnostic exercise.

ACTIVITIES

1. See "Square It" as described in the Activity Reservoir. Note variation (c).

2. Prepare and distribute a Power Puzzle such as the following. (Answers are given.)

a		b		c	d	e
1	2	5		1	2	1
		1			f	0
		g			h	0
		2	5		6	
i	j			k		
2	7			4		
	l	0	0	0		m
	2					8
n	4	9		0		0
		o	8	0	0	0

ACROSS

- a 5^3
c 11^2
f 10^1
g 5^2
h 6×10^1
i 3^3
l 2×10^3
n 7^2
o 8×10^4

DOWN

- b 8^3
d 6^3
e 10^2
j 9^3
k 4×10^3
m $2^3 \times 10^2$

3. If you have not already done so, see the suggested Activities on page 178.

EXTRA PRACTICE

1. Write as a product of factors.

(a) 7^4 (b) 3^5 (c) 9^2 (d) 10^6

2. Write as a power.

(a) $4 \times 4 \times 4 \times 4 \times 4$

(b) $6 \times 6 \times 6$

(c) $8 \times 8 \times 8 \times 8$

(d) 11×11

OBJECTIVE

To present problem solving involving visualization

PACING

- Level A 1-3
- Level B 3-5
- Level C 3, 5-7

MATERIALS

interlocking centimetre cubes (4 colours)

BACKGROUND

The teacher may want students to select any 2 problems to solve, using centimetre cubes.

SUGGESTIONS

Initial Activity Using coloured centimetre cubes and this legend, build a model of the $3 \times 3 \times 3$ cube:
3 painted surfaces — yellow
2 painted surfaces — blue
1 painted surface — red
0 painted surfaces — green.

Use the model to explain the “painted faces” problem.

Alternative (if coloured cubes are unavailable):

A committee of 2 students build a $3 \times 3 \times 3$ cuboid using centimetre cubes; suspend this cuboid and spray paint it yellow. Do this two days prior to the time it is needed. Use this cuboid to demonstrate the problem.

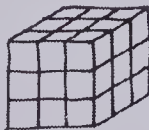
USING THE BOOK

Assign the problems, allowing the students to use coloured centimetre cubes to assist them with the visualization.

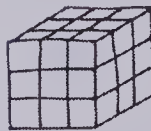
Painted Prisms

This large cube is made of 27 small cubes. The outside of the whole shape is painted yellow.

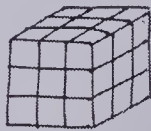
8 cubes have paint on 3 surfaces.



12 cubes have paint on only 2 surfaces.



6 cubes have paint on only 1 surface

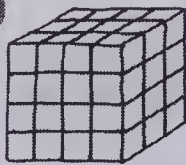


1 cube has no painted surfaces. (It is hidden in the centre of the large cube.)

Painted Surfaces	Number of Cubes
3	8
2	12
1	6
0	1
Total	27

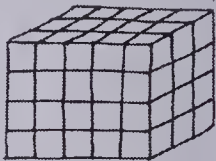
Exercises

Determine the painted surfaces to complete the charts. Calculate the volume of each cube.



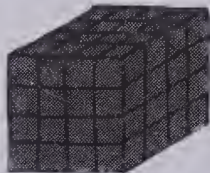
Painted Surfaces	Number of Cubes
3	8
2	20
1	16
0	4
Total	48

2.



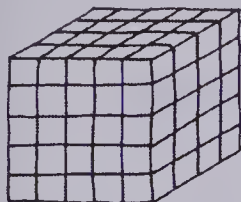
Painted Surfaces	Number of Cubes
3	8
2	24
1	22
0	6
Total	60

3.



Painted Surfaces	Number of Cubes
3	8
2	24
1	24
0	8
Total	64

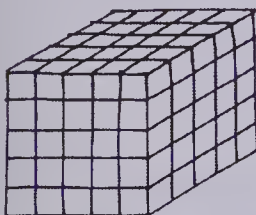
4.



Painted Surfaces	Number of Cubes
3	
2	
1	
0	
Total	

8
32
42
18
100

5.



Painted Surfaces	Number of Cubes
3	
2	
1	
0	
Total	

8
36
54
27
125

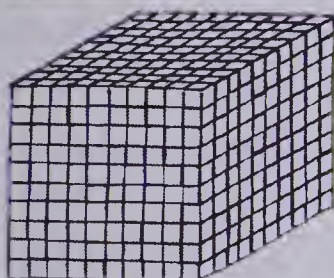
6.



Painted Surfaces	Number of Cubes
3	
2	
1	
0	
Total	

8
48
96
64
216

★7.



Painted Surfaces	Number of Cubes
3	
2	
1	
0	
Total	

8
96
384
512
1000



ACTIVITIES

1. This page can be used as an activity if students are provided with centimetre cubes.

2. Have the students draw a cuboid of their own design, and determine how many faces have paint on 3 faces, 2 faces, 1 face, and none.

3. If you have not already done so, see the "Polycube" idea listed as Activity 3 on page 49.

OBJECTIVE

To evaluate achievement of the chapter objectives

PACING

Level A All
Level B All
Level C All

RELATED AIDS

HMS — DM43.

USING THE BOOK

Each student should do this test independently under supervision. Assistance should be given only when the instructions are not understood. After the work has been corrected, you should provide appropriate remedial work. You may wish to reteach if a large number of students had difficulty with a particular topic or concept.

The following chart will help in this regard. The specific objectives are listed in the Chapter Overview (see page 156).

An alternate Chapter Test can be found in the Holt Mathematics System Duplicating Masters available for use with this grade level.

Test Item	Objective	Text Page Number
1, 2	A	158, 162
3, 4	B	166
5	C	170
6	D	171
7	E	174
8, 9	F	176-179
10, 11	G	175, 179-181

Chapter Test

- For each of the following, list all the possible *multiplication and division facts*.
(a) 10 (b) 32 (c) 54 (d) 60
- List the set of all factors for each of the following numbers.
(a) 12 *1, 2, 3, 4, 6, 12* (b) 20 *1, 2, 4, 5, 10, 20* (c) 36 *1, 2, 3, 4, 6, 9, 12, 18, 36* (d) 50 *1, 2, 5, 10, 25, 50*
- List the first 10 prime numbers greater than 1. *2, 3, 5, 7, 11, 13, 17, 19, 23, 29*
- List the first 10 composite numbers greater than 1. *4, 6, 8, 9, 10, 12, 14, 15, 16, 18*
- Find the prime factorization (product of prime factors) for each number.
(a) 12 *$12 = 2 \times 2 \times 3$* (b) 30 *$30 = 2 \times 3 \times 5$* (c) 42 *$42 = 2 \times 3 \times 7$* (d) 54 *$54 = 2 \times 3 \times 3 \times 3$* (e) 90 *$90 = 2 \times 3 \times 3 \times 5$*
- Find the greatest common factor for the following sets of numerals.
(a) 10, 35 *5* (b) 8, 20 *4* (c) 12, 42 *6* (d) 10, 32 *2*
- Find the least common multiple for the following sets of numerals.
(a) 6 and 10 *30* (b) 15 and 20 *60* (c) 30 and 40 *120* (d) 30 and 42 *210*
- Write as a single numeral.
(a) $10^2 = \blacksquare$ *100* (b) $2^3 = \blacksquare$ *8* (c) $3^4 = \blacksquare$ *81* (d) $3 \times 10^3 = \blacksquare$ *3000*
- Write each of the following numerals in expanded notation.
(a) 427 (b) 3795 (c) 8009
- Which is larger?
(a) 2^5 or 5^2 (b) 4^2 or 2^4 *Neither*
- Loose-leaf notebooks are shipped in cartons of 12.
Loose-leaf refills are shipped in cartons of 30.
How many of each should be ordered so that notebooks can be sold with 1 refill in each one? *60*

182 Chapter 6 test

ANSWERS:

- (a) $1 \times 10 = 10$, $10 \div 10 = 1$, $10 \div 1 = 10$; $2 \times 5 = 10$, $10 \div 5 = 2$, $10 \div 2 = 5$
(b) $1 \times 32 = 32$, $32 \div 32 = 1$, $32 \div 1 = 32$; $2 \times 16 = 32$, $32 \div 16 = 2$, $32 \div 2 = 16$; $4 \times 8 = 32$, $32 \div 8 = 4$
(c) $1 \times 54 = 54$, $54 \div 54 = 1$, $54 \div 1 = 54$; $2 \times 27 = 54$, $54 \div 27 = 2$, $54 \div 2 = 27$; $3 \times 18 = 54$, $54 \div 18 = 3$, $54 \div 3 = 18$; $6 \times 9 = 54$, $54 \div 9 = 6$, $54 \div 6 = 9$ (d) $1 \times 60 = 60$, $60 \div 60 = 1$, $60 \div 1 = 60$; $2 \times 30 = 60$, $60 \div 30 = 2$, $60 \div 2 = 30$; $3 \times 20 = 60$, $60 \div 20 = 3$, $60 \div 3 = 20$; $4 \times 15 = 60$, $60 \div 15 = 4$, $60 \div 4 = 15$; $5 \times 12 = 60$, $60 \div 12 = 5$, $60 \div 5 = 12$; $6 \times 10 = 60$, $60 \div 10 = 6$, $60 \div 6 = 10$
- (a) $427 = (4 \times 100) + (2 \times 10) + (7 \times 1) = (4 \times 10^2) + (2 \times 10^1) + (7 \times 1)$
(b) $3795 = (3 \times 1000) + (7 \times 100) + (9 \times 10) + (5 \times 1) = (3 \times 10^3) + (7 \times 10^2) + (9 \times 10^1) + (5 \times 1)$
(c) $8009 = (8 \times 1000) + (9 \times 1) = (8 \times 10^3) + (9 \times 1)$

Cumulative Review

Perform the individual operations

1. $1705 + 378 + 3927 + 53$ **6063**
2. $3579 - 982$ **2597**
3. 389×62 **24 118**
4. $2983 \div 21$ **142 R1**
5. $\$27.28 + \$107.92 + \$5.75$ **\$140.95**
6. $\$182.15 - \99.99 **\$ 82.16**
7. 450×3.8 **1710**
8. 17.8×0.3 **5.34**
9. $38.9 \div 0.01$ **3890**
10. $42.8 \div 0.2$ **214**
11. $\$30.08 - 12$ **\$ 2.51**
12. $\$72.50 - 0.5$ **\$145.00**
13. 7×10^3 **7000**
14. $5^2 \times 10^2$ **2500**

Express the following as decimals.

15. $3\frac{7}{10}$ **3.7**
16. $15\frac{32}{100}$ **15.32**
17. $2\frac{17}{1000}$ **2.017**
18. $\frac{6}{1000}$ **0.006**

Write the following in expanded notation.

19. 78 925
20. 800 723

Write the following as a product of prime factors

21. 18
22. 120
23. 84
24. 105

Identify which are divisible by 3 and which are divisible by 4 (or by both 3 and 4).

25. 66 **3**
26. 88 **4**
27. 321 **3**
28. 156 **3 and 4**

29. 920 students attended the area music night. Tickets cost \$1.75. How much money was collected for tickets? **\$1610.00**
30. Sergio saved \$2.30 per week from his allowance. For how many weeks would he have to save to purchase a mini-computer game that costs \$39.10, including sales tax? **17 weeks**

Chapters 1-6 cumulative review 183

OBJECTIVE

To review and test selected concepts and skills previously covered

PACING

Level A All
Level B All
Level C All

USING THE BOOK

This page may be used for diagnostic and remedial as well as review purposes. Students should check their work, correct any errors, and review the pages that contain any problems of the type they missed. Some students can do this on their own while others may need help. If a large number of students have a particular problem incorrect, you may want to reteach that topic to the groups, then assign a duplicated worksheet to reinforce that topic or, refer to an appropriate skill card in the BFA Computational Skills Kit II or BFA Problem Solving Kit II.

Test Item	Text Page Number
1, 2	4, 5
3	65
4	100
5	6
6	8
7	72
8	75
9	133
10	129
11	103
12	132
13, 14	179
15-18	2
19, 20	177
21-24	170
25-28	160
29	74
30	134

ANSWERS:

1. $(7 \times 10\,000) + (8 \times 1000) + (9 \times 100) + (2 \times 10) + (5 \times 1) = (7 \times 10^4) + (8 \times 10^3) + (9 \times 10^2) + (2 \times 10^1) + (5 \times 1)$
2. $(8 \times 100\,000) + (7 \times 100) + (2 \times 10) + (3 \times 1) = (8 \times 10^5) + (7 \times 10^2) + (2 \times 10^1) + (3 \times 1)$
3. $18 = 2 \times 3 \times 3$ 22. $120 = 2 \times 2 \times 2 \times 3 \times 5$ 23. $84 = 2 \times 2 \times 3 \times 7$
4. $105 = 3 \times 5 \times 7$

CHAPTER 7 OVERVIEW

This chapter reviews fraction concepts and uses the fraction line for addition and subtraction of like fractions.

The four operations — addition, subtraction, multiplication, and division — are studied using like and unlike fractions, mixed numerals, whole numbers, and combinations.

Equivalence is stressed throughout in determining least common denominators; lowest terms fractions; and decimal equivalents, with terminating and repeating quotients.

Fraction patterns and problem solving are also presented as a challenge for the students.

OBJECTIVES

- A To add and subtract both like and unlike fractions; to investigate the commutative and associative properties of addition of fractions
- B To generate equivalent fractions and decimal equivalents; to reduce fractions in lowest terms
- C To find common denominators for pairs of unlike fractions so that they may be compared, added, and subtracted
- D To add and subtract mixed numerals, with and without regrouping
- E To multiply and divide combinations of fractions, whole numbers, and mixed numbers
- F To create decimal equivalents using division; either terminating or repeating decimals
- G Solve equations involving fractions using inspection and inverse operations
- H To solve word problems

BACKGROUND

1. Both the numerator and the denominator of a fraction can be multiplied by the same amount to create equivalent fractions because $\frac{3}{3} = 1$, $\frac{5}{5} = 1$, $\frac{7}{7} = 1$, $\frac{a}{a} = 1$, and 1 is the identity element for multiplication. This means that the product of any number and one is the number itself.

Example

$$\frac{7}{10} \times \boxed{\frac{3}{3}} = \frac{21}{30}$$

$$\text{or } \frac{7 \times \boxed{3}}{10 \times \boxed{3}} = \frac{21}{30}$$

2. When adding fractions with unlike denominators, the least common denominator is either:

- (a) the largest denominator; or
- (b) a multiple of both denominators; or
- (c) the product of the denominators.

Check in the *above order*.

3. When determining reciprocals, encourage students to record answers on a chart. Otherwise statements such as:

$$\frac{1}{2} = \frac{2}{1}, \frac{3}{4} = \frac{4}{3} \text{ abound.}$$

These are incorrect and contain many misconceptions.

MATERIALS

graph paper
fractional number lines in halves, thirds, quarters, fifths, sixths, eighths, and tenths
blank playing cards, Bristol board
clear acetate for making transparencies (see pages 188, 190, 194, 197, 198, 210)
cubes for making dice
decimal place-value chart
centimetre cubes or bingo chips
several 4-function calculators (optional)

CAREER AWARENESS

Greenhouse Manager [202]

The manager of a greenhouse may be responsible for the growth of flowers, vegetables, or seedlings.

Greenhouses allow plants to grow during the colder winter season in Canada. If the sun is shining, the solar energy is usually sufficient to keep a greenhouse warm during the cold winter days. Auxiliary heating systems are required for nights and dull days. The greenhouse manager must monitor the temperatures and keep them within the acceptable growing range.

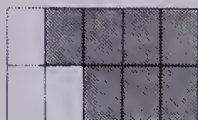
The manager must be aware of the growth needs of numerous plants, i.e., their water, soil, light and temperature requirements. Besides this, the manager requires acute business sense — how to organize manpower and available space to have the popular and most requested plants, shrubs, and saplings available when they are required.

Zoologist [214]

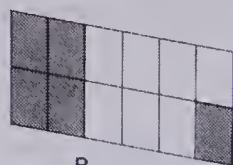
A zoologist is a highly trained specialist interested in the study of animals — their history (and future, if any!), physiology, classification, geographic distribution, habits, and behaviour. Zoologists study animals not just in isolation but in relation to their environment, that is — how animals affect and are effected by it.

Zoologists work with universities, zoological foundations, and governments to learn more about the animal kingdom. Many of their studies are presented in magazines (*National Geographic*, etc.) and in a wide variety of television shows.

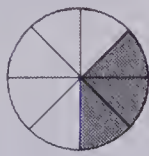
Fractions



A



B



C

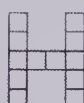
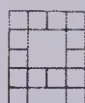
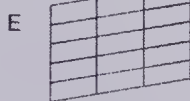
Shape	Number of Equal Parts (denominator)	Shaded Parts (numerator)	Fraction Shaded
A	10	7	$\frac{7}{10}$
B	12	5	$\frac{5}{12}$
C	8	3	$\frac{3}{8}$

The **numerator** states the number of equal parts selected

The **denominator** states the number of equal parts in the whole (or group).

Exercises

Copy and complete the chart.



Shape	Number of Equal Parts	Fraction Shaded	Fraction White
D			
E			
F			
M			
A			
T			
H			

$\frac{2}{5}, \frac{3}{5}, \frac{5}{5}$
 $\frac{1}{15}, \frac{2}{15}, \frac{8}{15}$
 $\frac{6}{11}, \frac{5}{11}, \frac{1}{11}$
 $\frac{8}{15}, \frac{7}{15}, \frac{1}{15}$
 $\frac{7}{14}, \frac{1}{14}, \frac{1}{14}$
 $\frac{5}{9}, \frac{4}{9}, \frac{1}{9}$
 $\frac{8}{12}, \frac{4}{12}, \frac{1}{12}$

Concept of a fraction 185

OBJECTIVE

To review the concept of a fraction

PACING

Level A All

Level B All

Level C All

BACKGROUND

The denominator denotes the total number of equal parts. Included in this total are the selected parts (numerator). Be watchful for those students who may have some difficulty with regard to this inclusion.

SUGGESTIONS

Initial Activity By referring to the grid designs in the display at the top of the page, determine:

- The number of *equal* parts in each.
- The number shaded in each.
- The fraction shaded.
- The number not shaded.
- The fraction not shaded.

Review the definition for numerator and denominator.

USING THE BOOK

Assign the exercises or discuss this exercise orally. You may wish to duplicate a blank chart either on paper for dispersal or on the chalkboard for discussion of the exercise.

ACTIVITIES

1. Have the children make Fraction Patterns. Provide materials and instructions such as:

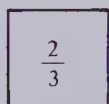
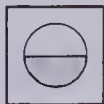
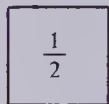
- Using square or triangular grids, draw some designs or figures.
- Divide the interior regions into equal-sized parts.
- Shade the design in a pleasing way.
- Record the "Fraction Shaded" and "Fraction Not Shaded".

2. Have the children play "Black and Red".

- Select some playing cards from a shuffled deck.

- Count the cards.
- What fraction are black cards? (i.e., clubs and spades)
- What fraction are red cards? (i.e., hearts and diamonds)
- Record this information and compare with other classmates.

3. Play "Concentration" as described in the Activity Reservoir. Use cards such as:



OBJECTIVE

To add and subtract using a fraction line

PACING

Level A 1-20

Level B 1-22

Level C All

MATERIALS

a demonstration-size *tenths* fraction line, paper plates

RELATED AIDS

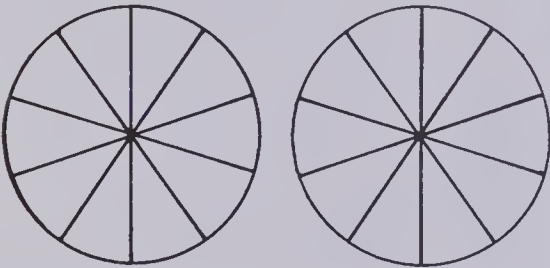
HMS — DM44.

BACKGROUND

The fraction line is a semiconcrete aid only. The concrete aids shown in the Initial Activity may well be necessary.

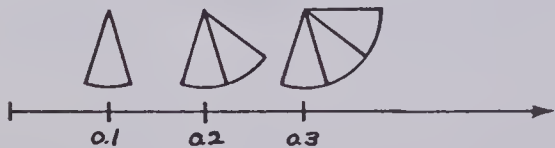
SUGGESTIONS

Initial Activity Prepare this set of aids to relate to the students' "Fraction" background:



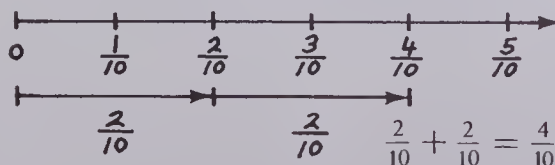
While cutting the pie plates into pieces, discuss: (a) the equality of size of the pieces; (b) the fact that each piece is $\frac{1}{10}$ or 0.1 of a whole plate; (c) that ten pieces can be written as $\frac{10}{10}$, 1.0, or "one whole"; and that (d) for example, fourteen pieces can be shown by "1.4", " $1\frac{4}{10}$ ", or "1.4".

Indicate how the sections match points on a tenths number line:

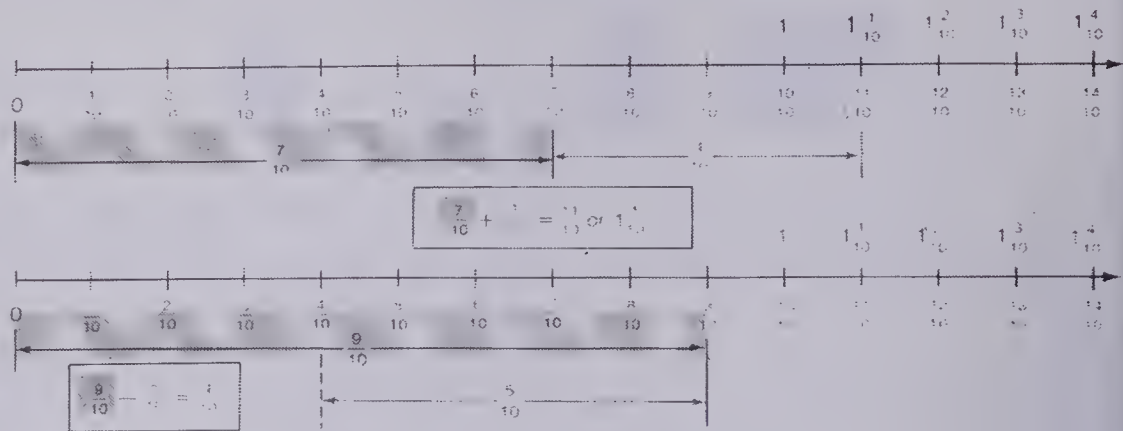


Show students a large *tenths* number line and have them count by *tenths* to 2.

Demonstrate how you could show $\frac{2}{10} + \frac{2}{10}$ with sections, then on the fraction number line.



Number Lines and Fractions



Like fractions have the same denominators $\frac{7}{10}$ and $\frac{4}{10}$ are like fractions.

Exercises

Add

1. $\frac{3}{10} + \frac{4}{10} = \frac{7}{10}$
2. $\frac{5}{10} + \frac{3}{10} = \frac{8}{10}$
3. $\frac{7}{10} + \frac{3}{10} = \frac{10}{10}$ or 1
4. $\frac{7}{10} + \frac{5}{10} = \frac{12}{10}$ or $1\frac{2}{10}$
5. $\frac{4}{10} + \frac{6}{10} = \frac{10}{10}$ or $1\frac{5}{10}$
6. $\frac{2}{10} + \frac{9}{10} = \frac{11}{10}$ or $1\frac{1}{10}$
7. $\frac{3}{10} + \frac{4}{10} = \frac{7}{10}$
8. $\frac{8}{10} + \frac{5}{10} = \frac{13}{10}$ or $1\frac{3}{10}$

Subtract

9. $\frac{3}{10} - \frac{2}{10} = \frac{1}{10}$
10. $\frac{9}{10} - \frac{5}{10} = \frac{4}{10}$
11. $\frac{11}{10} - \frac{4}{10} = \frac{7}{10}$
12. $\frac{12}{10} - \frac{5}{10} = \frac{7}{10}$
13. $\frac{13}{10} - \frac{7}{10} = \frac{6}{10}$
14. $\frac{10}{10} - \frac{3}{10} = \frac{7}{10}$

Calculate.

15. $1\frac{1}{10} - \frac{5}{10} = \frac{6}{10}$
16. $1\frac{2}{10} - \frac{4}{10} = \frac{8}{10}$
17. $1\frac{3}{10} - \frac{7}{10} = \frac{5}{10}$
18. $1\frac{3}{10} - \frac{9}{10} = \frac{4}{10}$
19. $1\frac{4}{10} - \frac{8}{10} = \frac{6}{10}$
20. $1\frac{4}{10} - 1 = \frac{4}{10}$
- ★ 21. $\frac{7}{10} - \frac{4}{10} = \frac{3}{10}$
- ★ 22. $\frac{5}{10} - \frac{2}{10} = \frac{3}{10}$
- ★ 23. $\frac{9}{10} - \frac{1}{10} - \frac{2}{10} - \frac{3}{10} = \frac{3}{10}$
- ★ 24. $\frac{3}{10} - \frac{3}{10} + \frac{1}{10} - \frac{5}{10} + \frac{2}{10} = \frac{4}{10}$

186 Number Lines (tenths)

Show other symmetrical questions such as: $\frac{1}{10} + \frac{1}{10}$, $\frac{3}{10} + \frac{3}{10}$, $\frac{4}{10} + \frac{4}{10}$, ...

USING THE BOOK

Demonstrate how the fractional number line is used to illustrate $\frac{7}{10} + \frac{4}{10} = \frac{11}{10}$ or $1\frac{1}{10}$ and how $\frac{9}{10} - \frac{5}{10} = \frac{4}{10}$.

Assign the exercises. The students are encouraged to use the number line to assist in finding the answers. Students can record the question and answers when done, e.g.,

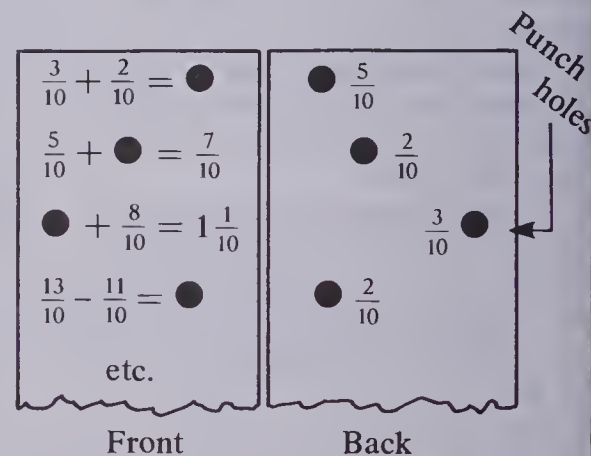
$$\frac{3}{10} + \frac{4}{10} = \frac{7}{10}$$

ACTIVITIES

1. Have those students who have difficulty use the sections of paper plates (see Initial Activity) to add and subtract tenths. Have them write the appropriate fraction number sentences.

2. To provide practice in addition and subtraction of fractions and finding missing addends, minuends, subtrahends, have the children prepare "Punch Cards" as shown. Use Bristol board cut into strips about

4.5 cm wide to allow a standard hole punch to reach centre when necessary. As indicated, answers are on the back, making the activity self-checking.



Front


Back

3. Provide an activity card such as:

- (a) Using a *tenths* number line, create 3 questions that can be answered.
- (b) Check your answers.
- (c) Trade questions with another classmate who is finished.

Like Fractions

Add.



$$\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$$

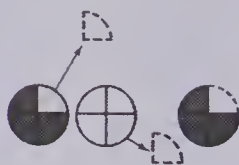
$$\begin{array}{r} 7 \\ 8 \\ + 2 \\ 8 \\ \hline 9 \\ 8 \end{array} \text{ or } 1 \frac{1}{8}$$

Think
 $\frac{9}{8} = \frac{8}{8} + \frac{1}{8}$

Improper fractions have a numerator greater than the denominator:

$$\frac{7}{4} \quad \frac{11}{6} \quad \frac{9}{8} \quad \frac{13}{10}$$

Subtract.



$$\frac{5}{4} - \frac{2}{4} = \frac{3}{4}$$

Proper fractions have a numerator less than the denominator.

$$\frac{3}{4} \quad \frac{5}{6} \quad \frac{7}{8} \quad \frac{9}{10}$$

$$\begin{array}{r} 5 \\ 6 \\ - 1 \\ 6 \\ \hline 4 \\ 6 \end{array}$$

Exercises

Add.

1. $\frac{1}{2} + \frac{1}{2} = \frac{2}{2} \text{ or } 1$ 2. $\frac{1}{3} + \frac{2}{3} = \frac{3}{3} \text{ or } 1$ 3. $\frac{1}{4} + \frac{3}{4} = \frac{4}{4}$ 4. $\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$ 5. $\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$

6. $\frac{3}{8} + \frac{4}{8} = \frac{7}{8}$ 7. $\frac{5}{8} + \frac{1}{8} = \frac{6}{8}$ 8. $\frac{3}{4} + \frac{1}{4} = \frac{4}{4} \text{ or } 1$ 9. $\frac{3}{5} + \frac{2}{5} = \frac{5}{5} \text{ or } 1$ 10. $\frac{4}{6} + \frac{2}{6} = \frac{6}{6}$

11. $\frac{5}{6} + \frac{3}{6} = \frac{8}{6} \text{ or } 1 \frac{2}{6}$ 12. $\frac{7}{8} + \frac{6}{8} = \frac{13}{8} \text{ or } 1 \frac{5}{8}$ 13. $\frac{5}{8} + \frac{3}{8} = \frac{8}{8} \text{ or } 1$ 14. $\frac{6}{10} + \frac{7}{10} = \frac{13}{10} \text{ or } 1 \frac{3}{10}$ 15. $\frac{4}{5} + \frac{4}{5} = \frac{8}{5} \text{ or } 1 \frac{3}{5}$

Subtract.

16. $\frac{2}{3} - \frac{1}{3} = \frac{1}{3}$ 17. $\frac{3}{4} - \frac{1}{4} = \frac{2}{4}$ 18. $\frac{4}{5} - \frac{1}{5} = \frac{3}{5}$ 19. $\frac{5}{6} - \frac{2}{6} = \frac{3}{6}$ 20. $\frac{7}{8} - \frac{2}{8} = \frac{5}{8}$

21. $\frac{3}{4} - \frac{1}{4} = \frac{2}{4} \text{ or } 1$ 22. $\frac{5}{8} - \frac{2}{8} = \frac{3}{8}$ 23. $\frac{7}{8} - \frac{4}{8} = \frac{3}{8}$ 24. $\frac{8}{10} - \frac{5}{10} = \frac{3}{10}$ 25. $\frac{10}{10} - \frac{3}{10} = \frac{7}{10}$

26. $\frac{7}{10} - \frac{1}{10} = \frac{6}{10}$ 27. $\frac{5}{4} - \frac{3}{4} = \frac{2}{4}$ 28. $\frac{7}{6} - \frac{4}{6} = \frac{3}{6}$ 29. $\frac{13}{10} - \frac{8}{10} = \frac{5}{10}$ 30. $\frac{13}{10} - \frac{7}{10} = \frac{6}{10}$

Adding and subtracting like fractions 187

OBJECTIVE

To add and subtract like fractions

PACING

Level A All
 Level B All
 Level C All

MATERIALS

fraction number lines showing halves, thirds, quarters, fifths, sixths, eighths, and tenths; paper plates cut into and labelled as above

RELATED AIDS

HMS — DM44.

BACKGROUND

Counting by fractions increases the awareness of sequence and relative position.

SUGGESTIONS

Initial Activity Have the students use the paper-plate sections to add and subtract fractions. When they have mastered this, have them use the halves fraction line to count by halves:

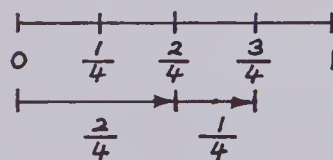
first — $\frac{1}{2}, \frac{2}{2}, \frac{3}{2}, \frac{4}{2}, \frac{5}{2}, \dots$

then — $\frac{1}{2}, 1, 1 \frac{1}{2}, 2, 2 \frac{1}{2}, 3, \dots$

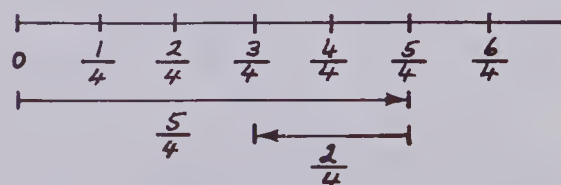
Repeat the (fraction) counting using thirds, quarters, fifths, and tenths.

USING THE BOOK

Using the quarters fraction line show $\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$.



Also show $\frac{5}{4} - \frac{2}{4} = \frac{3}{4}$.



Discuss proper and improper fractions. Emphasize that all the improper fractions are greater than 1 whole.

(Continued on page 193)

ACTIVITIES

1. Have the children play "Fraction Rummy" (2 players). The object of the game is to form sets of 2, 3, or 4 cards which have a fractional sum which is equivalent to a whole number.

Example

$$\frac{1}{5} + \frac{7}{5} + \frac{2}{5} = \frac{10}{5} \text{ or } 2$$

2 is a whole number.

(a) Make 3 cards for each of:

$$\frac{1}{5}, \frac{2}{5}, \dots, \frac{8}{5}, \frac{9}{5}$$

(b) Each player is dealt 3 cards. Rest of the cards are face down in a pile.

(c) Players take 1 card from the pile alternately and try to make whole number sets.

(d) No discards are allowed.

(e) Winner is the first player to use all cards.

Variations: Play as above using:

(a) 3 cards of each sixth from

$$\frac{1}{6}, \frac{2}{6}, \dots, \frac{11}{6}$$

(b) 3 cards of each eighth from

$$\frac{1}{8}, \frac{2}{8}, \dots, \frac{13}{8}$$

(c) 3 cards of each tenth from $\frac{1}{10}$ to $\frac{13}{10}$.

Note:

(i) Students can make up the cards for the games.

(ii) Some rules may have to be classified during play.

2. Have the students use the paper-plate sections to add like fractions. They may add more than two fractions. They should write their number sentences. Repeat for subtraction.

3. Have the children use graph paper to prepare pictorial representations of fraction cards used in "Fraction Rummy" (see Activity 1). These cards can be used together to play "Concentration". The general rules for "Concentration" may be found in the Activity Reservoir.

OBJECTIVE

To create equivalent fractions

PACING

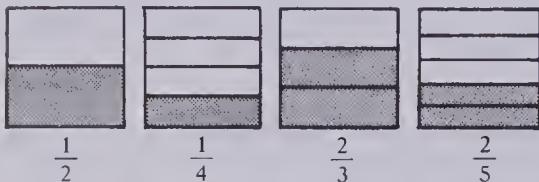
Level A 1-15

Level B 1-15

Level C 1-16

MATERIALS

1 of each on clear acetate. All squares are 10 cm by 10 cm.



and



RELATED AIDS

HMS — DM45.

BFA COMP LAB II — 60-62.

SUGGESTIONS

Initial Activity Use the shaded $\frac{1}{2}$ square on the overhead projector as a base transparency.

Superimpose the halves to show:

$$\frac{1}{2} = \frac{1 \times 2}{2 \times 2} = \frac{2}{4}$$

Superimpose the thirds to show:

$$\frac{1}{2} = \frac{3}{6}$$

Superimpose the quarters to show:

$$\frac{1}{2} = \frac{4}{8}$$

Superimpose the fifths to show:

$$\frac{1}{2} = \frac{5}{10}$$

Using the shaded $\frac{1}{4}$ and the clear halves, thirds, quarters, and fifths show:

$$\frac{1}{4} = \frac{2}{8} = \frac{3}{12} = \frac{4}{16} \text{ and } \frac{5}{20}$$

USING THE BOOK

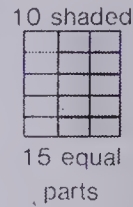
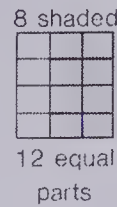
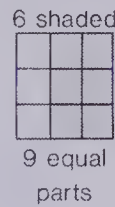
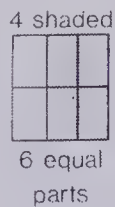
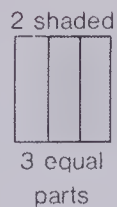
Using the shaded $\frac{2}{3}$ demonstrate each of the equivalent fractions shown in the display at the top of the pupil page.

Example

$$\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \frac{10}{15}$$

Discuss the fact that when showing this numerically, both the numerator

Equivalent Fractions



$$\frac{2 \times 2}{3 \times 2} = \frac{4}{6}$$

$$\frac{2 \times 3}{3 \times 3} = \frac{6}{9}$$

$$\frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

$$\frac{2 \times 5}{3 \times 5} = \frac{10}{15}$$

Exercises

Copy and complete to make equivalent fractions.

1. $\frac{3 \times 2}{4 \times 2} = \frac{6}{8}$

2. $\frac{3 \times 3}{4 \times 3} = \frac{9}{12}$

3. $\frac{3 \times 4}{4 \times 4} = \frac{12}{16}$

4. $\frac{4 \times 2}{5 \times 2} = \frac{8}{10}$

5. $\frac{4 \times 3}{5 \times 3} = \frac{12}{15}$

6. $\frac{4 \times 4}{5 \times 4} = \frac{16}{20}$

7. $\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$

8. $\frac{1}{4} = \frac{1 \times 5}{4 \times 5} = \frac{5}{20}$

9. $\frac{3}{8} = \frac{3 \times 3}{8 \times 3} = \frac{9}{24}$

10. $\frac{3}{10} = \frac{3 \times 6}{10 \times 6} = \frac{18}{60}$

11. $\frac{5}{8} = \frac{5 \times 4}{8 \times 4} = \frac{20}{32}$

12. $\frac{7}{8} = \frac{7 \times 5}{8 \times 5} = \frac{35}{40}$

13. $\frac{3}{2} = \frac{3 \times 6}{2 \times 6} = \frac{18}{12}$

14. $\frac{4}{3} = \frac{4 \times 7}{3 \times 7} = \frac{28}{21}$

15. $\frac{13}{10} = \frac{13 \times 2}{10 \times 2} = \frac{26}{20}$

16. Write 3 equivalent fractions for each.

(a) $\frac{3}{5}, \frac{6}{10}, \frac{9}{15}, \frac{12}{20}$

(b) $\frac{1}{6}, \frac{2}{12}, \frac{3}{18}, \frac{4}{24}$

(c) $\frac{7}{10}, \frac{14}{20}, \frac{21}{30}, \frac{28}{40}$

(d) $\frac{11}{12}, \frac{22}{24}, \frac{33}{36}, \frac{44}{48}$

188 Equivalent fractions

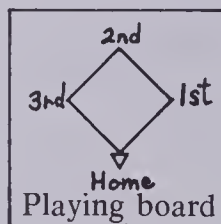
and denominator are multiplied by the same amount.

$$\frac{2 \times 2}{3 \times 2} = \frac{4}{6}; \frac{2 \times 3}{3 \times 3} = \frac{6}{9}; \frac{2 \times 4}{3 \times 4} = \frac{8}{12}; \text{etc.}$$

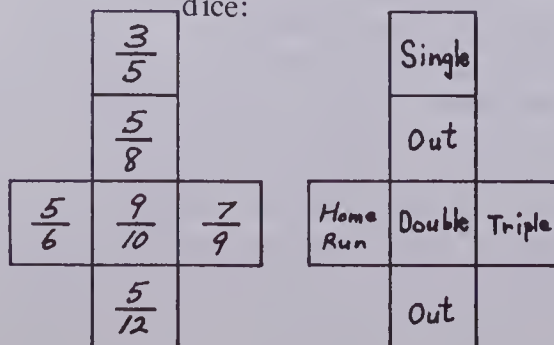
Assign the exercises.

ACTIVITIES

1. "Equivalence Baseball". The basic rules of baseball apply. Each player gets 3 outs per half inning — game is 3 innings.



Roll these dice:



When the dice are rolled the student must make the correct equivalent fraction.

Example

$\frac{3}{5}$ and **TRIPLE** means $\frac{3 \times 3}{5 \times 3} = \frac{9}{15}$.

If correct, the student moves his men through the correct number of bases.

Double $\rightarrow \frac{\square \times 2}{\square \times 2} = ?$

Home Run $\rightarrow \frac{\square \times 4}{\square \times 4} = ?$

The winner is the player with the most wins after 3 innings.

Answers:

	Single	Double	Triple	Home Run
$\frac{3}{5}$	$\frac{3}{5}$	$\frac{6}{10}$	$\frac{9}{15}$	$\frac{12}{20}$
$\frac{5}{8}$	$\frac{5}{8}$	$\frac{10}{16}$	$\frac{15}{24}$	$\frac{20}{32}$
$\frac{9}{10}$	$\frac{9}{10}$	$\frac{18}{20}$	$\frac{27}{30}$	$\frac{36}{40}$
$\frac{5}{6}$	$\frac{5}{6}$	$\frac{10}{12}$	$\frac{15}{18}$	$\frac{20}{24}$
$\frac{7}{9}$	$\frac{7}{9}$	$\frac{14}{18}$	$\frac{21}{27}$	$\frac{28}{36}$
$\frac{5}{12}$	$\frac{5}{12}$	$\frac{10}{24}$	$\frac{15}{36}$	$\frac{20}{48}$

(Continued on page 193)

Fractions in Lowest Terms

Many fractions can be written in lowest terms.

Divide the numerator and denominator by a common factor.

$$\frac{2}{6} \text{ in lowest terms is } \frac{1}{3} \quad \text{Why?} \quad \frac{2 \div 2}{6 \div 2} = \frac{1}{3}$$

$$\frac{6}{15} \text{ in lowest terms is } \frac{2}{5} \quad \text{Why?} \quad \frac{6 \div 3}{15 \div 3} = \frac{2}{5}$$

A fraction is in **lowest terms** if the only factor common to both numerator and denominator is 1.

Exercises

Express these fractions in lowest terms.

1. $\frac{5}{10} = \frac{1}{2}$ 2. $\frac{4}{8} = \frac{1}{2}$
3. $\frac{2}{8} = \frac{1}{4}$ 4. $\frac{6}{18} = \frac{1}{3}$
5. $\frac{9}{12} = \frac{3}{4}$ 6. $\frac{7}{21} = \frac{1}{3}$ 7. $\frac{10}{20} = \frac{1}{2}$
8. $\frac{12}{15} = \frac{4}{5}$ 9. $\frac{8}{10} = \frac{4}{5}$ 10. $\frac{6}{20} = \frac{3}{10}$
11. $\frac{4}{18} = \frac{2}{9}$ 12. $\frac{12}{21} = \frac{4}{7}$ 13. $\frac{6}{9} = \frac{2}{3}$
14. $\frac{9}{15} = \frac{3}{5}$ 15. $\frac{4}{14} = \frac{2}{7}$ 16. $\frac{6}{21} = \frac{2}{7}$
17. $\frac{1}{18} = \frac{1}{18}$ 18. $\frac{10}{15} = \frac{2}{3}$ 19. $\frac{8}{24} = \frac{1}{3}$
20. $\frac{15}{24} = \frac{5}{8}$ 21. $\frac{15}{30} = \frac{1}{2}$ 22. $\frac{11}{35} = \frac{11}{35}$
23. $\frac{10}{15} = \frac{2}{3}$ 24. $\frac{18}{27} = \frac{2}{3}$

BRAINTICKLER

Solve and look for a pattern.

$$\frac{1}{2} \times \frac{2}{3} = \frac{2}{6} = \frac{1}{3}$$

$$\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{6}{24} = \frac{1}{4}$$

$$\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} = \frac{24}{120} = \frac{1}{5}$$

$$\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \frac{5}{6} = \frac{1}{6}$$

Remember: Reduce to lowest terms.

Use a pattern to predict the answers.

$$\frac{1}{2} \times \frac{2}{3} \times \dots \times \frac{7}{8} = \frac{1}{8}$$

$$\frac{1}{2} \times \frac{2}{3} \times \dots \times \frac{9}{10} = \frac{1}{10}$$

$$\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \dots \times \frac{19}{20} = \frac{1}{20}$$

$$\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \dots \times \frac{99}{100} = \frac{1}{100}$$

Fractions — equivalent, lowest terms 189

OBJECTIVE

To express equivalent lowest terms fractions

PACING

Level A All

Level B All

Level C All

MATERIALS

Use the transparencies described on page 188.

RELATED AIDS

HMS — DM46.

BFA COMP LAB II — 63.

SUGGESTIONS

Initial Activity Use the shaded $\frac{1}{2}$ and the superimposed clear $\frac{1}{2}$.



By removing the clear half we see that $\frac{2}{4} = \frac{1}{2}$. $\frac{2 \div 2}{4 \div 2} = \frac{1}{2}$

By using shaded $\frac{1}{2}$ and thirds, we can also see that

$$\frac{3}{6} = \frac{3 \div 3}{6 \div 3} = \frac{1}{2}$$

By using the shaded $\frac{1}{2}$ and quarters, we see that

$$\frac{4}{8} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2}$$

Using the shaded $\frac{2}{5}$ and the half, third, quarter, and fifth overlays show that:

$$(a) \frac{4}{10} = \frac{4 \div 2}{10 \div 2} = \frac{2}{5};$$

$$(b) \frac{6}{15} = \frac{6 \div 3}{15 \div 3} = \frac{2}{5};$$

$$(c) \frac{8}{20} = \frac{8 \div 4}{20 \div 4} = \frac{2}{5}; \text{ and}$$

$$(d) \frac{10}{25} = \frac{10 \div 5}{25 \div 5} = \frac{2}{5}.$$

After each graphic demonstration on the overhead, summarize what has happened by listing the numerical process on the chalkboard.

USING THE BOOK

Using the $\frac{2}{5}$ shaded square and the thirds square show $\frac{6}{15}$ by removing the thirds square.

$$\frac{6}{15} = \frac{6 \div 3}{15 \div 3} = \frac{2}{5}$$

Emphasize that, when reducing fractions numerically, the numerator and denominator are divided by the same amount.

Assign the exercises. You may wish to complete Exercises 1, 2, and 3 orally with some groups.

ACTIVITIES

1. Prepare an activity card as follows: Use these patterns to make up 10 fractions which could be reduced to lowest terms.

$$\frac{\square \times 5}{\square \times 5} = ? \quad \frac{\square \times 6}{\square \times 6} = ?$$

$$\frac{\square \times 8}{\square \times 8} = ? \quad \frac{\square \times 9}{\square \times 9} = ?$$

(a) Select a different fraction to go in this place $\frac{\square}{\square}$ in each of the above.

(b) Multiply to create the equivalent fraction which could be reduced to lowest terms.

(c) Write your 10 fractions which can be reduced to lowest terms on an activity work card under the following heading:

Express each fraction in lowest terms.

Place your activity card in the exchange centre.

2. Use a sampling of the fractions created in Activity 1 above. Have various fractions written on cards. Group the cards in sets of 5 to 7 with an instruction card: "Arrange these fractions in order from smallest to largest."

3. See the "Fraction Cards" ideas in the Activity Reservoir.

EXTRA PRACTICE

Have each student do a minimum of 5 questions from the work cards.

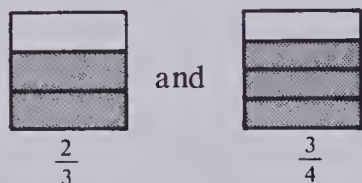
OBJECTIVE

To compare fractional quantities

PACING

Level A 1-13
Level B 1-15
Level C All

MATERIALS



and the clear thirds and fourths from page 188.

RELATED AIDS

HMS — DM45.

SUGGESTIONS

Initial Activity Use the $\frac{2}{3}$ shaded square and the clear quarters overlay to show:

$$\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

Use the $\frac{3}{4}$ shaded square and the clear thirds overlay to show:

$$\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12}$$

USING THE BOOK

Discuss the problem in the display at the top of the page and show how forming fractions having the same denominators makes the comparison easier.

In Exercise 1, thirds and fifths can both be expressed as “fifteenths”.

In Exercise 2, quarters and fifths can both be expressed as “twentieths”.

Review the meaning of each of these signs:

$>$ means “is greater than”;
 $<$ means “is less than”;
 $=$ means “is equal to”.

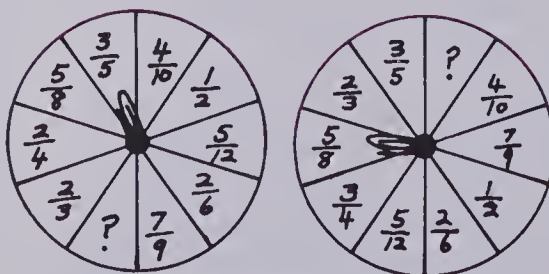
ACTIVITIES

1. Prepare 2 spinners as shown and have the children play “Spinner Winner”. Players (in twos) take turns choosing a spinner and simultaneously twirling the arrows. Players must determine who has the greater fraction. That player scores 1 point. If one pointer indicates a “?”, the first player to name an equivalent fraction wins the point. The winner is the player with the most points after a predetermined number of rounds.

190 Comparing fractions

ANSWERS:

4. $\frac{7}{8} > \frac{2}{3}$ 5. $\frac{7}{10} < \frac{3}{4}$ 6. $\frac{2}{5} > \frac{3}{8}$ 7. $\frac{3}{4} < \frac{5}{6}$ 8. $\frac{1}{4} = \frac{3}{12}$ 9. $\frac{1}{3} > \frac{3}{10}$
10. $\frac{2}{3} > \frac{6}{10}$ 11. $\frac{1}{2} > \frac{7}{15}$ 12. $\frac{3}{5} > \frac{7}{12}$ 13. $\frac{4}{5} < \frac{5}{6}$ 14. $\frac{1}{3} < \frac{3}{8}$ 15. $\frac{8}{9} < \frac{9}{10}$



Use a split fastener and a large paper clip as a pointer.

2. See Activity 2, page 189.
3. See the “Fraction Cards” ideas in the Activity Reservoir.

The Show

Anita is a game-show winner.
She has to choose a prize:

either
 $\frac{2}{3}$ of her mass in silver coins
or
 $\frac{3}{4}$ of her mass in silver coins.

Which should she choose?

Compare these fractions: $\frac{2}{3}$ and $\frac{3}{4}$.

Change both fractions to twelfths:

$$\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

$$\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12}$$

Since $\frac{9}{12}$ is greater than $\frac{8}{12}$
then $\frac{3}{4}$ is greater than $\frac{2}{3}$.



Anita should choose $\frac{3}{4}$ of her mass in silver dollars.

Exercises

Compare these fractions. (Use $>$, $<$, or $=$.)

1. $\frac{2}{3}$ and $\frac{1}{6}$

2. $\frac{1}{4}$ and $\frac{1}{9}$

3. $\frac{1}{5}$ and $\frac{2}{6}$

4. $\frac{2}{3} = \frac{2 \times 5}{3 \times 5} = \frac{10}{15}$

5. $\frac{1}{4} = \frac{1 \times 5}{4 \times 5} = \frac{5}{20}$

6. $\frac{1}{3} = \frac{1 \times 2}{3 \times 2} = \frac{2}{6}$

7. $\frac{4}{5} = \frac{4 \times 3}{5 \times 3} = \frac{12}{15}$

8. $\frac{1}{2} = \frac{1 \times 4}{2 \times 4} = \frac{4}{8}$

9. $\frac{2}{6} = \frac{2}{6}$

10. $\frac{1}{2} < \frac{4}{6}$

11. $\frac{1}{4} > \frac{1}{9}$

12. $\frac{1}{3} = \frac{2}{6}$

4. $\frac{7}{8}$ and $\frac{2}{3}$

5. $\frac{7}{10}$ and $\frac{3}{4}$

6. $\frac{2}{5}$ and $\frac{3}{8}$

7. $\frac{3}{4}$ and $\frac{5}{6}$

8. $\frac{1}{4}$ and $\frac{3}{10}$

9. $\frac{1}{3}$ and $\frac{3}{10}$

Which is larger? (Use $>$.)

10. $\frac{2}{3}$ or $\frac{6}{10}$

11. $\frac{7}{15}$ or $\frac{1}{2}$

12. $\frac{2}{3}$ or $\frac{7}{10}$

Which is smaller? (Use $<$.)

13. $\frac{4}{5}$ or $\frac{5}{6}$

14. $\frac{5}{8}$ or $\frac{1}{3}$

15. $\frac{3}{10}$ or $\frac{8}{9}$

★16. Arrange these fractions in order from smallest to largest.

(a) $\frac{2}{3}, \frac{5}{6}, \frac{3}{4}, \frac{7}{10}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{5}{6}$

(b) $\frac{1}{10}, \frac{7}{8}, \frac{3}{5}, \frac{1}{4}, \frac{9}{8}, \frac{3}{5}, \frac{7}{10}$

EXTRA PRACTICE

Arrange these fractions in order from smallest to largest.

- $\frac{1}{2}, \frac{1}{3},$ and $\frac{1}{5}$
- $\frac{2}{3}, \frac{1}{4},$ and $\frac{4}{5}$
- $\frac{2}{5}, \frac{3}{10},$ and $\frac{1}{2}$
- $\frac{3}{4}, \frac{7}{10},$ and $\frac{3}{5}$
- $\frac{7}{12}, \frac{2}{3},$ and $\frac{3}{5}$

Adding and Subtracting Fractions

$$\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

Addition

$$\frac{1}{2} = \frac{1 \times 2}{2 \times 2} = \frac{2}{4}$$

$$+ \frac{1}{4} = \frac{1}{4}$$

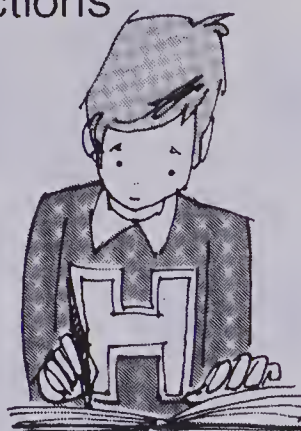
$$\hline \frac{3}{4}$$

Subtraction

$$\frac{1}{2} = \frac{1 \times 2}{2 \times 2} = \frac{2}{4}$$

$$- \frac{1}{4} = \frac{1}{4} \text{ (Subtract)}$$

$$\hline \frac{1}{4}$$



Exercises
Calculate.

$$\frac{1}{5} = \frac{1 \times 2}{5 \times 2} = \frac{2}{10}$$

$$+ \frac{1}{10} = \frac{1}{10}$$

$$\hline \frac{3}{10}$$

$$\frac{1}{3} = \frac{1 \times 2}{3 \times 2} = \frac{2}{6}$$

$$+ \frac{1}{6} = \frac{1}{6}$$

$$\hline \frac{3}{6}$$

$$\frac{2}{3} = \frac{2 \times 3}{3 \times 3} = \frac{6}{9}$$

$$+ \frac{2}{9} = \frac{2}{9}$$

$$\hline \frac{8}{9}$$

$$\frac{1}{6} + \frac{1}{6} = \frac{2}{6}$$

$$\frac{1}{8} + \frac{3}{8} = \frac{4}{8}$$

$$\frac{2}{10} + \frac{1}{10} = \frac{3}{10}$$

$$\frac{1}{9} + \frac{2}{9} = \frac{3}{9}$$

$$\frac{1}{10} + \frac{1}{10} = \frac{2}{10}$$

Find the differences.

$$\frac{4}{5} = \frac{4 \times 2}{5 \times 2} = \frac{8}{10}$$

$$- \frac{3}{10} = \frac{3}{10}$$

$$\hline \frac{5}{10}$$

$$\frac{2}{3} = \frac{2 \times 3}{3 \times 3} = \frac{6}{9}$$

$$- \frac{3}{9} = \frac{3}{9}$$

$$\hline \frac{3}{9}$$

$$\frac{3}{4} = \frac{3 \times 2}{4 \times 2} = \frac{6}{8}$$

$$- \frac{5}{8} = \frac{5}{8}$$

$$\hline \frac{1}{8}$$

$$\frac{8}{9} - \frac{3}{9} = \frac{5}{9}$$

$$\frac{4}{10} - \frac{1}{10} = \frac{3}{10}$$

$$\frac{3}{8} - \frac{1}{8} = \frac{2}{8}$$

$$\frac{5}{6} - \frac{2}{6} = \frac{3}{6}$$

$$\frac{14}{15} - \frac{3}{15} = \frac{11}{15}$$

Unlike denominators. 191

OBJECTIVE

To add and subtract fractions with unlike denominators

PACING

Level A All
Level B All
Level C All

RELATED AIDS

BFA COMP LAB II — 69, 70.

BACKGROUND

For the exercises on this page, the least common denominator is the larger denominator.

Example

Exercise 3:

$$\frac{2}{3} = \frac{2 \times 3}{3 \times 3} = \frac{6}{9}$$

$$+ \frac{2}{9} = \frac{2}{9}$$

$$\hline \frac{8}{9}$$

SUGGESTIONS

Initial Activity Tell the children that in order to add or subtract fractions with unlike denominators, the common denominator is either the larger denominator or a multiple of the larger denominator.

Demonstrate:

$$\frac{5}{6} + \frac{1}{3}$$

$$\frac{5}{6} + \frac{1}{3} = \frac{1 \times 2}{3 \times 2} = \frac{2}{6}$$

$$\hline \frac{7}{6} \text{ or } 1 \frac{1}{6}$$

and

$$\frac{5}{6} - \frac{1}{3}$$

$$\frac{5}{6} - \frac{1}{3} = \frac{1 \times 2}{3 \times 2} = \frac{2}{6}$$

$$\hline \frac{3}{6} \text{ or } \frac{1}{2}$$

USING THE BOOK

Encourage the students to determine the common denominator first. Have them make equivalent fractions and then add or subtract as indicated.

Students may be able to write the equivalence directly if they understand what they are doing.

Example

Exercise 9:

$$\frac{4}{5} = \frac{8}{10}$$

$$- \frac{3}{10} = \frac{3}{10}$$

$$\hline \frac{5}{10} \text{ or } \frac{1}{2}$$

If a student fails to use the lowest possible denominator, do not say that the method is wrong.

Example

Exercise 15:

$$\frac{5}{6} = \frac{10}{12}$$

$$- \frac{1}{2} = \frac{6}{12}$$

$$\hline \frac{4}{12} \text{ or } \frac{1}{3}$$

This is an acceptable solution.

ACTIVITIES

1. When students are finished this exercise they might wish to play the "Fraction 'Golf'" game on pages 192 and 193.

2. Have the children play the "Spinner Winner" game as described in Activity 1, page 190.

3. If you have not already done so, see the "Fraction Cards" ideas in the Activity Reservoir.

Assign the exercises.

OBJECTIVE

To provide a fraction activity

PACING

Level A All
Level B All
Level C All

MATERIALS

small wooden or plastic cubes to make a die

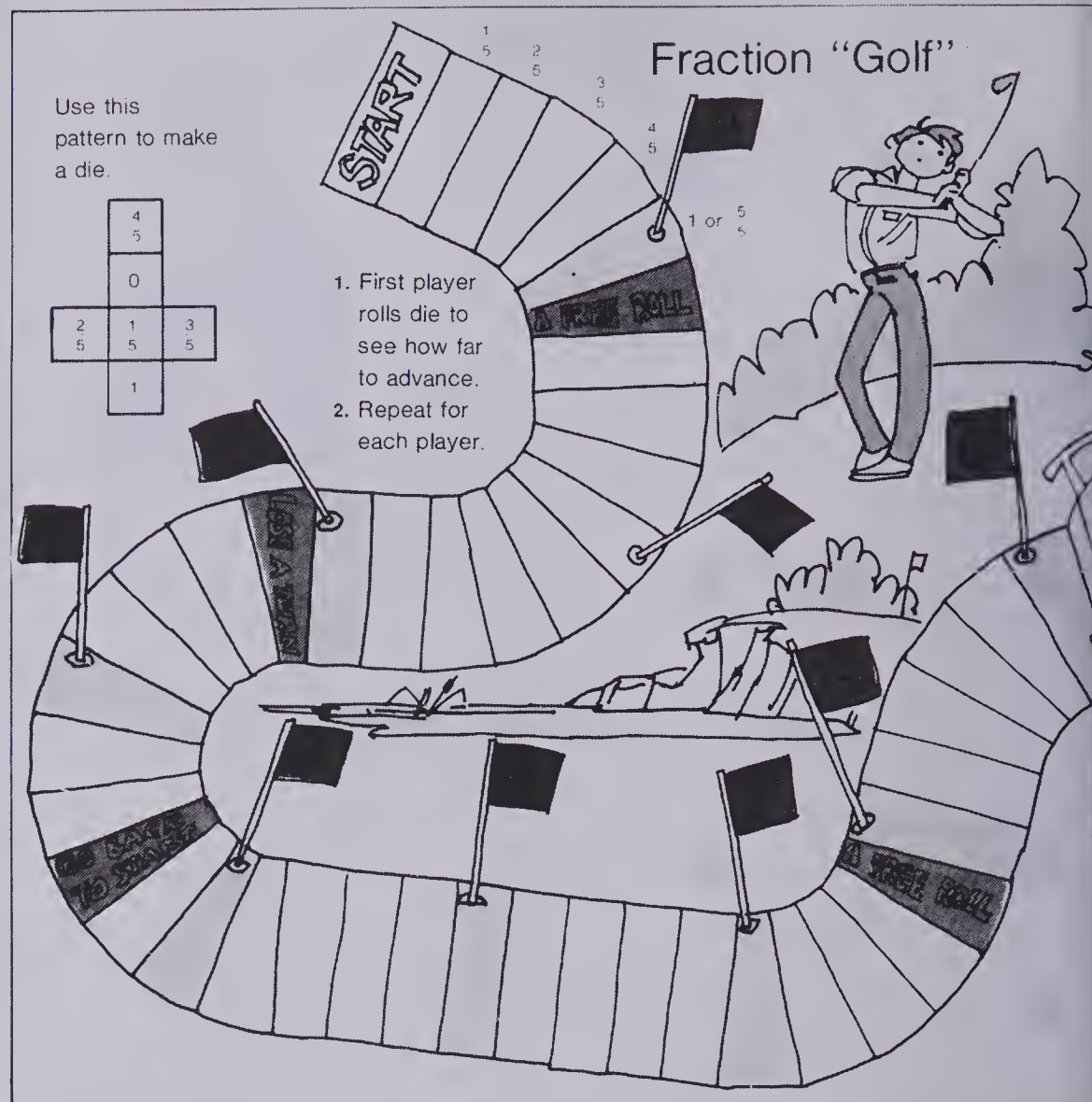
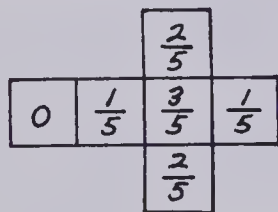
BACKGROUND

In this game each space represents $\frac{1}{5}$ of the "par" for the golf hole.

USING THE BOOK

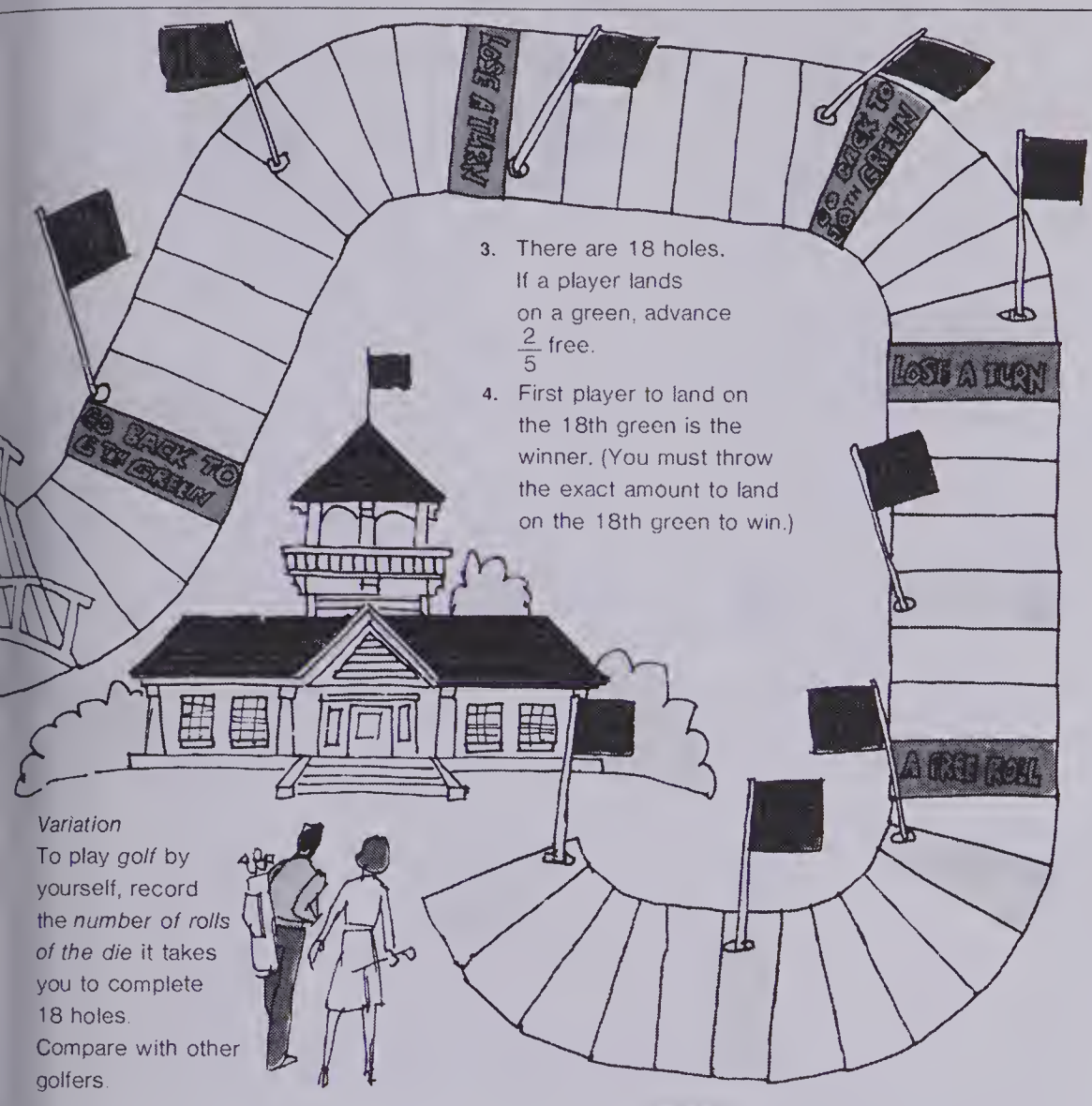
- Make the die by marking the faces as indicated on the cube net.
- Decide who will start the game.
- Each player moves the indicated "fifths" and follows any instructions on the spaces when landing on them.
- When the die shows "0" the player does not move.
- All other rules are on the game board.

Another Variation: Play the game using a different die.



192 Activity

What difference does this make in the playing of the game?



Activity 193

ACTIVITY

Have the children design their own "Golf" game board. (They may wish to have some very long holes and some that are quite short.)

(Continued from page 187)

Demonstrate how questions can be recorded in a horizontal or vertical model.

$$\frac{7}{8} + \frac{2}{8} = \frac{9}{8} \text{ or } 1\frac{1}{8}$$

HORIZONTAL

$$\begin{array}{r} \frac{7}{8} \\ + \frac{2}{8} \\ \hline \frac{9}{8} \end{array} \text{ or } 1\frac{1}{8}$$

V
E
R
T
I
C
A
L

Assign the exercises, allowing the students to use number lines when required.

2. Variation:
Use this die instead:

	$\frac{4}{5}$	
	$\frac{7}{10}$	
$\frac{8}{9}$	$\frac{11}{12}$	$\frac{8}{15}$
	$\frac{7}{8}$	

If the equivalent fraction is incorrect the batter is out.

Answers:

	Single	Double	Triple	Home Run
$\frac{4}{5}$	$\frac{4}{5}$	$\frac{8}{10}$	$\frac{12}{15}$	$\frac{16}{20}$
$\frac{7}{10}$	$\frac{7}{10}$	$\frac{14}{20}$	$\frac{21}{30}$	$\frac{28}{40}$
$\frac{8}{9}$	$\frac{8}{9}$	$\frac{16}{18}$	$\frac{24}{27}$	$\frac{32}{36}$

(Continued from page 188)

$$\frac{11}{12}, \frac{11}{12}, \frac{22}{24}, \frac{33}{36}, \frac{44}{48}$$

$$\frac{8}{15}, \frac{8}{15}, \frac{16}{30}, \frac{24}{45}, \frac{32}{60}$$

$$\frac{7}{8}, \frac{7}{8}, \frac{14}{16}, \frac{21}{24}, \frac{28}{32}$$

3. Prepare a number of Equivalent Fraction puzzles such as these. Each letter stands for only 1 digit. In some instances, answers may vary.

$$(1) \frac{D}{E} = \frac{E}{R} = \frac{X}{L} = \frac{R}{N}$$

Note: D = 1, N = 8, R = 4, X = 3.

$$\left[\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} \right]$$

$$(2) \frac{A}{CA} = \frac{CD}{BD} = \frac{CA}{HA}$$

Note: A = 5, B = 3, C = 1.

$$\left[\frac{5}{15} = \frac{10}{30} = \frac{15}{45} \right]$$

OBJECTIVE

To add and subtract unlike fractions

PACING

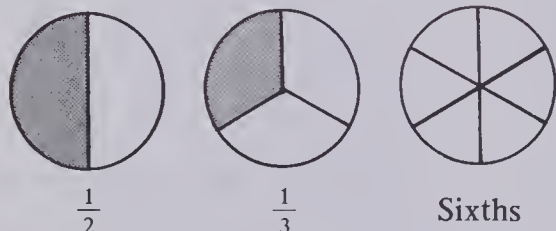
Level A 1-17

Level B 1-20

Level C 4-24

MATERIALS

acetate circles (radius 4 cm)
showing:



overhead projector

RELATED AIDS

HMS — DM45.

BFA COMP LAB II — 64.

SUGGESTIONS

Initial Activity (a) Read the "pizza problem" to the class from the book.
(b) Using the clear acetate "pizzas" ask the students to speculate how much pizza the boys ate in total. (Most should see that the answer is less than 1 pizza.)

To formally add these unlike fractional amounts, a common denominator is required. Ask the students for suggestions.

When they have decided on "sixths", use the overlays to show that $\frac{1}{2} = \frac{3}{6}$ and $\frac{1}{3} = \frac{2}{6}$.



USING THE BOOK

Show the formal solution to the addition and subtraction problems in the display at the top of the pupil page.

Indicate that in many cases, the required common denominator is the product of the denominators.

Example

$\frac{1}{2} + \frac{1}{3}$ The common denominator is 2×3 or 6.

In Exercise 1 the common denominator is 4×5 or 20.

Assign the exercises. Some students can write the equivalent fractions directly.

Pizzas

Derek ate $\frac{1}{2}$ of one pizza.

Keith ate $\frac{1}{3}$ of another pizza.



$$\frac{1}{2} = \frac{3}{6}$$



$$\frac{1}{3} = \frac{2}{6}$$

(a) How much pizza did the two boys eat altogether?

$$\begin{array}{r} \frac{1}{2} = \frac{1 \times 3}{2 \times 3} = \frac{3}{6} \\ + \frac{1}{3} = \frac{1 \times 2}{3 \times 2} = \frac{2}{6} \\ \hline \frac{5}{6} \end{array}$$

They ate $\frac{5}{6}$ of a pizza

(b) How much more pizza did Derek eat than Keith?

$$\begin{array}{r} \frac{1}{2} = \frac{1 \times 3}{2 \times 3} = \frac{3}{6} \\ - \frac{1}{3} = \frac{1 \times 2}{3 \times 2} = \frac{2}{6} \\ \hline \frac{1}{6} \end{array}$$

Derek ate $\frac{1}{6}$ of a pizza more than Keith

Exercises

Find the sums.

$$\begin{array}{r} 3 = \frac{3 \times 5}{4 \times 5} = \frac{15}{20} \\ 4 = \frac{4 \times 5}{5 \times 4} = \frac{20}{20} \\ + 5 = \frac{2 \times 4}{5 \times 4} = \frac{8}{20} \\ \hline 23 \end{array}$$

$$\begin{array}{r} 6. \quad \frac{1}{4} \\ + \frac{3}{5} \\ \hline \frac{17}{20} \end{array}$$

$$\begin{array}{r} 7. \quad \frac{9}{10} \\ + \frac{1}{3} \\ \hline \frac{37}{30} \end{array}$$

$$\begin{array}{r} 8. \quad \frac{3}{4} \\ + \frac{1}{5} \\ \hline \frac{19}{20} \end{array}$$

$$\begin{array}{r} 9. \quad \frac{5}{5} \\ + \frac{1}{3} \\ \hline \frac{23}{24} \end{array}$$

$$\begin{array}{r} 10. \quad \frac{2}{4} \\ + \frac{3}{5} \\ \hline \frac{19}{20} \end{array}$$

Calculate.

$$\begin{array}{r} 11. \quad \frac{2}{3} \\ - \frac{1}{4} \\ \hline \frac{5}{12} \end{array}$$

$$\begin{array}{r} 12. \quad \frac{7}{2} \\ - \frac{3}{5} \\ \hline \frac{11}{10} \end{array}$$

$$\begin{array}{r} 13. \quad \frac{1}{10} \\ - \frac{2}{3} \\ \hline \frac{1}{30} \end{array}$$

$$\begin{array}{r} 14. \quad \frac{3}{4} \\ - \frac{1}{5} \\ \hline \frac{5}{12} \end{array}$$

$$\begin{array}{r} 15. \quad \frac{4}{5} \\ - \frac{3}{4} \\ \hline \frac{1}{20} \end{array}$$

$$\begin{array}{r} 16. \quad \frac{5}{8} \\ - \frac{1}{3} \\ \hline \frac{7}{24} \end{array}$$

$$\begin{array}{r} 17. \quad \frac{13}{10} \\ - \frac{3}{4} \\ \hline \frac{11}{20} \end{array}$$

$$\begin{array}{r} 18. \quad \frac{8}{5} \\ - \frac{2}{3} \\ \hline \frac{14}{15} \end{array}$$

$$\begin{array}{r} 19. \quad \frac{4}{10} \\ - \frac{1}{4} \\ \hline \frac{1}{20} \end{array}$$

$$\begin{array}{r} 20. \quad \frac{5}{8} \\ - \frac{1}{3} \\ \hline \frac{7}{24} \end{array}$$

$$\star 21. \quad \frac{1}{2} + \frac{1}{4} + \frac{1}{5} = \frac{19}{20}$$

$$\star 22. \quad \frac{2}{3} + \frac{1}{2} + \frac{7}{10} = \frac{56}{30}$$

$$\star 23. \quad \frac{14}{15} - \frac{3}{10} = \frac{19}{30}$$

$$\star 24. \quad \frac{1}{9} - \frac{1}{11} = \frac{2}{99}$$

194 Common denominators, product of denominators

Example

Exercise 6:

$$\begin{array}{r} \frac{1}{4} = \frac{5}{20} \\ + \frac{3}{5} = \frac{12}{20} \\ \hline \frac{17}{20} \end{array}$$

ACTIVITIES

1. Provide a challenge activity such as:

Subtract the following *unit* fraction "neighbours".

$$(1) \frac{1}{2} - \frac{1}{3} = \square \quad (2) \frac{1}{3} - \frac{1}{4} = \square$$

$$(3) \frac{1}{4} - \frac{1}{5} = \square \quad (4) \frac{1}{5} - \frac{1}{6} = \square$$

(5) Can you discover a pattern for the answers for questions 1, 2, 3, and 4?

Predict answers for these differences. Check your predictions.

$$(6) \frac{1}{7} - \frac{1}{8} = \square \quad (7) \frac{1}{8} - \frac{1}{9} = \square$$

$$(8) \frac{1}{9} - \frac{1}{10} = \square$$

2. Provide a challenge activity such as:

Calculate the answers to these unit fraction differences.

$$(1) \frac{1}{2} - \frac{1}{5} = \square \quad (2) \frac{1}{3} - \frac{1}{5} = \square$$

(3) Can you discover a pattern for the answers to questions 1 and 2?

Predict the answers for these differences. Check your predictions.

$$(4) \frac{1}{3} - \frac{1}{7} = \square \quad (5) \frac{1}{5} - \frac{1}{8} = \square$$

$$(6) \frac{1}{4} - \frac{1}{6} = \square \quad (7) \frac{1}{5} - \frac{1}{10} = \square$$

3. Use the spinners described in Activity 1 on page 190. Have the players take turns twirling the spinners for each other to generate fraction subtraction exercises with like and unlike denominators.

Example

Player A twirls spinners indicating $\frac{2}{3}$ and $\frac{3}{4}$.

(Continued on page 199)

Using Least Common Denominators

Often a common denominator is a multiple of the *larger* denominator.

$$\begin{array}{r} \frac{3}{4} = \frac{9}{12} \\ + \frac{1}{12} = \frac{1}{12} \\ \hline \frac{10}{12} \text{ or } \frac{5}{6} \end{array}$$

The least common denominator is 12

$$\begin{array}{r} \frac{9}{10} = \frac{18}{20} \\ - \frac{3}{4} = \frac{15}{20} \\ \hline \frac{3}{20} \end{array}$$

The least common denominator is 20.

$$\begin{array}{r} \frac{13}{15} = \frac{26}{30} \\ - \frac{1}{6} = \frac{5}{30} \\ \hline \frac{21}{30} \text{ or } \frac{7}{10} \end{array}$$

The least common denominator is 30.

Exercises

Add or subtract

Use least common denominators.

$$\begin{array}{r} \frac{1}{4} = \frac{5}{20} \\ + \frac{7}{10} = \frac{14}{20} \\ \hline \frac{19}{20} \end{array}$$

$$\begin{array}{r} \frac{7}{8} = \frac{7}{40} \\ - \frac{1}{4} = \frac{10}{40} \\ \hline \frac{3}{40} \end{array}$$

$$\begin{array}{r} \frac{11}{15} = \frac{47}{30} \text{ or } \frac{17}{10} \\ + \frac{2}{3} = \frac{14}{15} \\ \hline \frac{61}{30} \end{array}$$

$$\begin{array}{r} \frac{3}{4} = \frac{33}{20} \text{ or } \frac{13}{8} \\ + \frac{9}{10} = \frac{18}{10} \\ \hline \frac{51}{20} \end{array}$$

$$\begin{array}{r} \frac{5}{6} = \frac{5}{6} \\ - \frac{1}{2} = \frac{3}{6} \\ \hline \frac{2}{6} \text{ or } \frac{1}{3} \end{array}$$

$$\begin{array}{r} \frac{1}{2} = \frac{3}{6} \\ - \frac{1}{6} = \frac{1}{6} \\ \hline \frac{2}{6} \text{ or } \frac{1}{3} \end{array}$$

$$\begin{array}{r} \frac{11}{10} = \frac{11}{10} \\ - \frac{3}{4} = \frac{7}{20} \\ \hline \frac{11}{20} \end{array}$$

$$\begin{array}{r} \frac{5}{6} = \frac{19}{12} \text{ or } \frac{7}{4} \\ + \frac{3}{4} = \frac{9}{12} \\ \hline \frac{28}{12} \text{ or } \frac{7}{3} \end{array}$$

$$\begin{array}{r} \frac{7}{8} = \frac{16}{30} \text{ or } \frac{8}{15} \\ - \frac{1}{6} = \frac{5}{30} \\ \hline \frac{11}{30} \end{array}$$

$$\begin{array}{r} \frac{7}{10} = \frac{16}{30} \text{ or } \frac{8}{15} \\ - \frac{1}{6} = \frac{5}{30} \\ \hline \frac{11}{30} \end{array}$$

$$\begin{array}{r} \frac{4}{5} = \frac{10}{15} \text{ or } \frac{2}{3} \\ + \frac{2}{3} = \frac{10}{15} \\ \hline \frac{20}{15} \text{ or } \frac{4}{3} \end{array}$$

$$\begin{array}{r} \frac{3}{16} = \frac{13}{16} \\ + \frac{9}{16} = \frac{9}{16} \\ \hline \frac{22}{16} \text{ or } \frac{11}{8} \end{array}$$

$$\begin{array}{r} \frac{7}{8} = \frac{14}{16} \\ + \frac{3}{4} = \frac{12}{16} \\ \hline \frac{26}{16} \text{ or } \frac{13}{8} \end{array}$$

$$\begin{array}{r} \frac{3}{4} = \frac{13}{8} \text{ or } \frac{5}{2} \\ + \frac{2}{8} = \frac{2}{8} \\ \hline \frac{15}{8} \end{array}$$

$$\begin{array}{r} \frac{9}{8} = \frac{13}{24} \\ - \frac{1}{2} = \frac{12}{24} \\ \hline \frac{1}{24} \end{array}$$

$$\begin{array}{r} \frac{11}{12} = \frac{13}{24} \\ - \frac{3}{8} = \frac{9}{24} \\ \hline \frac{2}{24} \text{ or } \frac{1}{12} \end{array}$$

16. Carol is making a suit. She needs $\frac{3}{8}$ m of fabric for the jacket and $\frac{7}{10}$ m for the skirt. How much fabric must she purchase? $\frac{63}{40} \text{ m or } \frac{23}{10} \text{ m}$

Least common denominators 195

OBJECTIVE

To determine the least common denominator

PACING

Level A All
Level B All
Level C All

RELATED AIDS

HMS — DM47.

BACKGROUND

The approach on this page summarizes the approaches presented on pages 191 and 194. Some of the exercises will be found to contain the common denominator as a multiple of the larger denominator while others will involve identifying the common denominator as a product of denominators.

SUGGESTIONS

Initial Activity (a) Demonstrate a solution to the question $\frac{3}{4} + \frac{1}{12}$ using a common denominator of 48.

Example

$$\begin{array}{r} \frac{3}{4} = \frac{36}{48} \\ + \frac{1}{12} = \frac{4}{48} \\ \hline \frac{40}{48} \text{ or } \frac{5}{6} \end{array}$$

(b) Demonstrate a solution to the same problem using a common denominator of 24.

$$\begin{array}{r} \frac{3}{4} = \frac{18}{24} \\ + \frac{1}{12} = \frac{2}{24} \\ \hline \frac{20}{24} \text{ or } \frac{5}{6} \end{array}$$

(c) Finally demonstrate a solution using twelfths.

$$\begin{array}{r} \frac{3}{4} = \frac{9}{12} \\ + \frac{1}{12} = \frac{1}{12} \\ \hline \frac{10}{12} \text{ or } \frac{5}{6} \end{array}$$

All approaches lead to a correct answer.

2. Finding the least common denominator is essentially the same arithmetic process as finding the least common multiple. See the activities listed on pages 173 and 174 for practice ideas in this area.

3. If you have not already done so, see the "Fraction Cards" ideas suggested in the Activity Reservoir.

USING THE BOOK

Introduce a technique for finding the *least* common denominator by testing multiples of the larger denominator.

Example

$$\frac{9}{10} - \frac{3}{4}$$

Can *quarters* be expressed as *tenths*? (the first multiple of ten) No!

Can quarters be expressed as twentieths? (the second multiple of ten) Yes!

$$\begin{array}{r} \frac{9}{10} = \frac{18}{20} \\ - \frac{3}{4} = \frac{15}{20} \\ \hline \frac{3}{20} \end{array}$$

This technique always covers the *least* common denominator.

Assign the exercises.

ACTIVITIES

1. Provide an Activity Instruction card such as:

- Make up your own test which contains 5 challenging addition questions involving fractions.
- Answer each of your 5 addition questions. If any of your questions are too difficult, you may revise the question.
- Exchange your questions with a classmate. Do solutions for each other's questions.
- If you both agree, then make up an Activity Test card with the title:
"_____ 's TEST"
↑
Print your name here.
- Place all tests in a central location. Have each student choose the test he or she wishes to complete.
- Each test is marked by the student who made the test.

OBJECTIVE

To recognize and use the commutative and associative properties when adding fractions

PACING

Level A 1-8

Level B 1-9

Level C 1-9

MATERIALS

the following fractions printed on large cards:



masking tape, blank playing cards or Bristol board for the Activity

SUGGESTIONS

Initial Activity Using the cards, make up two different looking questions. Use the same fractions but change the order of the addends.

Example

$$(a) \quad \boxed{\frac{1}{2}} + \boxed{\frac{1}{3}} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$

$$(b) \quad \boxed{\frac{1}{3}} + \boxed{\frac{1}{2}} = \frac{2}{6} + \frac{3}{6} = \frac{5}{6}$$

Complete the computations together, emphasizing that since both questions generate the same answers, the addition of fractions is commutative.

Students will look at this property as: "The order can be changed when adding fractions."

Demonstrate the associative property using the cards to show that $(\frac{1}{2} + \frac{1}{3}) + \frac{1}{3}$ is equivalent to $\frac{1}{2} + (\frac{1}{3} + \frac{1}{3})$.

Stress that the order has *not* been changed in this arrangement but that the grouping (pairing) of the fractions has.

Note that this solution is easier to follow.

$$\begin{aligned} \frac{1}{2} + (\frac{1}{3} + \frac{1}{3}) &= \frac{1}{2} + \frac{2}{3} \\ &= \frac{3}{6} + \frac{4}{6} \\ &= \frac{7}{6} \text{ or } 1\frac{1}{6} \end{aligned}$$

USING THE BOOK

Have the students look at the two examples in the display at the top of

Properties of Addition of Fractions

Compare:

$$(a) \quad \frac{1}{2} + \frac{3}{4} = \frac{2}{4} + \frac{3}{4}$$

$$= \frac{5}{4} = 1\frac{1}{4}$$

$$(b) \quad \frac{3}{4} + \frac{1}{2} = \frac{3}{4} + \frac{2}{4}$$

$$= \frac{5}{4} = 1\frac{1}{4}$$

Compare

What is the rule?

Compare:

$$(a) \quad (\frac{1}{2} + \frac{1}{3}) + \frac{1}{3} = (\frac{3}{6} + \frac{2}{6}) + \frac{1}{3}$$

$$= \frac{5}{6} + \frac{2}{6}$$

$$= \frac{7}{6} = 1\frac{1}{6}$$

$$(b) \quad \frac{1}{2} + (\frac{1}{3} + \frac{1}{3}) = \frac{1}{2} + \frac{2}{3}$$

$$= \frac{3}{6} + \frac{4}{6}$$

$$= \frac{7}{6} = 1\frac{1}{6}$$

Compare

What is the rule?

Exercises

They are the same.

Add each pair of questions and compare the answers. Write a rule.

$$1. (a) \quad \frac{1}{8} + \frac{1}{4} = \frac{3}{8}$$

$$2. (a) \quad \frac{3}{4} + \frac{2}{5} = \frac{17}{20}$$

$$3. (a) \quad \frac{5}{8} + \frac{1}{3} = \frac{23}{24}$$

Changing the order of the addends does not change the sum.

$$4. (a) \quad \frac{2}{3} + \frac{3}{10} = \frac{29}{30}$$

$$(b) \quad \frac{1}{4} + \frac{1}{8} = \frac{3}{8}$$

$$(b) \quad \frac{2}{5} + \frac{3}{4} = \frac{17}{20}$$

$$(b) \quad \frac{1}{3} + \frac{5}{8} = \frac{23}{24}$$

$$(b) \quad \frac{3}{10} + \frac{2}{3} = \frac{29}{30}$$

Add each pair of questions and compare the answers. Write a rule.

They are the same.

$$5. (a) \quad (\frac{1}{8} + \frac{1}{8}) + \frac{1}{4} = \frac{1}{2}$$

$$6. (a) \quad (\frac{1}{2} + \frac{1}{4}) + \frac{1}{4} = 1$$

$$17. (a) \quad (\frac{1}{4} + \frac{1}{6}) + \frac{1}{3} = \frac{3}{4}$$

Changing the grouping of the addends does not change the sum.

$$18. (a) \quad (\frac{2}{5} + \frac{1}{6}) + \frac{1}{5} = 1$$

$$(b) \quad \frac{1}{8} + (\frac{1}{8} + \frac{1}{4}) = \frac{1}{2}$$

$$(b) \quad \frac{1}{2} + (\frac{1}{4} + \frac{1}{4}) = 1$$

$$(b) \quad \frac{1}{4} + (\frac{1}{6} + \frac{1}{3}) = \frac{3}{4}$$

$$(b) \quad \frac{2}{5} + (\frac{1}{6} + \frac{1}{5}) = 1$$

- ★9. What happens if you try these rules with subtraction? Do you get the same answers in these pairs of questions? *No*

$$(a) \quad \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

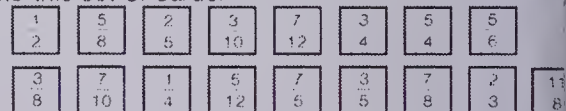
$$(b) \quad \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$$

$$(a) \quad (\frac{7}{10} - \frac{1}{5}) - \frac{1}{10} = \frac{2}{5}$$

$$(b) \quad \frac{7}{10} - (\frac{1}{5} - \frac{1}{10}) = \frac{3}{5}$$

Activity

Make this set of cards.



- Select 2 fraction cards. Add these fractions.
- Make up 5 addition questions this way. Solve.
- Make up 5 subtraction questions in a similar way. Solve.

the page as a short review of the concepts presented in the Initial Activity.

Have each student write a rule and share these rules with the class. Assign the pairs of exercises.

ACTIVITIES

1. Provide blank cards or Bristol board for the Activity described on the pupil page.

When the students are doing part (c), encourage them to select a large fraction first and then select a smaller fraction for subtraction from the larger one.

2. Play "Concentration" as described in the Activity Reservoir. Use cards such as:

$$\boxed{\frac{1}{2} + \frac{1}{3}}$$

$$\boxed{\frac{1}{3} + \frac{1}{2}}$$

$$\boxed{\frac{1}{4} + (\frac{2}{3} + \frac{1}{3})}$$

$$\boxed{(\frac{1}{4} + \frac{2}{3}) + \frac{1}{3}}$$

3. See the "Coded Riddles" idea as described in the Activity Reservoir. Use (or have the children use) addition and subtraction of fractions as part of the decoding process.

Mixed Numerals

To change an improper fraction into a **mixed numeral**:

Divide the numerator by the denominator

Improper fraction



Divide:

$$\begin{array}{r} 2 \\ 3 \overline{)7} \\ \underline{6} \end{array}$$

Mixed numeral:



The remainder becomes the numerator of the fraction

$$\begin{array}{r} 1 \\ 5 \overline{)19} \\ \underline{15} \\ 4 \end{array}$$

$$\frac{19}{5}$$

To change a **mixed numeral** into an improper fraction

Multiply the whole number by the denominator. Then add the numerator

Mixed numeral

Multiply and add

Improper fraction



$$3 \times 4 + 1 = 13$$

Keep the same denominator



Exercises

Change each to an equivalent mixed numeral

1. $2\frac{1}{2}$ 2. $1\frac{2}{3}$ 3. $1\frac{3}{4}$ 4. $1\frac{4}{5}$ 5. $1\frac{5}{8}$ 6. $1\frac{7}{10}$ 7. $5\frac{1}{2}$ 8. $6\frac{2}{3}$
9. $5\frac{3}{4}$ 10. $6\frac{3}{5}$ 11. $6\frac{3}{8}$ 12. $7\frac{2}{10}$ 13. $2\frac{1}{3}$ 14. $1\frac{3}{4}$ 15. $15\frac{3}{5}$

Change each to an equivalent improper fraction

16. $3\frac{1}{2}$ 17. $1\frac{3}{4}$ 18. $1\frac{5}{8}$ 19. $2\frac{1}{3}$ 20. $1\frac{7}{10}$
21. $2\frac{13}{5}$ 22. $5\frac{17}{3}$ 23. $4\frac{17}{4}$ 24. $3\frac{31}{8}$ 25. $5\frac{59}{10}$
26. $7\frac{22}{3}$ 27. $8\frac{42}{5}$ 28. $12\frac{25}{2}$ 29. $9\frac{39}{4}$ 30. $5\frac{43}{8}$

Improper fractions and mixed numerals 197

USING THE BOOK

(a) Using seven $\frac{1}{3}$ pieces, show that

$$\frac{7}{3} = 2\frac{1}{3}$$

Demonstrate also that, if 7 is divided by 3, the same answer can be arrived at.

$$\begin{array}{r} 2 \text{ R}1 \rightarrow 2\frac{1}{3} \\ 3 \overline{)7} \\ \underline{6} \\ 1 \end{array}$$

(b) Using nineteen $\frac{1}{5}$ pieces, show

$$\text{that } \frac{19}{5} = 3\frac{4}{5}$$

But if 19 is divided by 5:

$$\begin{array}{r} 3 \text{ R}4 \rightarrow 3\frac{4}{5} \\ 5 \overline{)19} \\ \underline{15} \\ 4 \end{array}$$

When discussing this *numerical* method of changing improper fractions to mixed numerals, emphasize that, as shown at the top of the page, the remainder becomes the numerator of the fraction part of the mixed number.

(c) Point out that, in $2\frac{1}{3}$ each whole

$$\begin{aligned} &\text{is equivalent to } \frac{3}{3} \text{ so } 2\frac{1}{3} \\ &= \boxed{\frac{3}{3}} + \boxed{\frac{3}{3}} + \frac{1}{3} \\ &= \frac{7}{3} \end{aligned}$$

A way to calculate this is to: multiply the whole number by the denominator and add the numerator, to get your proper fraction.

Example

$$2\frac{1}{3} \rightarrow \begin{array}{c} \text{Denominator} \\ \uparrow \\ \text{Whole No.} \end{array} \frac{2 \times 3 + 1}{3} \begin{array}{c} \text{Numerator} \\ \uparrow \end{array}$$

Same denominator

(d) Demonstrate this for $3\frac{4}{5}$.

(e) Assign the exercises.

ACTIVITIES

1. Play "Mixed Numeral Draw". (2 to 4 players)

Prepare 4 each of 1, 2, 3, 4, and 5 on red cards. Prepare 1 each of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, $\frac{1}{6}$, $\frac{5}{6}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, $\frac{1}{10}$, $\frac{3}{10}$, $\frac{7}{10}$, and $\frac{9}{10}$ on white cards. To

(Continued on page 198)

OBJECTIVE

To change improper fractions to mixed numerals and vice versa

PACING

Level A 1-12, 16-25

Level B 1-12, 16-28

Level C All

MATERIALS

overhead projector, circles (radius = 4 cm) made out of acetate and cut into their fraction parts showing



two of these three of these four of these

RELATED AIDS

HMS — DM45.

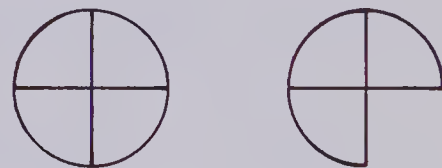
BFA COMP LAB II — 67, 68, 72.

BACKGROUND

Neither improper fractions nor mixed numerals are preferable to the other. Mixed numerals show quantity related to wholes, and improper fractions are essential for the operations of multiplication and division.

SUGGESTIONS

Initial Activity (a) Show seven $\frac{1}{4}$'s on the overhead using the acetate "pie" pieces.



Rearrange the pieces to show

$$\frac{7}{4} = 1\frac{3}{4}. \text{ Break the } 1\frac{3}{4} \text{ into quarters again to show } 1\frac{3}{4} = \frac{7}{4}.$$

(b) Show that $\frac{5}{3}$ equals $1\frac{2}{3}$ in a similar manner.

(c) Show that $\frac{12}{5}$ equals $2\frac{2}{5}$ and $2\frac{2}{5}$ equals $\frac{12}{5}$ using the acetate "pie" pieces.

OBJECTIVE

To provide an activity involving fraction equivalents and predicting addition patterns

PACING

Level A Optional
Level B All
Level C All

MATERIALS

a transparency of the Fraction Tower for demonstration using the overhead projector

RELATED AIDS

HMS — DM45.

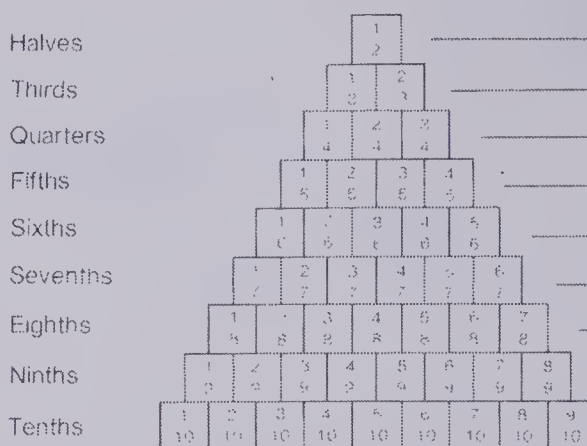
BACKGROUND

This *tower* shows all the proper fractions (less than 1) whose denominators range from 2 through 10. Note that pages 198 and 199 represent a two-page presentation and that the addends of the addition pattern exercise at the top of page 199 come from the tower shown on page 198.

SUGGESTIONS

Initial Activity Show the fraction tower to the class using the overhead projector. Ask the class to look for any patterns in this tower. List any patterns suggested.

Fraction Towers



Copy this Fraction Tower into your notebook.

1. Colour groups of equivalent fractions the same colour.
2. Which row has no fractions equivalent to another fraction in the chart? *Sevenths*
3. Which equivalent fractions are lined up vertically? $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}$
4. Use different coloured pairs of fractions from different levels of the Fraction Tower.
 - (a) Make up 5 addition questions and solve them.
 - (b) Make up 5 subtraction questions and solve them.

(Pick the first fraction from the right side of the tower. Choose the second fraction from those that are to the left of the first fraction. For Example: $\frac{5}{8}$ is from the right side of the tower. $\frac{5}{8}$ is to the left of $\frac{6}{8}$.)

$$\begin{array}{r} \frac{5}{8} = \frac{20}{24} \\ \frac{6}{8} = \frac{24}{24} \\ - \frac{3}{8} = \frac{9}{24} \\ \hline \frac{5}{8} = \frac{24}{24} \\ \hline 11 \\ 24 \end{array}$$

198 Equivalent fractions

(Continued from page 197)

play, each player: (a) draws a red and a white card; together these cards represent a mixed numeral; (b) expresses the mixed numeral as an improper fraction; (c) writes this improper fraction on a slip of paper, passes this paper to the player on his/her left, and asks them to discover what the two cards are.

Players get a point for each correct formation of the improper fraction, and a point for each card guessed correctly. The players with the most points after 5 draws are the winners.

2. See the "Fraction Cards" idea in the Activity Reservoir.

3. See "Scramble" as described in the Activity Reservoir. Be sure to use cards such as:

A

Front

$\frac{7}{4} = \frac{\blacksquare}{\blacksquare}$

Back

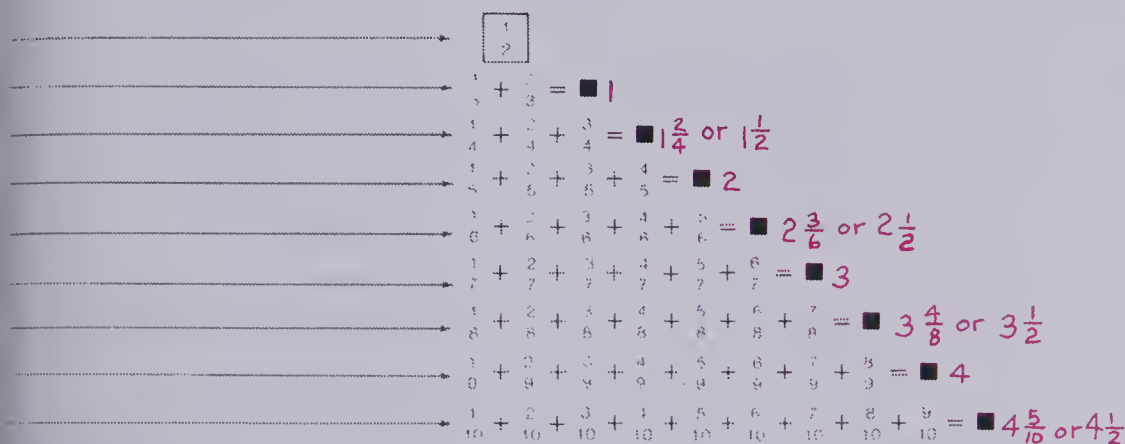
T

$\frac{12}{5} = \frac{\blacksquare}{\blacksquare}$

S

$3\frac{1}{7} = \frac{\blacksquare}{\blacksquare}$

Fraction-Tower Sums



1. Copy and calculate the sum of each row in the fraction tower above. Express answers as mixed numerals.

2. What pattern do you notice in the answers?

Explain this pattern *Each row increases by $\frac{1}{2}$.*

3. Without adding, predict the answers to the following.

(a) $\frac{1}{11} + \frac{2}{11} + \frac{10}{11} = \blacksquare 5$ (b) $\frac{1}{12} + \frac{2}{12} + \frac{11}{12} = \blacksquare 5\frac{1}{2}$

(c) $\frac{1}{13} + \frac{2}{13} + \frac{12}{13} = \blacksquare 6$ (d) $\frac{1}{14} + \frac{2}{14} + \frac{13}{14} = \blacksquare 6\frac{1}{2}$

(e) $\frac{1}{15} + \frac{2}{15} + \frac{14}{15} = \blacksquare 7$

(f) Check your predictions by adding.



Addition patterns 199

USING THE BOOK

Using the overhead projector and the "Fraction Tower", ask the students to find all the fractions equivalent to $\frac{1}{2}$.

List these.

Example

$$\frac{1}{2} = \frac{2}{4}$$

$$\frac{1}{2} = \frac{4}{8}$$

$$\frac{1}{2} = \frac{3}{6}$$

$$\frac{1}{2} = \frac{5}{10}$$

Colour all 5 fractions blue.

Suggest that they continue finding equivalent fractions, working from the top down.

Example

Next $\frac{1}{3}$, then $\frac{2}{3}$, then $\frac{1}{4}$, then $\frac{3}{4}$, etc.

Use a different colour for each group of equivalents.

Show how the fractions are selected for Exercise 4(b) so that negative rational answers are avoided.

This activity summarizes the concept of equivalent fractions, addition and subtraction of like and unlike fractions, and involves some pattern problem-solving techniques.

Assign all the exercises on pages 198 and 199. Be available to assist the students with the purpose of each question. Be careful not to do the thinking for them.

ACTIVITIES

1. Prepare pictorial fraction puzzles as shown. Have the students identify the pattern and predict what the next picture should be.



etc.

2. Prepare numeric fraction puzzles for completion by students.

(a) $1\frac{1}{2}, 3\frac{1}{2}, 5\frac{1}{2}, \rightarrow \underline{\hspace{1cm}}, \underline{\hspace{1cm}}$

(b) $2\frac{1}{4}, \frac{9}{4}, 4\frac{1}{4}, \frac{17}{4}, 6\frac{1}{4}, \rightarrow \underline{\hspace{1cm}}, \underline{\hspace{1cm}}$

(c) $\frac{4}{2}, 4, \frac{12}{2}, 8, \rightarrow \underline{\hspace{1cm}}, \underline{\hspace{1cm}}$

etc.

3. Have the children prepare their own pictorial and/or numeric pattern puzzles for exchange with other classmates, groups, or classes in the school.

(Continued from page 194)

Player B writes:

$$\begin{array}{r} \frac{3}{4} \\ - \frac{2}{3} \\ \hline \end{array} \rightarrow \begin{array}{r} \frac{9}{12} \\ - \frac{8}{12} \\ \hline \frac{1}{12} \end{array}$$

Player B scores 1 point (or, to add an element of chance, 12 points—equal to the final reduced denominator). Player B twirls spinners for Player A, etc.

OBJECTIVE

To express fractions whose denominators are 10, 100, 1000, or 10 000 as decimals

PACING

Level A 1-15, 21, 22
Level B 1-24
Level C 6-26

MATERIALS

a large decimal place-value chart and cards with the digits 0 through 9

BACKGROUND

When expressing quantities less than 1 whole as decimals, write a zero in the unit's column preceding the decimal point.

Example

$$\frac{7}{10} = 0.7$$

$$\frac{9}{100} = 0.09$$

$$\frac{57}{100} = 0.57$$

SUGGESTIONS

Initial Activity Using the large place-value chart, review what decimals such as 0.3, 0.36, and 0.365 mean so that students are reminded that the first column is tenths, the second is hundredths, the third is thousandths, and the fourth is ten thousandths.

Place some decimal amounts on the place-value chart and have students read the amounts.

USING THE BOOK

Show each of the fractions in the display on the large place-value chart. For $\frac{7}{100}$, explain why the seven is in the hundredth's column.

$$\frac{7}{100} = 0.07$$

For $\frac{58}{10\,000}$, explain why the eight is in the ten thousandth's column.

$$\frac{58}{10\,000} = 0.0058$$

Remind the students that, whether we write $\frac{525}{1000}$ or 0.525, we are expressing the same amount.

Assign the exercises.

ACTIVITIES

1. See "The P.V. Game" as described in the Activity Reservoir. Use blanks such as $\underline{\quad}.\underline{\quad}\underline{\quad}\underline{\quad}$ and have as the object, to make the smallest in value number possible.

Decimal Equivalence

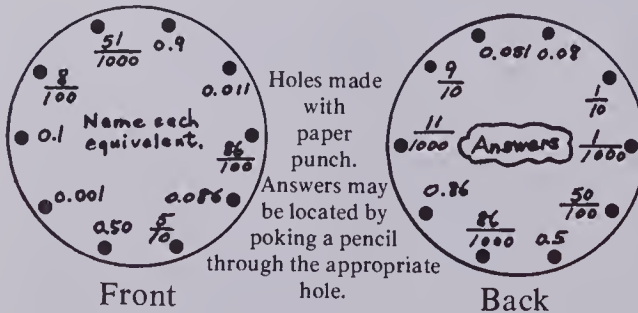
	ones	tenths	hundredths	thousandths	ten thousandths	
$\frac{3}{10}$	0	3				three tenths
$\frac{8}{10}$	0	8				eight tenths
$\frac{36}{100}$	0	3	6			thirty-six hundredths
$\frac{7}{100}$	0	0	7			seven hundredths
$\frac{525}{1000}$	0	5	2	5		five hundred twenty-five thousandths
$\frac{86}{1000}$	0	0	8	6		eighty-six thousandths
$\frac{3275}{10\,000}$	0	3	2	7	5	three thousand, two hundred seventy-five ten thousandths
$\frac{58}{10\,000}$	0	0	0	5	8	fifty-eight ten thousandths

Exercises
Express each as a decimal fraction.

1. $\frac{5}{10}$ 0.5	2. $\frac{9}{10}$ 0.9	3. $\frac{6}{10}$ 0.6	4. $\frac{10}{10}$ 1.0	5. $\frac{17}{100}$ 0.17
6. $\frac{75}{100}$ 0.75	7. $\frac{8}{100}$ 0.08	8. $\frac{1}{100}$ 0.01	9. $\frac{875}{1000}$ 0.875	10. $\frac{25}{1000}$ 0.025
11. $\frac{3}{1000}$ 0.003	12. $\frac{456}{1000}$ 0.456	13. $\frac{38}{1000}$ 0.038	14. $\frac{2}{100}$ 0.02	15. $\frac{399}{1000}$ 0.399
16. $\frac{99}{10\,000}$ 0.0099	17. $\frac{2539}{10\,000}$ 0.2539	18. $\frac{725}{10\,000}$ 0.0725	19. $\frac{65}{10\,000}$ 0.0065	20. $\frac{4}{10\,000}$ 0.0004
21. seven hundred twenty-three thousandths 0.723	22. twenty-nine hundredths 0.29	23. seven hundred thirteen thousandths 0.713	24. four hundred eighty-five ten thousandths 0.0485	
★ 25. one hundred two and one tenth 102.1	★ 26. sixty-six and ninety-two thousandths 66.092			

200 Decimal equivalence

2. Have the students challenge and test each other by having them prepare pie plate punch cards as shown.



denominators), equivalent fractions, reducing fractions, improper and mixed fractions, and fraction-decimal equivalence).

EXTRA PRACTICE

Express as decimal fractions.

- | | | |
|---------------------------|--------------------------|----------------------|
| 1. $\frac{3}{100}$ | 2. $\frac{71}{100}$ | 3. $\frac{97}{1000}$ |
| 4. $\frac{29}{10\,000}$ | 5. $\frac{735}{10\,000}$ | 6. $\frac{14}{1000}$ |
| 7. $\frac{8325}{10\,000}$ | 8. $\frac{739}{10\,000}$ | 9. $\frac{15}{100}$ |
| 10. $\frac{53}{10\,000}$ | | |

3. See "Football" as described in the Activity Reservoir. Use cards which briefly review the fraction skills covered so far (i.e., recognition, simple addition and subtraction (use like

Adding and Subtracting Decimals

$$0.7 + 0.3816 + 0.925 + 0.93 + 0.32148$$

Remember
Adding and
subtracting decimals
is like adding and
subtracting whole numbers.

$$\begin{array}{r} 0.7 \\ 0.3816 \\ 0.925 \\ 0.93 \\ + 0.32148 \\ \hline 3.25808 \end{array}$$

$$0.93847 - 0.72546$$

$$\begin{array}{r} 0.93847 \\ - 0.72546 \\ \hline 0.21301 \end{array}$$

Always line up the decimal points to maintain the correct decimal-place values.

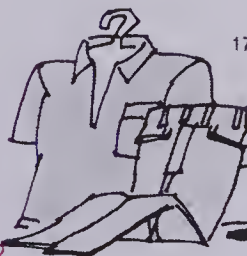
Exercises

Solve.

1. $0.92 + 0.71 + 0.32$ **1.95** 2. $0.85 - 0.16$ **0.69** 3. $0.783 + 0.123 + 0.857$ **1.763**
 4. $0.673 - 0.128$ **0.545** 5. $0.94 + 0.5 + 0.71$ **2.15** 6. $0.5371 - 0.28$ **0.2571**
 7. $0.7 + 0.389 + 0.9513$ **2.0403** 8. $0.63 - 0.318$ **0.312** 9. $\$16.75 + \3.65 **$\$20.40$**
 10. $\$12.93 - \6.17 **$\$6.76$** 11. $\$52.03 + \2.08 **$\$54.11$** 12. $\$20.15 - \13.85 **$\$6.30$**
 13. $0.63842 + 0.270547$ **0.908967** 14. $0.9352 - 0.2519$ **0.6833**
 ★15. $0.1235 + 0.73 + 0.82719 + 8.3451 + 0.579864 + 0.325074$ **10.930728**

16. Calculate the total cost of new spring clothes:

Shirt	—\$ 7.95
Jeans	—\$24.95
Socks	—\$ 3.50
Shoes	—\$18.75
Sweater	—\$16.95 $\\72.10



17. Four \$20 bills were used to pay for the clothes in Exercise 16. What was the change? **$\$7.90$**



Adding and subtracting decimals 201

ACTIVITIES

1. Prepare a "Sum Fun" activity card:

- (a) Write all the decimal numbers from Exercises 1 through 12 on a small card, (1 decimal number per card)

e.g., 0.92 0.71

- (b) Shuffle the cards. Select 3. Calculate the sum.
 (c) Create and solve as many questions as you can.
 (d) Check your answers using a calculator.

2. See the "Flippin' Subtraction" idea described as Activity 3 on page 9.
 3. Use catalogues from local department stores. Have the students (without pencil or paper) try to select 5 items that will yield a total closest to \$200.00 (without tax). Have them check their selections.

OBJECTIVE

To review the addition and subtraction of decimals

PACING

Level A 1-14, 16
 Level B 7-17
 Level C 7-17

MATERIALS

graph paper

BACKGROUND

This page reviews the concepts first presented on pages 6 to 9. This exercise should be done in conjunction with page 200 as a joint assignment to review equivalence, place value, and addition and subtraction.

SUGGESTIONS

Initial Activity "Anna wrote 4 cheques during the month of February; \$1.87, \$8.70, \$12.05, and \$3.99. Calculate the total of all cheques written."

Ask students how they would solve this problem. What are some mistakes that could be made?

Solve the problem cooperatively.

Example

$$\begin{array}{r} 12.2 \\ \$1.87 \\ 8.70 \\ 12.05 \\ + 3.99 \\ \hline \$26.61 \end{array}$$

Discuss *reasons* why the decimal points are aligned.

USING THE BOOK

Have the students: (a) cover up the solutions to the 2 examples in the display; (b) write both questions in a *vertical* format and calculate the answers; and (c) check their answers with those in the display.

Assign the exercises. You may wish to have the exercises completed on graph paper. Working on grids with these types of computations helps keep work legible and organized.

OBJECTIVE

To add and subtract without regrouping mixed numerals

PACING

Level A All
Level B All
Level C All

RELATED AIDS

BFA COMP LAB II—71, 75.
BFA PROB. SOLVING LAB II—134.

BACKGROUND

See the Career Awareness notes in the Chapter Overview, page 184.

A *flat* is a square low box which contains a number of potted flowers or plants.

SUGGESTIONS

Initial Activity Review addition of unlike fractions using the example:
 $\frac{1}{2} + \frac{2}{5}$. Show how the common denominator is determined.

Record the solution:

$$\begin{array}{r} \frac{1}{2} = \frac{5}{10} \\ + \frac{2}{5} = \frac{4}{10} \\ \hline \frac{9}{10} \end{array}$$

USING THE BOOK

Discuss the problem in the display at the top of the page. Determine that addition is required.

Compare the solution of $1\frac{1}{2} + 3\frac{2}{5}$ to the above solution.

(a) The common denominators are the same.

(b) The fraction equivalents are the same.

$$\begin{array}{r} \text{Hence } 1\frac{1}{2} = 1\frac{5}{10} \\ + 3\frac{2}{5} = 3\frac{4}{10} \\ \hline 4\frac{9}{10} \end{array}$$

Steps:

- (1) Form equivalent common fractions.
- (2) Add fractional parts.
- (3) Add whole numbers.

Before assigning the exercises, complete Exercises 1 and 2 orally.

ACTIVITIES

1. Have the children design a problem involving mixed numerals about "Things That Grow".

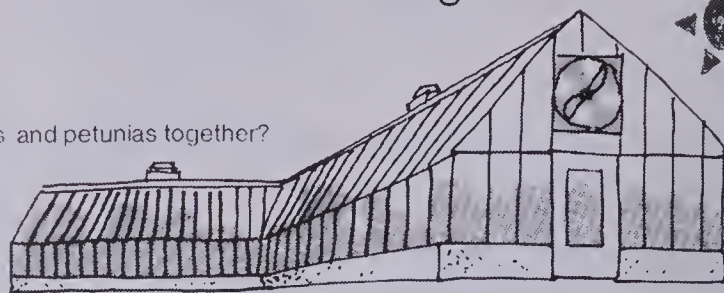
The Greenhouse Manager

$1\frac{1}{2}$ flats of carnations.

$3\frac{2}{5}$ flats of petunias

How many flats of carnations and petunias together?

$$\begin{array}{r} 1\frac{1}{2} \longrightarrow 1\frac{5}{10} \\ + 3\frac{2}{5} \longrightarrow + 3\frac{4}{10} \\ \hline 4\frac{9}{10} \end{array}$$



There are $4\frac{9}{10}$ flats of carnations and petunias

Exercises

Copy and complete.

$$\begin{array}{r} 3\frac{1}{2} = 3\frac{3}{6} \\ + 1\frac{1}{3} = 1\frac{2}{6} \\ \hline = 4\frac{5}{6} \end{array}$$

$$\begin{array}{r} 5\frac{4}{5} = 5\frac{8}{10} \\ - 2\frac{1}{10} = 2\frac{1}{10} \\ \hline = 3\frac{7}{10} \end{array}$$

$$\begin{array}{r} 3\frac{2}{3} = 3\frac{10}{15} \\ - 2\frac{3}{5} = 2\frac{9}{15} \\ \hline = 1\frac{1}{15} \end{array}$$

$$\begin{array}{r} 4. \quad 2\frac{3}{4} \\ + 4\frac{1}{5} \\ \hline 6\frac{19}{20} \end{array}$$

$$\begin{array}{r} 5. \quad 5\frac{7}{8} \\ - 3\frac{1}{4} \\ \hline 2\frac{5}{8} \end{array}$$

$$\begin{array}{r} 6. \quad 3\frac{3}{10} \\ + 4\frac{1}{2} \\ \hline 7\frac{8}{10} \text{ or } 7\frac{4}{5} \end{array}$$

$$\begin{array}{r} 7. \quad 5\frac{2}{5} \\ - 1\frac{3}{10} \\ \hline 4\frac{7}{10} \end{array}$$

$$\begin{array}{r} 8. \quad 12\frac{1}{3} \\ + 7\frac{1}{4} \\ \hline 19\frac{7}{12} \end{array}$$

$$\begin{array}{r} 9. \quad 18\frac{4}{5} \\ - 7\frac{1}{2} \\ \hline 11\frac{3}{10} \end{array}$$

$$\begin{array}{r} 10. \quad 11\frac{1}{8} \\ + 8\frac{3}{4} \\ \hline 19\frac{17}{24} \end{array}$$

$$\begin{array}{r} 11. \quad 12\frac{1}{5} \\ - 6\frac{1}{3} \\ \hline 6\frac{7}{15} \end{array}$$

12. The manager had $3\frac{3}{5}$ rows of pink alyssum and $2\frac{1}{4}$ rows of blue alyssum. How many rows of alyssum in all does the manager have? $5\frac{13}{20}$

13. There are $6\frac{4}{5}$ flats of marigolds. The manager sold $2\frac{1}{3}$ flats of marigolds. How many flats of marigolds were left? $4\frac{7}{15}$

2. Have the children design a problem involving mixed numerals about a hobby. For each of the problems have the children:

- (a) write solutions for both of the problems that were designed;
- (b) write each problem on a file card and sign their name to the card. Put problems in a file for sharing.

3. Prepare a deck of about 25 cards, each one showing a reduced mixed numeral (i.e., $1\frac{1}{2}$, $2\frac{1}{5}$, $7\frac{3}{10}$, etc.). Shuffle the deck and deal 3 cards per

player (3 to 6). On the word "Go!" have players (a) arrange them in order smallest to largest and (b) write an equivalent mixed numeral for each (i.e., $1\frac{3}{6}$, $2\frac{2}{10}$, $7\frac{6}{20}$). The first player to meet both requirements (a) and (b) scores 1 point. The first player to score five points overall is the winner.

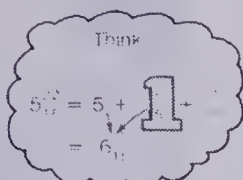
EXTRA PRACTICE

Select and solve 2 problems from the file described in Activities.

The Science Fair

Jennifer made a model of a volcano for the science fair. She spent $3\frac{2}{3}$ h making the plaster model on Saturday and $2\frac{4}{5}$ h painting it on Monday. How much time did she spend on her project?

$$\begin{array}{r} 3\frac{2}{3} = 3\frac{10}{15} \\ + 2\frac{4}{5} = 2\frac{12}{15} \\ \hline 5\frac{22}{15} = 6\frac{7}{15} \end{array}$$



Jennifer spent $6\frac{7}{15}$ h on the project.



Exercises

Add

$$\begin{array}{r} 2\frac{1}{2} = 2\frac{2}{4} \\ + 3\frac{3}{4} = 3\frac{3}{4} \\ \hline 5\frac{5}{4} = 6\frac{1}{4} \end{array}$$

$$\begin{array}{r} 5\frac{1}{2} = 5\frac{10}{20} \\ + 1\frac{3}{4} = 1\frac{15}{20} \\ \hline 6\frac{25}{20} = 7\frac{5}{4} \end{array}$$

$$\begin{array}{r} 3\frac{2}{3} = 3\frac{4}{6} \\ + 5\frac{1}{2} = 5\frac{3}{6} \\ \hline 8\frac{5}{6} \end{array}$$

$$\begin{array}{r} 2\frac{3}{8} = 2\frac{3}{8} \\ + 3\frac{1}{4} = 3\frac{2}{8} \\ \hline 5\frac{4}{8} = 5\frac{1}{2} \end{array}$$

$$\begin{array}{r} 7\frac{3}{8} = 7\frac{3}{8} \\ + 1\frac{1}{4} = 1\frac{2}{8} \\ \hline 8\frac{4}{8} = 9\frac{1}{2} \end{array}$$

$$\begin{array}{r} 2\frac{9}{10} = 2\frac{9}{10} \\ + 2\frac{1}{5} = 2\frac{2}{10} \\ \hline 4\frac{10}{10} = 5 \end{array}$$

$$\begin{array}{r} 3\frac{5}{6} = 3\frac{10}{12} \\ + 2\frac{1}{4} = 2\frac{3}{12} \\ \hline 5\frac{13}{12} = 6\frac{1}{12} \end{array}$$

$$\begin{array}{r} 2\frac{3}{10} = 2\frac{3}{10} \\ + 1\frac{1}{5} = 1\frac{2}{10} \\ \hline 3\frac{4}{10} = 3\frac{2}{5} \end{array}$$

$$\begin{array}{r} 4\frac{1}{8} = 4\frac{1}{8} \\ + 3\frac{3}{8} = 3\frac{3}{8} \\ \hline 7\frac{4}{8} = 7\frac{1}{2} \end{array}$$

$$\begin{array}{r} 1\frac{7}{8} = 1\frac{7}{8} \\ + 4\frac{6}{8} = 4\frac{6}{8} \\ \hline 5\frac{13}{8} = 6\frac{5}{8} \end{array}$$

- *11. Duncan spent $3\frac{1}{2}$ h on Monday, $2\frac{1}{3}$ h on Tuesday, and $1\frac{1}{6}$ h on Wednesday making his science-fair project.

How long did he spend making his project? $7\frac{5}{12}$ h

Adding mixed numerals (regrouping) 203

USING THE BOOK

Discuss the volcano model problem and demonstrate the solution using the cut-out “ones” if necessary.

Emphasize that the fraction component in $5\frac{22}{15}$ (i.e., $\frac{22}{15}$) is improper and that it can be changed to $\frac{15}{15}$ (which equals 1) with $\frac{7}{15}$ left over.

$$\begin{aligned} \text{Therefore } 5\frac{22}{15} &= 5 + \frac{15}{15} + \frac{7}{15} \\ &= 6 + \frac{7}{15} \\ &= 6\frac{7}{15} \end{aligned}$$

To ensure that the students understand, have them express these as simplified mixed numerals:

- (a) $1\frac{5}{4}$ (b) $3\frac{7}{6}$ (c) $2\frac{17}{12}$
(d) $5\frac{27}{20}$ (e) $6\frac{13}{8}$ (f) $3\frac{35}{24}$

Answers:

- (a) $2\frac{1}{4}$ (b) $4\frac{1}{6}$ (c) $3\frac{5}{12}$
(d) $6\frac{7}{20}$ (e) $7\frac{5}{8}$ (f) $4\frac{11}{24}$

If necessary, complete Exercises 1 and 2 orally before assigning the exercises.

ACTIVITIES

1. Have the children supply the correct numerals for such puzzles as:

$$\begin{array}{r} 2\frac{3}{4} = \square\frac{15}{20} \\ + \square\frac{1}{5} = 4\square\frac{\square}{\square} \\ \hline \square\frac{\square}{\square} \end{array}$$

$$\begin{array}{r} 2\frac{\square}{5} = \square\frac{8}{\square} \\ + \square\frac{\square}{\square} = 9\frac{5}{10} \\ \hline \square\frac{\square}{\square} = \square\frac{\square}{\square} \end{array}$$

2. Use a checker or chessboard and a coin to play “Flippin’ Mixed Addition”. Write mixed numerals as shown on slips of paper.

$8\frac{1}{2}$	$6\frac{3}{5}$	$2\frac{1}{3}$	$3\frac{3}{4}$	$5\frac{1}{6}$	$4\frac{3}{8}$	$7\frac{1}{2}$	$9\frac{2}{3}$

Start

OBJECTIVE

To add mixed numerals and express the answer as a mixed numeral

PACING

Level A 1-10
Level B 1-10
Level C All

MATERIALS

6 large cutouts in the shape of “one” like this:



Acetate “pies” suggested on page 197.

RELATED AIDS

BFA COMP LAB II — 73.

SUGGESTIONS

Initial Activity Review fractions equivalent to one whole using the acetate “pies” on the overhead projector. Be sure to:

- (a) Ask, “How many quarters in one whole?” $\frac{4}{4} = 1$. Prepare one of

the cut-out “ones” to show: $\frac{4}{4}$.

- (b) Show $\frac{3}{3} = 1$ and $\frac{5}{5} = 1$ using pies and “ones” cutouts.

- (c) Ask, “How many fifteenths in one whole?” $\frac{15}{15}$.

Players take turns flipping a coin and moving a chess or checker piece one space forward to the right for heads and one space forward to the left for tails. When a player arrives at a number, it becomes his/her score and is added to that player’s cumulative total. The player with the greatest total after a predetermined number of trips across the board wins.

3. Have the children try a creative problem-solving assignment such as:

(Continued on page 212)

OBJECTIVE

To subtract mixed numerals (with regrouping)

PACING

Level A 1-8

Level B 1-5, 7, 8, 10-12

Level C All

RELATED AIDS

HMS — DM48.

BFA COMP LAB II — 75.

SUGGESTIONS

Initial Activity Give this question to the students to solve. (Allow 3 min.)

$$3\frac{1}{2} - 1\frac{2}{3} = ?$$

Ask if there are any problems. Elicit such responses as:

"I can't take $\frac{4}{6}$ from $\frac{3}{6}$."

"I don't have enough sixths on top." etc.

Show that regrouping is required.

$$\begin{array}{r} 3\frac{1}{2} = 3\frac{3}{6} = 2\frac{9}{6} \\ - 1\frac{2}{3} = 1\frac{4}{6} = 1\frac{4}{6} \\ \hline \end{array}$$

Borrow 1 whole from the 3, leaving 2 wholes. The 1 borrowed is equivalent to $\frac{6}{6}$ so $3\frac{3}{6}$ becomes $2 + \frac{6}{6} + \frac{3}{6}$ or $2\frac{9}{6}$.

Remind the children that 1 whole equals $\frac{4}{4}$, $\frac{6}{6}$, $\frac{10}{10}$, $\frac{12}{12}$, $\frac{20}{20}$, $\frac{30}{30}$, etc.

$$\begin{array}{r} 2\frac{9}{6} \\ - 1\frac{4}{6} \\ \hline 1\frac{5}{6} \end{array}$$

USING THE BOOK

Explain the regrouping step in the display problem at the top of the page and in Exercises 1 and 2 of the exercises.

Example

$$\begin{aligned} \text{Display: } 5\frac{5}{15} &= (4 + \frac{15}{15}) + \frac{5}{15} \\ &= 4 + (\frac{15}{15} + \frac{5}{15}) \\ &= 4\frac{20}{15} \end{aligned}$$

$$\begin{aligned} \text{Exercise 1: } 8\frac{3}{12} &= (7 + \frac{12}{12}) + \frac{3}{12} \\ &= 7(\frac{12}{12} + \frac{3}{12}) \\ &= 7\frac{15}{12} \end{aligned}$$

$$\begin{aligned} \text{Exercise 2: } 7\frac{8}{20} &= (6 + \frac{20}{20}) + \frac{8}{20} \\ &= 6 + (\frac{20}{20} + \frac{8}{20}) \\ &= 6\frac{28}{20} \end{aligned}$$

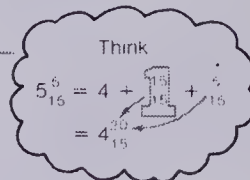
Building Models

Clover spent $5\frac{1}{3}$ h building her model, and Stephanie spent $3\frac{4}{5}$ h making her project. How much longer did Clover spend?

In this example, regrouping is required.

$$\begin{array}{r} 5\frac{1}{3} = 5\frac{5}{15} = 4\frac{20}{15} \\ - 3\frac{4}{5} = 3\frac{12}{15} = 3\frac{12}{15} \\ \hline \end{array}$$

$$1\frac{8}{15}$$



Clover spent $1\frac{8}{15}$ h longer on her project.

Exercises

Subtract. (Use regrouping when required.)

$$\begin{array}{r} 8\frac{1}{4} = 8\frac{3}{12} = 7\frac{15}{12} \\ - 2\frac{2}{3} = 2\frac{8}{12} = 2\frac{8}{12} \\ \hline 5\frac{7}{12} \end{array}$$

$$\begin{array}{r} 7\frac{2}{5} = 7\frac{8}{20} = 6\frac{28}{20} \\ - 1\frac{3}{4} = 1\frac{15}{20} = 1\frac{15}{20} \\ \hline 5\frac{13}{20} \end{array}$$

$$\begin{array}{r} 9\frac{3}{10} \\ - 3\frac{3}{4} \\ \hline 5\frac{11}{20} \end{array}$$

$$\begin{array}{r} 8\frac{1}{4} \\ - 2\frac{1}{2} \\ \hline 5\frac{3}{4} \end{array}$$

$$\begin{array}{r} 6\frac{3}{8} \\ - 3\frac{2}{3} \\ \hline 2\frac{17}{24} \end{array}$$

$$\begin{array}{r} 5\frac{1}{3} \\ - 2\frac{7}{10} \\ \hline 2\frac{19}{30} \end{array}$$

$$\begin{array}{r} 12\frac{1}{8} \\ - 4\frac{3}{4} \\ \hline 7\frac{3}{8} \end{array}$$

$$\begin{array}{r} 14\frac{2}{3} \\ - 9\frac{1}{4} \\ \hline 5\frac{5}{12} \end{array}$$

$$\begin{array}{r} 13\frac{2}{5} \\ - 8\frac{1}{2} \\ \hline 4\frac{9}{10} \end{array}$$

$$\begin{array}{r} 10\frac{1}{5} \\ - 3\frac{5}{8} \\ \hline 6\frac{23}{40} \end{array}$$

11. Which question did not involve regrouping? Why not? *The minuend was greater than the subtrahend.*

★12. Scott, Sandra, and Gina worked as a group on a science model. Scott worked $3\frac{1}{4}$ h. Sandra worked $2\frac{1}{5}$ h, and Gina worked $6\frac{3}{5}$ h. How much longer did Gina work than Scott and Sandra combined? $1\frac{3}{20}$ h

204 Subtracting mixed numerals (regrouping)

Assign the exercises. Encourage the students to follow the model solutions in the display and in Exercises 1 and 2.

ACTIVITIES

1. See the "Fraction Cards" ideas described in the Activity Reservoir. Note especially the "Fraction Snap" idea. Use cards such as:

$$\boxed{6\frac{1}{2}} \quad \boxed{5\frac{3}{2}} \quad \boxed{8\frac{1}{7}} \quad \boxed{7\frac{9}{7}} \quad \text{etc.}$$

2. Have the students prepare puzzles such as these for exchange with classmates:

$$3\frac{3}{4} = \boxed{\frac{15}{4}}; \quad \boxed{\frac{15}{5}} = 8\frac{9}{5}; \quad \text{etc.}$$

3. Use the card sets supplied for Activity 1 to play "Concentration".

EXTRA PRACTICE

Regroup each of these mixed numerals so that the whole number is one less.

Example

$$\begin{aligned} 7\frac{3}{10} &= 6 + \frac{10}{10} + \frac{3}{10} \\ &= 6 + (\frac{10}{10} + \frac{3}{10}) \\ &= 6\frac{13}{10} \end{aligned}$$

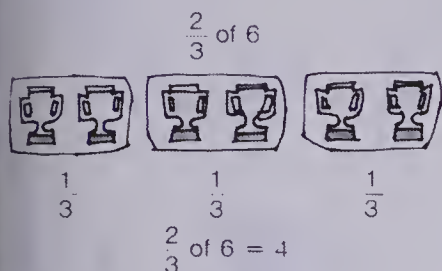
$$\begin{array}{llll} 1. 5\frac{2}{3} & 2. 8\frac{4}{5} & 3. 4\frac{1}{2} & 4. 3\frac{5}{8} \\ 5. 7\frac{3}{10} & 6. 15\frac{7}{12} & 7. 10\frac{3}{4} & 8. 6\frac{8}{15} \\ 9. 3\frac{9}{20} & 10. 6\frac{13}{24} & & \end{array}$$

The Trophy Shop

Centennial School ordered 6 trophies.

They want $\frac{2}{3}$ of them delivered for the spring awards assembly.

How many should be sent?



$$\frac{2}{3} \times 6$$

$$= \frac{2}{3} \times \frac{6}{1}$$

$$= \frac{12}{3}$$

$$= 4$$

Hint
 $6 = \frac{6}{1}$



Exercises

Copy and complete.

$$\frac{1}{3} \text{ of } 6 = \frac{1}{3} \times \frac{6}{1} = \frac{6}{3} = 2$$

$$\frac{1}{5} \text{ of } 15 = \frac{1}{5} \times \frac{15}{1} = \frac{15}{5} = 3$$

$$\frac{3}{4} \text{ of } 12 = \frac{3}{4} \times \frac{12}{1} = \frac{36}{4} = 9$$

Compute as above.

$$\frac{1}{4} \text{ of } 16 = 4$$

$$\frac{1}{5} \text{ of } 10 = 2$$

$$\frac{1}{10} \text{ of } 30 = 3$$

$$\frac{1}{6} \text{ of } 18 = 3$$

$$\frac{3}{4} \text{ of } 16 = 12$$

$$\frac{3}{5} \text{ of } 10 = 6$$

$$\frac{7}{10} \text{ of } 30 = 21$$

$$\frac{5}{6} \text{ of } 18 = 15$$

$$\frac{2}{3} \times 12 = 8$$

$$\frac{3}{4} \times 20 = 15$$

$$\frac{1}{5} \times 24 = 3$$

$$\frac{3}{8} \times 24 = 9$$

$$\frac{2}{5} \times 20 = 8$$

$$\frac{1}{6} \times 24 = 4$$

$$\frac{3}{10} \times 20 = 6$$

$$\frac{7}{8} \times 16 = 14$$

$$\frac{4}{5} \times 25 = 20$$

$$\frac{2}{3} \times 21 = 14$$

$$\frac{3}{2} \times 10 = 15$$

$$\frac{7}{5} \times 10 = 14$$

$$\frac{5}{4} \times 12 = 15$$

$$\frac{6}{5} \times 10 = 12$$

Fractional parts of whole numbers 205

OBJECTIVE

To multiply whole numbers by fractions

PACING

Level A 1-21

Level B All

Level C 4-25

MATERIALS

centimetre cubes (bingo chips), an overhead projector

RELATED AIDS

BFA PROB. SOLVING LAB II — 135.

BACKGROUND

Review the fact that whole numbers can be expressed as 7 or $\frac{7}{1}$ or 3 or $\frac{3}{1}$, etc.

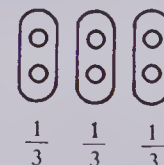
SUGGESTIONS

Initial Activity Using the centimetre cubes (bingo chips) on the overhead projector, demonstrate some parts of groups.

Example

$$\frac{1}{3} \text{ of } 6 = 2$$

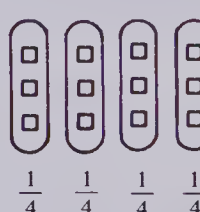
$$\frac{2}{3} \text{ of } 6 = 4$$



Divide
6
into
3 equal
groups.

$$\frac{1}{4} \text{ of } 12 = 3$$

$$\frac{3}{4} \text{ of } 12 = 9$$



Divide
12
into
4 equal
groups.

Provide pairs of students with bingo chips. Assign a problem. One student illustrates it as shown, the second writes and solves a corresponding number sentence.

grid to show the likes of " $\frac{2}{5} \times 20$ ", " $\frac{3}{4} \times 40$ ", " $\frac{1}{9} \times 18$ ", etc., at various coordinate locations. Have the players roll both dice once to yield a location, compute the question, and add the answer to their point total. The player with the greatest total after a predetermined number of turns wins.

USING THE BOOK

Read through the Trophy Shop example at the top of the page.

Discuss why $\frac{2}{3}$ of 6 equals 4. Point out that this information is shown using pictures (at the left of the page) and numbers (at the right of the page).

Identify and record on the chalkboard, the steps involved in multiplying fractions and whole numbers:

1. Change the whole number to a fraction expressed over 1;
2. Multiply numerators;
3. Multiply denominators;
4. Reduce by dividing the denominator into the numerator.

Point out that "of" still means "multiply" (just as it did on page 71) and that the fraction line in the fractions to be reduced means "divide".

Complete Exercises 1, 2, and if necessary, 3 orally before assigning the exercises. Have those children who need to, refer to the steps which are

listed for them on the chalkboard.

Advise the children that their answers for Exercises 22 to 25 will be "surprising". Ask: "What is surprising?" [The answers are greater than the whole numbers that they started with.] and "Why is this so?" [They were multiplying by improper fractions, i.e., fractions greater than one.]

ACTIVITIES

1. Play "Bingo" as described in the Activity Reservoir. Have the children write these 24 numbers randomly on their blank Bingo sheets: 2, 3, 3, 3, 3, 4, 4, 6, 6, 8, 8, 9, 9, 12, 12, 14, 14, 14, 15, 15, 15, 20, 21, 30. Call out (randomly) Exercises 2 to 25 from this page (i.e., $\frac{1}{5} \times 15$; $\frac{3}{4} \times 12$; etc.). Have the children cross off *one* corresponding answer on their grid (i.e., call $\frac{1}{5} \times 15$, cross out one 3, not all four of them).

2. See the "500 Grand" idea in the Activity Reservoir. Change the

OBJECTIVE

To multiply fractions by whole numbers

PACING

Level A All
Level B All
Level C All

MATERIALS

acetate "pies" from page 197

RELATED AIDS

BFA COMP LAB II—76.

BACKGROUND

Even though the calculation of the answers is similar to the work on page 205, the context of these examples is quite different.

On page 205, fractional parts of whole amounts were determined (dividing the whole into groups).

On this page (page 206), whole sets of fractional parts are being combined. (Here we are reminded that multiplication is like repeated addition.)

SUGGESTIONS

Initial Activity Using the "pies" on the overhead projector show:

(a) 3 groups of $\frac{1}{4}$ or $3 \times \frac{1}{4}$.



Total: $\frac{3}{4}$

(b) 3 groups of $\frac{2}{3}$.



Total: $\frac{6}{3}$

Show the calculated answers to the above.

$$(a) 3 \times \frac{1}{4} = \frac{3}{1} \times \frac{1}{4} = \boxed{\frac{3}{4}}$$

$$(b) 3 \times \frac{2}{3} = \frac{3}{1} \times \frac{2}{3} = \frac{6}{3} = \boxed{2}$$

USING THE BOOK

Discuss the two problems at the top of the page.

Note that there are 3 groups of $\frac{1}{2}$ a block of clay.

$$3 \times \frac{1}{2} = \frac{3}{1} \times \frac{1}{2} = \frac{3}{2} \text{ or } 1\frac{1}{2}$$

The Art Class

Each student used $\frac{1}{2}$ block of modelling clay.

How much did 3 students use?

$$3 \times \frac{1}{2} = \frac{3}{1} \times \frac{1}{2} = \frac{3}{2} \text{ or } 1\frac{1}{2}$$

They used $1\frac{1}{2}$ blocks in all.

$$3 \times \frac{1}{2} = \frac{3}{1} \times \frac{1}{2} = \frac{3}{2} \text{ or } 1\frac{1}{2}$$



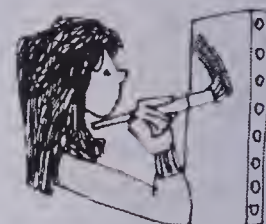
Gabrielle used $\frac{2}{5}$ jar of 4 different colours of tempera paint.

How much did she use in all?



She used $1\frac{2}{5}$ jars in all.

$$4 \times \frac{2}{5} = \frac{4}{1} \times \frac{2}{5} = \frac{8}{5} \text{ or } 1\frac{3}{5}$$



Exercises

$$5 \times \frac{1}{3} = \frac{5}{1} \times \frac{1}{3} = \frac{5}{3} = 1\frac{2}{3}$$

$$2 \times \frac{3}{4} = \frac{2}{1} \times \frac{3}{4} = \frac{6}{4} = 1\frac{2}{4} \text{ or } 1\frac{1}{2}$$

$$3 \times \frac{2}{5} = \frac{3}{1} \times \frac{2}{5} = \frac{6}{5} = 1\frac{1}{5}$$

Compute as above.

$$5 \times \frac{3}{4} = 3\frac{3}{4}$$

$$5. 3 \times \frac{3}{8} = 1\frac{1}{8}$$

$$6. 2 \times \frac{6}{10} = \frac{6}{5} \text{ or } 1\frac{1}{5}$$

$$7. 7 \times \frac{1}{2} = 3\frac{1}{2}$$

$$8. 3 \times \frac{2}{3} = 2$$

$$9. 6 \times \frac{4}{5} = 4\frac{4}{5}$$

$$10. 4 \times \frac{5}{8} = 2\frac{4}{8} \text{ or } 2\frac{1}{2}$$

$$11. 3 \times \frac{7}{10} = 2\frac{1}{10}$$

$$12. 7 \times \frac{5}{6} = 5\frac{5}{6}$$

$$13. 2 \times \frac{3}{10} = \frac{6}{10} \text{ or } \frac{3}{5}$$

$$14. 5 \times \frac{2}{4} = 2$$

$$15. 4 \times \frac{7}{8} = 3\frac{4}{8} \text{ or } 3\frac{1}{2}$$

16. How many balls of cord should the art teacher order for the school macramé display? There are 36 art students, and each will use about $\frac{2}{3}$ of a ball. **24**

206 Whole numbers multiplied by fractions

Note that Gabrielle used 4 groups of $\frac{2}{5}$ of a jar (i.e., the parts of the jars which are empty or *unshaded*).

$$4 \times \frac{2}{5} = \frac{4}{1} \times \frac{2}{5} = \frac{8}{5} \text{ or } 1\frac{3}{5}$$

Assign the exercises. Encourage the students to follow the steps as illustrated in the display and in Exercises 1 to 3.

ACTIVITIES

1. "Multiplication Race". (A game for 2 players)

(a) Copy 2 of each numeral on a red card:

1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

(b) Copy 1 of each fraction on a green card:

$\frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{1}{4}, \frac{3}{4}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \frac{1}{6}, \frac{5}{6}, \frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8}, \frac{1}{9}, \frac{2}{9}, \frac{4}{9}, \frac{5}{9}, \frac{7}{9}, \frac{8}{9}, \frac{1}{10}, \frac{3}{10}, \frac{7}{10}, \text{ and } \frac{9}{10}$.

(c) Place cards in 2 decks.

(d) Each player picks up a red and

green card, and writes the product of the 2 cards.

(e) Each player repeats the above to calculate more products.

(f) The winner is the first player with 5 correct products.

(g) One error and the other player wins. Questions should be recorded as:

$$7 \times \frac{3}{4} = \frac{21}{4} \text{ or } 5\frac{1}{4},$$

given these cards:



Red

Green

2. See Activity 2 on page 205.

3. See "Square It" as described in the Activity Reservoir. Instead of whole numbers showing in the squares on the dot paper, use quantities such as $2 \times \frac{3}{10}$, etc. Players must compute these expressions to find out what their score will be.

Missing Fractions

When Jackie found her exercise book, which had been lost for a week, she noticed that some fractions were covered up by water marks

$$\begin{array}{ll} 1. \frac{3}{4} + \frac{2}{3} = 3 & 2. \frac{5}{8} - \frac{1}{8} = 4 \\ 3. 3 \times \frac{1}{2} = 3 & 4. \frac{1}{3} \times 4 = 3 \end{array}$$

She was able to figure out the missing fractions



Exercises

Find the missing fractions. (Express answers in lowest terms.)

$$\begin{array}{llll} 1. \frac{1}{4} + \frac{1}{4} = 1 & 2. \frac{1}{5} - \frac{1}{5} = 0 & 3. \frac{1}{2} + \frac{1}{2} = 1 & 4. \frac{1}{2} + \frac{1}{2} = 1 \\ 5. 5 \times \frac{1}{3} = \frac{5}{3} & 6. \frac{1}{15} - \frac{1}{15} = 0 & 7. \frac{2}{5} + \frac{2}{5} = \frac{4}{5} & 8. \frac{6}{6} - \frac{6}{6} = 0 \\ 9. \frac{1}{15} \times \frac{1}{15} = \frac{1}{225} & 10. \frac{7}{15} \div \frac{7}{15} = 1 \end{array}$$

Solve for n (n represents the missing fraction)

$$\begin{array}{llll} 11. n + \frac{1}{4} = 1 & 12. n - \frac{1}{5} = 0 & 13. n \times \frac{1}{2} = 1 & 14. \frac{5}{10} \div n = \frac{1}{2} \\ n = \frac{3}{4} & n = \frac{1}{5} & n = 2 & n = \frac{5}{10} \text{ or } \frac{1}{2} \\ 15. \frac{1}{15} \times n = \frac{1}{15} & 16. \frac{9}{12} - n = \frac{1}{12} & 17. \frac{4}{12} \div n = \frac{1}{3} & 18. \frac{4}{2} \div n = 2 \\ n = \frac{1}{15} & n = \frac{8}{12} \text{ or } \frac{2}{3} & n = \frac{4}{12} \text{ or } \frac{1}{3} & n = \frac{4}{2} \text{ or } 2 \end{array}$$

Solve for n (Use common denominators)

$$\begin{array}{llll} 20. \frac{6}{15} \div n = \frac{2}{5} & 21. n + \frac{1}{10} = \frac{12}{20} & 22. n - \frac{1}{6} = \frac{1}{6} & 23. \frac{1}{3} - n = \frac{1}{4} \\ n = \frac{6}{15} \text{ or } \frac{2}{5} & n = \frac{11}{20} & n = \frac{2}{6} \text{ or } \frac{1}{3} & n = \frac{1}{12} \end{array}$$

Solving equations 207

OBJECTIVE

To solve equations involving fractions

PACING

Level A 1-15

Level B 1-19

Level C All

BACKGROUND

This exercise encourages the guessing and inspection method for solving equations and a technique involving number properties and inverse operations.

SUGGESTIONS

Initial Activity On the overhead projector or the chalkboard show these questions.

(a) $\blacksquare + 3 = 8$

Ask: "What number goes in the box to make this equation true?"

$\blacksquare = 5$

because $5 + 3 = 8$.

(b) $\blacksquare - 5 = 2$

Ask: "What number goes in the box to make this equation true?"

$\blacksquare = 7$

because $7 - 5 = 2$.

Point out that this "guess and test" approach is called solving equations by inspection. It is a very useful technique.

Inverse operations could also be used. Addition and subtraction are inverse operations.

Show related true statements.

$$\begin{array}{ll} 5 + 3 = 8 & \text{and} \quad 8 - 3 = 5 \\ 7 - 5 = 2 & \text{and} \quad 2 + 5 = 7 \end{array}$$

These inverse relationships can be used to solve equations.

$n + 3 = 8$ and $8 - 3 = n$ Thus $n = 5$.

$n - 5 = 2$ and $2 + 5 = n$ Thus $n = 7$.

EXTRA PRACTICE

Have each student solve 10 equations which were designed by 2 of his or her classmates.

USING THE BOOK

Look at the four questions in the display and have students suggest some fractional solutions for the missing fractions.

Possible solutions:

1. $\blacksquare + \frac{2}{3} = \frac{3}{3}$

$\blacksquare = \frac{1}{3}$

because

$\frac{1}{3} + \frac{2}{3} = \frac{3}{3}$

Inspection

$\blacksquare + \frac{2}{3} = \frac{3}{3}$

or

$\frac{3}{3} - \frac{2}{3} = \blacksquare$

$\blacksquare = \frac{1}{3}$

Using inverse operations

3. $3 \times \blacksquare = \frac{3}{2}$

Think: 3 times what = $\frac{3}{2}$.

$\blacksquare = \frac{1}{2}$

because

$\frac{3}{1} \times \frac{1}{2} = \frac{3}{2}$.

Inspection

4. $\blacksquare \times 4 = \frac{4}{3}$

What times 4 equals $\frac{4}{3}$?

$\blacksquare = \frac{1}{3}$

because

$\frac{1}{3} \times 4 = \frac{4}{3}$.

Inspection

$n + \frac{1}{10} = \frac{7}{10}$

$n = \frac{6}{10}$

because

$\frac{6}{10} + \frac{1}{10} = \frac{7}{10}$

Inspection

$n + \frac{1}{10} = \frac{7}{10}$

or $\frac{7}{10} - \frac{1}{10} = n$

$\frac{6}{10} = n$

Inverse

Operations

Assign the exercises.

ACTIVITIES

1. "What's My Fraction?" Have students (a) write 5 true addition or subtraction statements involving fractions; (b) replace either the first or second fraction with an "n", e.g., $\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$ becomes $n + \frac{2}{6} = \frac{5}{6}$ or $\frac{3}{6} + n = \frac{5}{6}$; (c) write the 5 equations thus formed on a file card, sign their names to it, and place it in a central file for sharing.

2. See the "Fraction Cards" ideas in the Activity Reservoir.

3. Have the children prepare punch cards as described in Activity 2, page 186. Have them use equations of the sort shown on this page.

Before assigning the exercises, demonstrate two solutions for Exercise 11.

OBJECTIVE

To multiply a fraction by a fraction

PACING

Level A 1-14, 19

Level B All

Level C All

MATERIALS

transparencies as described on page 188

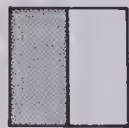
RELATED AIDS

HMS — DM49.

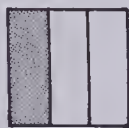
BFA COMP LAB II — 77.

SUGGESTIONS

Initial Activity Use a water colour marker to colour in the fully clear transparencies described on page 188 to show:



Half



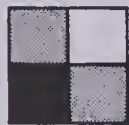
Third



Fourth

Overlay these newly coloured transparencies on top of the other originally coloured transparencies to show that $\frac{1}{2}$ of $\frac{1}{2}$ is $\frac{1}{4}$.

Example



Repeat this for various other combinations, showing that, by watching for the double shaded portions, $\frac{1}{2}$ of $\frac{1}{4}$ is $\frac{1}{8}$, $\frac{1}{3}$ of $\frac{1}{4}$ is $\frac{1}{12}$, $\frac{1}{4}$ of $\frac{2}{3}$ is $\frac{2}{12}$, etc. Once these are shown graphically, relate them to the numerical calculations. Emphasize that, when multiplying fractions, simply multiply numerators together and denominators together to yield the product.

USING THE BOOK

Read through the pizza example at the top of the page. Point out that, as Gail and Bryan each take their one half of what's left (i.e., $\frac{3}{4}$), that they are in fact each receiving one half of three fourths or $\frac{1}{2} \times \frac{3}{4}$ or $\frac{3}{8}$ of the original pizza.

Complete Exercises 1 and 2 orally, relating the graphic to the numeric representations. Emphasize again that numerators are multiplied together and

The Pizza Party

Bryan and Gail were late. $\frac{3}{4}$ of a rectangular pizza was saved for them. They each ate $\frac{1}{2}$ of that. What part of a whole pizza did Bryan eat?



Bryan ate $\frac{3}{8}$ of a rectangular pizza

$$\frac{1}{2} \text{ of } \frac{3}{4} = ? \longrightarrow \frac{1}{2} \times \frac{3}{4} = \frac{1 \times 3}{2 \times 4} = \frac{3}{8}$$

Exercises

Compute.

$$\frac{1}{2} \text{ of } \frac{3}{5} = \frac{1}{2} \times \frac{3}{5} = \frac{1 \times 3}{2 \times 5} = \frac{3}{10}$$



$$\frac{1}{3} \text{ of } \frac{3}{4} = \frac{1}{3} \times \frac{3}{4} = \frac{1 \times 3}{3 \times 4} = \frac{3}{12} \text{ or } \frac{1}{4}$$



$$\frac{1}{2} \text{ of } \frac{4}{5} = \frac{4}{10} \text{ or } \frac{2}{5}$$

$$\frac{1}{2} \text{ of } \frac{3}{10} = \frac{3}{20}$$

$$\frac{1}{3} \text{ of } \frac{5}{8} = \frac{5}{24}$$

$$\frac{1}{3} \text{ of } \frac{3}{5} = \frac{3}{15} \text{ or } \frac{1}{5}$$

$$7. \frac{1}{4} \text{ of } \frac{5}{6} = \frac{5}{24}$$

$$8. \frac{1}{4} \text{ of } \frac{7}{10} = \frac{7}{40}$$

$$9. \frac{2}{3} \text{ of } \frac{3}{4} = \frac{6}{12} \text{ or } \frac{1}{2}$$

$$10. \frac{3}{4} \text{ of } \frac{2}{3} = \frac{6}{12} \text{ or } \frac{1}{2}$$

$$11. \frac{1}{2} \times \frac{5}{6} = \frac{5}{12}$$

$$12. \frac{1}{3} \times \frac{9}{10} = \frac{9}{30}$$

$$13. \frac{4}{5} \times \frac{5}{6} = \frac{20}{30} \text{ or } \frac{2}{3}$$

$$14. \frac{3}{10} \times \frac{2}{3} = \frac{6}{30} \text{ or } \frac{1}{5}$$

$$15. \frac{1}{2} \times \frac{5}{3} = \frac{5}{6}$$

$$16. \frac{1}{3} \times \frac{7}{5} = \frac{7}{15}$$

$$17. \frac{2}{3} \times \frac{5}{4} = \frac{10}{12} \text{ or } \frac{5}{6}$$

$$18. \frac{3}{4} \times \frac{3}{2} = \frac{9}{8} \text{ or } 1\frac{1}{8}$$

19. A large motorcycle has $\frac{2}{5}$ as much mass as the pizza-delivery car. The car has a mass of $\frac{3}{4}$ t.

What is the mass of the motorcycle? $\frac{3}{10}$ t

20. A motorcycle has a mass of $\frac{1}{12}$ t. A racing bicycle has a mass that is $\frac{2}{9}$ as much. What is the mass of the bicycle? $\frac{1}{54}$ t

208 Multiplying fractions

denominators are multiplied by each other.

Assign the exercises. You may wish to allow unreduced answers at this point to help emphasize the arithmetic process and reduce possible confusion (e.g., Exercises 6, 9, 10, 12 to 14, 17 to 20). The answer for Exercise 18 will be greater than 1.

ACTIVITIES

1. Have the children play "Multiplication Madness". (A game for 2 or 3 players)

(a) Create a set of fraction cards containing 2 each of:

$$\frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{1}{4}, \frac{3}{4}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \frac{1}{6}, \frac{5}{6}, \frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8}, \frac{1}{9}, \frac{2}{9}, \frac{4}{9}, \frac{5}{9}, \frac{7}{9}, \frac{8}{9}, \frac{1}{10}, \frac{3}{10}, \frac{7}{10}, \text{ and } \frac{9}{10}.$$

(b) Shuffle the cards.

(c) Each player selects 3 pairs of cards and places them face down.

(d) At the "Start" signal, each player

turns over a pair of cards, writes the multiplication question using the two fractions, and calculates the fully reduced answer.

(e) Repeat this until all 3 reduced products are found.

(f) The winner is the first person finished with all answers correct.

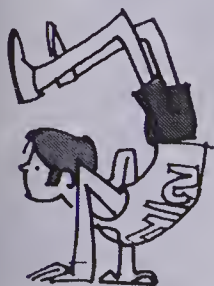
2. See "500 Grand" as described in the Activity Reservoir. Replace the numbers in the grid with various fraction amounts. Have the players keep a cumulative total of their points (this will involve addition of unlike fractions too). The winner is the player with the greatest total after a predetermined number of turns.

EXTRA PRACTICE

Each student could create 10 questions by drawing 10 pairs of fraction cards, copying down the questions, and finding the products.

Tumbling Fractions

Reciprocals are like tumbling fractions.



The reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$

The reciprocal of $\frac{4}{3}$ is $\frac{3}{4}$

The product of two reciprocals is always 1.

$$\frac{3}{4} \times \frac{4}{3} = \frac{12}{12} = 1$$

The reciprocal of 4 is $\frac{1}{4}$ because $\frac{4}{1} \times \frac{1}{4} = \frac{4}{4} = 1$.



Exercises

1. Copy and complete the chart for each.

$\frac{2}{3}, 1, \frac{3}{5}, \frac{5}{8}, 1, \frac{8}{5}, \frac{9}{4}, 1, \frac{4}{9}, \frac{3}{10}, \frac{10}{7}, \frac{7}{10}, 1, \frac{2}{1}, \frac{1}{2}, 1, \frac{1}{3}, 3, 1, 5, \frac{1}{5}, 1, \frac{1}{8}, 8, 1, 3, \frac{1}{3}, 1, \frac{1}{4}, 4, 1$

Given fraction	Reciprocal	Product of both
$\frac{2}{3}$	$\frac{3}{2}$	$\frac{2}{3} \times \frac{3}{2} = \frac{6}{6} = 1$
$\frac{3}{5}$	$\frac{5}{3}$	$\frac{3}{5} \times \frac{5}{3} = \frac{15}{15} = 1$

Solve for n .

$$n \times \frac{3}{4} = 1$$

$$n = \frac{4}{3}$$

$$n \times 5 = 1$$

$$n = \frac{1}{5}$$

$$\frac{1}{2} \times n = 1$$

$$n = 2$$

$$5. \frac{4}{5} \times n = 1 \quad \frac{5}{4}$$

$$6. \frac{7}{10} \times n = 1 \quad \frac{10}{7}$$

$$7. \frac{7}{8} \times n = 1 \quad \frac{8}{7}$$

$$8. \frac{10}{3} \times n = 1 \quad \frac{3}{10}$$

$$9. 6 \times n = 1 \quad \frac{1}{6}$$

$$10. 4 \times n = 1 \quad \frac{1}{4}$$

$$11. \frac{5}{8} \times \frac{8}{5} = n \quad 1$$

$$12. \frac{2}{1} \times \frac{1}{2} = n \quad 1$$

$$13. \frac{7}{6} \times \frac{6}{7} = n \quad 1$$

$$14. n \times \frac{1}{3} = 1 \quad 3$$

$$15. n \times \frac{1}{8} = 1 \quad 8$$

$$16. 14 \times n = 1 \quad \frac{1}{14}$$

Reciprocals 209

OBJECTIVE

To introduce reciprocals

PACING

Level A 1-12

Level B All

Level C All

MATERIALS

flash cards showing reciprocals: $\frac{1}{2}$ on one side, $\frac{2}{1}$ on the other

BACKGROUND

Reciprocals are taught so that the formal algorithm for division of fractions can be presented.

SUGGESTIONS

Initial Activity Define the reciprocal using examples and flash cards.

Note that the product of 2 reciprocals is 1. Mention that any whole number can be expressed as a fraction with 1 as its denominator.

Thus 5 is $\frac{5}{1}$ and the reciprocal of $\frac{5}{1}$ is $\frac{1}{5}$.

USING THE BOOK

Encourage the students to record their reciprocals in a chart. This overcomes a common error when students write $\frac{1}{2} = \frac{2}{1}$ which is *not* a true statement. Solving the equations in Exercises 2 through 16 applies the students' knowledge of reciprocals.

ACTIVITIES

1. Arrange working teams of 3 or 4 students to help in the preparation of "Reciprocal Flash Cards". Have the students:

(a) Make the reciprocal flash cards using black and coloured magic markers.

(b) Write all the fractions from the "Fraction Tower" on page 198 on cards using a black felt pen.

(c) Flip each card over and write the reciprocal using coloured felt pen.

(d) Write the whole numbers from 1 through 10 on cards.

(e) Write the reciprocals of the whole numbers on the flip side in colour.

Shuffle the cards and use them to drill each other, or teach another group about reciprocals.

2. Use the cards described in Activity 1 to play "Flips". Players, in twos, should each hold an equal number of cards. Players take turns calling out either "match" or "no match" as both simultaneously flip a card. (Players should play in standing position, releasing their cards so that they flip several times before landing on the floor.) If both cards do in fact correspond to what was called (i.e., match — $\frac{5}{1}$, $\frac{7}{1}$ or $\frac{1}{6}$, $\frac{1}{3}$ or not match — $\frac{1}{5}$, $\frac{2}{1}$), the calling player wins those cards. If the cards do not correspond to what was called, they remain on the floor, thereby building the jackpot. The winner is the player with the most cards after a predetermined number of turns or amount of time.

OBJECTIVE

To divide a whole number by a fraction and a fraction by a whole number

PACING

Level A 1-10, 23


Level B All


Level C All



RELATED AIDS

HMS — DM45 and DM50.

MATERIALS

transparencies showing six 

and 12  which, when fit together,

show two  and 3 

respectively

BACKGROUND

The purpose of this page is to have students visualize how many fractional parts are in a whole amount.

Unit fraction amounts are easy.

Example

$2 \div \frac{1}{4} \rightarrow$ How many quarters in 1? [4]

How many quarters in 2? [8]

$2 \div \frac{1}{4} = 8$

SUGGESTIONS

Initial Activity Use the above grids to show:

(a) There are 6 “one thirds” in 2.

Thus $2 \div \frac{1}{3} = 6$.

(b) There are 12 “quarters” in 3.

Thus $3 \div \frac{1}{4} = 12$.

USING THE BOOK

Read through the information at the top of the page noting that

$2 \div \frac{1}{3} = 6$ but

$2 \times \frac{3}{1}$ also equals 6

and

$3 \div \frac{1}{4} = 12$ but

$3 \times \frac{4}{1}$ also equals 12.

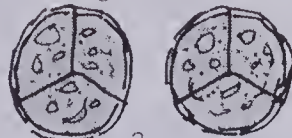
Point out that a faster way to calculate the answer is to invert the divisor and multiply.

Do Exercise 1 with the class.

Remind the students that the question asks: “How many quarters in 5?”

Division Using Reciprocals

How many $\frac{1}{3}$'s of a pie in 2 whole pies?



But $2 \times \frac{3}{1} = 6$

There are 6 one thirds in 2.

$$2 \div \frac{1}{3} = 6$$

Therefore: $2 \div \frac{1}{3} = \frac{2}{1} \times \frac{3}{1}$

Reciprocals of the divisor can be used as a short cut when dividing.

For $3 \div \frac{1}{4}$

write: $\frac{3}{1} \times \frac{4}{1} = 12$.

To divide by a fraction, multiply by its reciprocal.



Exercises

Rewrite each division question as a multiplication question and solve

1. $5 \div \frac{1}{2} = \frac{5}{1} \times \frac{2}{1}$
= **10**

2. $5 \div \frac{1}{3} = \frac{5}{1} \times \frac{3}{1}$
= **15**

3. $3 \div \frac{1}{4} = \frac{3}{1} \times \frac{4}{1}$
= **12**

4. $4 \div \frac{2}{3} = \frac{4}{1} \times \frac{3}{2} = 6$

5. $3 \div \frac{3}{4} = \frac{3}{1} \times \frac{4}{3} = 4$

6. $2 \div \frac{2}{3} = \frac{2}{1} \times \frac{3}{2} = 3$

7. $5 \div \frac{5}{6} = \frac{5}{1} \times \frac{6}{5} = 6$

8. $\frac{1}{2} \div 2 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

9. $\frac{1}{3} \div 2 = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

10. $\frac{1}{4} \div 3 = \frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$

11. $10 \div 2 = \frac{10}{1} \times \frac{1}{2} = 5$

12. $10 \div 3 = \frac{10}{1} \times \frac{1}{3} = \frac{10}{3}$

13. $10 \div 3 = \frac{10}{1} \times \frac{1}{3} = \frac{10}{3}$

14. $\frac{1}{8} \div 3 = \frac{1}{8} \times \frac{1}{3} = \frac{1}{24}$

15. $\frac{2}{5} \div 2 = \frac{2}{5} \times \frac{1}{2} = \frac{1}{5}$

16. $\frac{7}{8} \div 5 = \frac{7}{8} \times \frac{1}{5} = \frac{7}{40}$

17. $7 \div \frac{7}{1} = \frac{7}{1} \times \frac{1}{7} = 1$

18. $\frac{3}{8} \div 2 = \frac{3}{8} \times \frac{1}{2} = \frac{3}{16}$

19. $9 \div \frac{9}{1} = \frac{9}{1} \times \frac{1}{9} = 1$

20. $\frac{9}{10} \div 2 = \frac{9}{10} \times \frac{1}{2} = \frac{9}{20}$

21. $10 \div \frac{10}{1} = \frac{10}{1} \times \frac{1}{10} = 1$

22. $\frac{4}{5} \div 3 = \frac{4}{5} \times \frac{1}{3} = \frac{4}{15}$

23. A group of campers ate 5 dozen eggs for breakfast. Each camper ate $\frac{1}{4}$ dozen eggs. How many campers were there? **20**

210 Division using reciprocals

$$5 \div \frac{1}{4} = 20 \rightarrow 5 \div \frac{1}{4} = 5 \times \frac{4}{1}$$

$$= \frac{5 \times 4}{1}$$

$$= 20$$

This shows the division algorithm.

Point out also that: there are

(a) four $\frac{1}{4}$'s in 1,

(b) eight $\frac{1}{4}$'s in 2,

(c) twelve $\frac{1}{4}$'s in 3,

(d) sixteen $\frac{1}{4}$'s in 4,

(e) twenty $\frac{1}{4}$'s in 5.

This shows a logical investigation.

Complete Exercises 1 to 3, 8, and, if necessary, Exercise 9 orally before assigning the exercises.

ACTIVITIES

1. Play “Bingo” as described in the Activity Reservoir. Call out expressions such as $\frac{4}{5} \div 3$, $7 \div \frac{1}{3}$, etc. and have corresponding whole number amounts crossed off the Bingo grid.

2. See “Road Rally” as described in the Activity Reservoir. Use Pit Stop cards which reflect the skills from this lesson.

3. See “500 Grand” as described in the Activity Reservoir. Use a grid which shows expressions such as $\frac{7}{10} \div 2$, $9 \div \frac{3}{5}$, etc. and have players throw just 1 set of dice to yield a location. The player performs the calculation indicated at that grid location and adds the quotient to his/her cumulative total. The player with the greatest point total after a predetermined number of turns wins.

Dividing Fractions

How many $\frac{1}{8}$'s of a pizza are in $\frac{1}{2}$ pizza?

$$\begin{aligned}\frac{1}{2} \div \frac{1}{8} &= \frac{1}{2} \times \frac{8}{1} \\ &= \frac{8}{2} \\ &= 4\end{aligned}$$



There are 4 pieces

There are four $\frac{1}{8}$'s in $\frac{1}{2}$

To divide by a fraction, multiply by its reciprocal.



Exercises

Write each division question as a multiplication question and solve

Express all answers in lowest terms

$$\begin{aligned}1. \quad \frac{7}{8} \div \frac{1}{4} &= \frac{7}{8} \times \frac{4}{1} \\ &= \frac{56}{8} \\ &= 7\end{aligned}$$

$$\begin{aligned}2. \quad \frac{3}{5} \div \frac{1}{2} &= \frac{3}{5} \times \frac{2}{1} \\ &= \frac{6}{5} \\ &= 1\frac{1}{5}\end{aligned}$$

$$3. \quad \frac{1}{4} \div \frac{1}{2} = \frac{1}{4} \times \frac{2}{1} = \frac{1}{2}$$

$$4. \quad \frac{3}{10} \div \frac{1}{2} = \frac{3}{10} \times \frac{2}{1} = \frac{6}{10} = \frac{3}{5}$$

$$5. \quad \frac{7}{10} \div \frac{1}{4} = \frac{7}{10} \times \frac{4}{1} = \frac{28}{10} = \frac{14}{5}$$

$$6. \quad \frac{2}{3} \div \frac{1}{10} = \frac{2}{3} \times \frac{10}{1} = \frac{20}{3}$$

$$7. \quad \frac{4}{5} \div \frac{1}{10} = \frac{4}{5} \times \frac{10}{1} = 8$$

$$8. \quad \frac{7}{8} \div \frac{10}{3} = \frac{7}{8} \times \frac{3}{10} = \frac{21}{80}$$

$$9. \quad \frac{9}{10} \div \frac{4}{3} = \frac{9}{10} \times \frac{3}{4} = \frac{27}{40}$$

$$10. \quad \frac{11}{8} \div \frac{2}{1} = \frac{11}{8} \times \frac{1}{2} = \frac{11}{16}$$

$$11. \quad \frac{1}{2} \div \frac{4}{3} = \frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$$

$$12. \quad \frac{1}{3} \div \frac{3}{2} = \frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$$

$$13. \quad \frac{1}{3} \div \frac{7}{10} = \frac{1}{3} \times \frac{10}{7} = \frac{10}{21}$$

$$14. \quad \frac{3}{10} \div \frac{8}{5} = \frac{3}{10} \times \frac{5}{8} = \frac{15}{80} = \frac{3}{16}$$

$$15. \quad \frac{1}{2} \div \frac{6}{5} = \frac{1}{2} \times \frac{5}{6} = \frac{5}{12}$$

$$16. \quad \frac{4}{10} \div \frac{3}{4} = \frac{4}{10} \times \frac{4}{3} = \frac{16}{30} = \frac{8}{15}$$

$$17. \quad \frac{2}{4} \div \frac{8}{8} = \frac{2}{4} \times \frac{8}{8} = \frac{16}{32} = \frac{1}{2}$$

$$18. \quad \frac{1}{10} \div \frac{2}{1} = \frac{1}{10} \times \frac{1}{2} = \frac{1}{20}$$

$$19. \quad \frac{3}{8} \div \frac{5}{3} = \frac{3}{8} \times \frac{3}{5} = \frac{9}{40}$$

$$20. \quad \frac{1}{2} \div \frac{9}{10} = \frac{1}{2} \times \frac{10}{9} = \frac{10}{18} = \frac{5}{9}$$

★ 21. Explain why the answers to Questions 1 through 10 are greater than 1. *The dividend is greater than the divisor.*

★ 22. Explain why the answers to Questions 11 through 20 are less than 1. *The divisor is greater than the dividend.*

Dividing fractions 211

OBJECTIVE

To divide fractions using the formal algorithm

PACING

Level A All (21 and 22 are optional)

Level B All (21 and 22 are optional)

Level C All

RELATED AIDS

HMS — DM50.

BFA COMP LAB II — 79.

BACKGROUND

The main purpose of this lesson is to practise the formal division of fractions by fractions algorithm.

SUGGESTIONS

Initial Activity Show a pizza divided into eight equal parts.

Ask: "How many 'eighths' in one half pizza?" Answer graphically and numerically:



$$\frac{1}{2} \div \frac{1}{8} = 4$$

or

$$\begin{aligned}\frac{1}{2} \div \frac{1}{8} &= \frac{1}{2} \times \frac{8}{1} \\ &= \frac{1 \times 8}{2 \times 1} \\ &= \frac{8}{2} \text{ or } 4\end{aligned}$$

USING THE BOOK

Review the terms "dividend", "divisor", and "quotient".

Example

Dividend		Divisor		Quotient
$\frac{1}{2}$	\div	$\frac{1}{8}$	$=$	4

Note that the divisor is the number that comes after the division sign.

Assign the exercises. You may wish to advise students to check their answers to Exercises 1 to 3 when they are finished before proceeding.

Students who answer Exercises 21 and 22 correctly have good number sense and a sense of pattern.

ACTIVITIES

1. Have the children play "Division Madness". Use the cards from "Multiplication Madness" (Activity 1 on page 208) using similar rules.

Example

$$\begin{aligned}\frac{7}{8} \div \frac{1}{2} &= \frac{7}{8} \times \frac{2}{1} \\ &= \frac{14}{8} \\ &= \frac{7}{4} \text{ or } 1\frac{3}{4}\end{aligned}$$

2. Have the students use "Multiplication Madness" cards to create 10 random division questions for exchange with classmates.

3. See the "Fraction Cards" ideas in the Activity Reservoir.

OBJECTIVE

To solve word problems involving fractions

PACING

Level A 1-5

Level B 1-5

Level C 1-5

VOCABULARY

volunteered, committee

BACKGROUND

Review the steps in problem solving with the class. (See Professor Q, pages 17 and 22.)

Exercise 6 is optional at the teacher's discretion.

USING THE BOOK

Read the situation of the Class Party through together. Discuss some of the pertinent facts required.

Example

There are 36 students.

$\frac{1}{9}$ of the students brought ice cream.

How many brought ice cream?

$$\frac{1}{9} \times 36 = \frac{36}{9}$$

4

4 students brought ice cream.

Mention that Exercise 1 is really 6 questions in one. Have the children label each part (a), (b), (c), (d), (e), and (f).

Assign the page. Later problems presume that each student got previous problems correct.

ACTIVITIES

1. Use Exercise 6 as an activity.

These questions and many others are worthy of discussion.

(a) Research how much certain items cost at the grocery store before deciding how many students bring certain items. Why is it easier to have the class collect equal amounts from each student and have a committee buy the food for the party?

(b) What other planning must be done for a class luncheon or picnic? List these. Assign volunteers to research the possibilities and report back to the class.

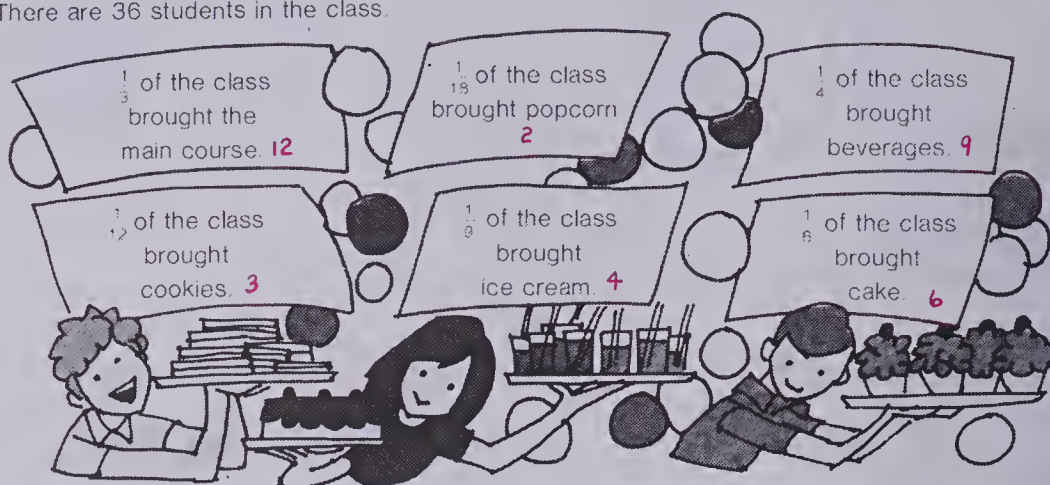
2. Have the students each write 3 true statements about the class. Have them written in word-problem format. Discuss these and solve them together.

The Class Party!

Mr. Williamson's class decided to have a class lunch.

Each member of the class volunteered to bring one item of the lunch.

There are 36 students in the class.



1. How many students brought each of the above items? *See above.*
2. Did every student participate by bringing something for the lunch? *Yes*
3. Each student who volunteered to bring ice cream brought 2 L. How many litres of ice cream were used? *8 L*
4. Two food groups formed the clean-up committee. There were 6 people. Which food groups formed the clean-up committee? *Popcorn and ice cream*
5. $\frac{1}{3}$ of a food group formed the games committee. Which group did the games committee come from if there were 4 on this committee? *Main course*
- ★ 6. Plan your own class luncheon or picnic and determine what the class would like to eat. How many students should provide each type of food so that the cost is shared equally? What fraction of the class is providing each type of food?

212 Problem solving

(Continued from page 203)

Design a "Science Problem" involving addition of mixed numerals.

- (a) Write a complete solution.
- (b) Write the problem on a file card and sign your name to the card.
- (c) Place problems in a file for sharing.

EXTRA PRACTICE

Select and solve at least one of the problems designed by a classmate. When completed, have the author check your work.

Example

$\frac{1}{2}$ of our class are girls. How many girls?

$\frac{1}{4}$ of the people in our class live in apartments. How many people is this? etc.

Mixed Numerals

$$1\frac{1}{2} \times 2\frac{2}{5} = \frac{3}{2} \times \frac{12}{5}$$

$$= \frac{36}{10}$$

$$= 3\frac{6}{10}$$

$$= 3\frac{3}{5}$$

- Step 1 Change mixed numerals to improper fractions.
- Step 2 Multiply the improper fraction.
- Step 3 Change the improper fraction to a mixed numeral and reduce.



$$2\frac{3}{4} - 1\frac{1}{2} = \frac{11}{4} - \frac{3}{2}$$

$$= \frac{11}{4} \times \frac{2}{3}$$

$$= \frac{22}{12}$$

$$= 1\frac{10}{12}$$

$$= 1\frac{5}{6}$$

- Step 1 Change to improper fractions.
- Step 2 Use reciprocal of divisor and change operation.
- Step 3 Multiply.
- Step 4 Change improper fraction to mixed numeral and reduce.

Exercises

Express answers in lowest terms.

$$1\frac{3}{4} \times 1\frac{1}{3} = \frac{7}{4} \times \frac{4}{3}$$

$$= \frac{28}{12}$$

$$2 = \frac{24}{12}$$

$$2 = \frac{4}{12}$$

$$3\frac{1}{2} \times 1\frac{2}{3} = \frac{7}{2} \times \frac{5}{3}$$

$$= \frac{35}{6}$$

$$5 = \frac{30}{6}$$

$$5 = \frac{5}{6}$$

3. $3\frac{3}{5} \times 2\frac{1}{2}$ 9 4. $2\frac{1}{10} \times 1\frac{3}{7}$ 3 5. $1\frac{1}{5} \times 4\frac{4}{9}$ $5\frac{1}{3}$ 6. $5\frac{1}{3} \times 2\frac{1}{4}$ 12
7. $3\frac{1}{2} \times 1\frac{5}{7}$ 6 8. $3\frac{1}{5} \times 1\frac{1}{4}$ 4 9. $2\frac{1}{3} \times 2\frac{7}{10}$ $6\frac{3}{10}$ 10. $1\frac{7}{8} \times 6\frac{2}{5}$ 12

Calculate.

11. $1\frac{1}{4} - 2\frac{1}{2} = \frac{5}{4} - \frac{5}{2}$ 12. $3\frac{1}{2} + 1\frac{1}{4} = \frac{7}{2} + \frac{5}{4}$ 13. $4\frac{1}{2} - 2\frac{1}{3}$ $1\frac{13}{6}$ 14. $3\frac{1}{2} \div 2\frac{1}{2}$ $1\frac{2}{5}$

$= \frac{5}{4} \times \frac{2}{5} = 1\frac{1}{2}$ $= \frac{7}{2} \times \frac{4}{5} = 2\frac{4}{5}$ 15. $5\frac{1}{4} \div 2\frac{1}{3}$ $2\frac{1}{4}$ 16. $5\frac{1}{3} \div 4\frac{1}{4}$ $1\frac{13}{51}$

Mixed numerals 213

OBJECTIVE

To multiply and divide mixed numerals

PACING

Level A 1-6, 11, 12
Level B 1-8, 11, 12
Level C All

RELATED AIDS

HMS — DM51.
BFA COMP LAB II — 80.

BACKGROUND

It is assumed that by now all students know how to multiply and divide a pair of fractions.

SUGGESTIONS

Initial Activity After reviewing

$$\frac{1}{2} \times \frac{2}{3} = \frac{2}{6}$$

$$= \frac{1}{3}$$

allow students to suggest how they would solve $\frac{3}{2} \times \frac{4}{3} = \blacksquare$.

Most will agree that $\frac{3}{2} \times \frac{4}{3} = \frac{12}{6}$
 $= 2$.

But this question is the same question in disguise:

$$1\frac{1}{2} \times 1\frac{1}{3} = \blacksquare$$

Change mixed numerals to improper fractions.

$$\frac{3}{2} \times \frac{4}{3} = \frac{12}{6}$$

$$= 2$$

Note: $1\frac{1}{2} = \frac{3}{2}$ and $1\frac{1}{3} = \frac{4}{3}$.

3. See the "500 Grand" idea in the Activity Reservoir. Replace the whole numbers in the grid with mixed numerals.

USING THE BOOK

Demonstrate the steps as indicated in the display at the top of the page.

The only step which is new to these questions is the first one, i.e., changing mixed numerals to improper fractions.

The teacher may wish to review this skill using examples such as:

1. $1\frac{1}{2} = \frac{3}{2}$ 2. $2\frac{2}{5} = \frac{12}{5}$
3. $2\frac{3}{4} = \frac{11}{4}$ 4. $1\frac{3}{4} = \frac{7}{4}$
5. $1\frac{1}{3} = \frac{4}{3}$ 6. $3\frac{1}{2} = \frac{7}{2}$
7. $1\frac{2}{3} = \frac{5}{3}$ 8. $3\frac{3}{5} = \frac{18}{5}$
9. $2\frac{1}{10} = \frac{21}{10}$ 10. $4\frac{4}{9} = \frac{40}{9}$

Remind students to follow the steps as outlined while completing the exercises. You may wish to complete Exercises 1, 2, 11, and 12 on the chalkboard first. Note especially that Exercises 11 and 12 provide only partial hints and clues as to the steps involved. " $\frac{5}{4} \times \frac{2}{5}$ " and " $\frac{7}{2} \times \frac{4}{3}$ "

respectively are not complete answers. Have the students proceed through all of the necessary steps.

ACTIVITIES

1. To review the mental operations involved in changing mixed numerals to improper fractions, practise oral drill of such expressions as:

" $2 \times 4 + 1$ "; " $2 \times 6 + 3$ "; " $9 \times 3 + 5$ "; etc.

2. Prepare puzzles as shown to help reinforce the steps involved in multiplying and dividing mixed numerals.

(a) $1\frac{1}{\blacksquare} \times \blacksquare\frac{\blacksquare}{5} = \frac{\blacksquare}{4} \times \frac{12}{5}$

$$= \frac{\blacksquare}{\blacksquare}$$

$$= 3$$

(b) $\blacksquare\frac{1}{\blacksquare} \div 2\frac{\blacksquare}{8} = \frac{7}{2} \div \frac{21}{\blacksquare}$

$$= \frac{\blacksquare}{\blacksquare} \times \frac{\blacksquare}{\blacksquare}$$

$$= \frac{56}{42}$$

$$= 1\frac{\blacksquare}{\blacksquare} = 1\frac{\blacksquare}{\blacksquare} = \blacksquare\frac{\blacksquare}{\blacksquare}$$

OBJECTIVE

To solve word problems

PACING

Level A All

Level B All

Level C All

VOCABULARY

zoologist, Thomson's gazelle, conceived, conception

BACKGROUND

Define the term "conception" for the students as the time when the small animal begins to grow inside the mother.

You may wish to see the Career Awareness notes in the Chapter Overview, page 184.

SUGGESTIONS

Initial Activity Review the steps in problem solving. (See Professor Q, pages 17 and 22.)

You may wish to demonstrate the solution of a problem such as:

A case of pop contains 24 bottles.

$2\frac{3}{4}$ cases will be needed for the graduation party.

How many bottles are needed?

While completing the problem, remind the students that $24 = \frac{24}{1}$.

USING THE BOOK

Assign the problems. Be available to discuss the context of the problem with individual children. Clarify and interpret, but do not tell the student how to do the problem.

When taking up the problems you may wish to discuss:

(a) why some animals eat more than other animals;

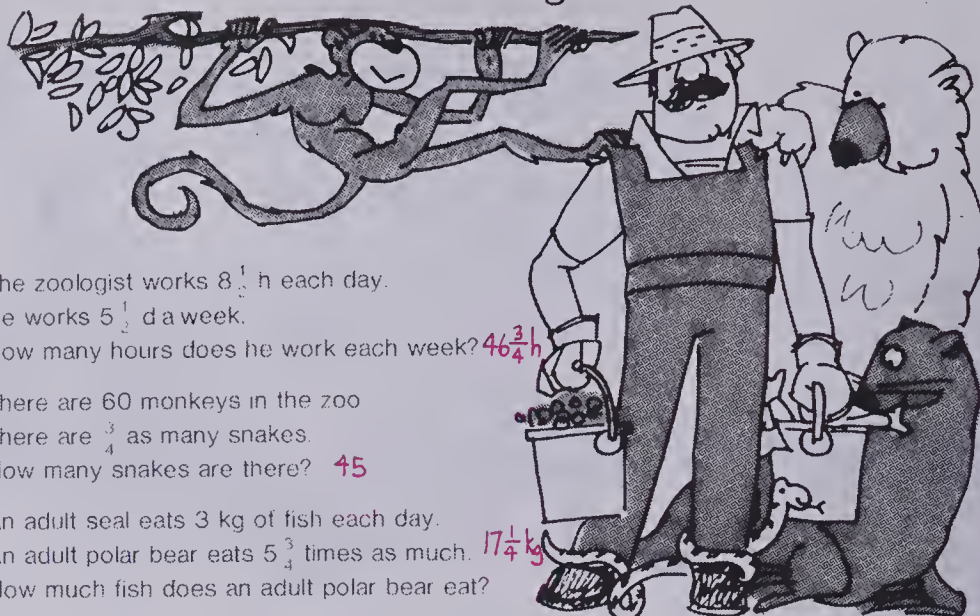
(b) why some animals develop for a longer period before they are born; etc.

ACTIVITIES

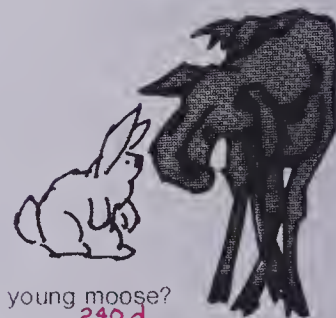
1. If you have not already done so, see Activity 1 for pages 22 and 23 for an idea to help practise choosing the correct operation. Use problems which contain fractions.

2. Have the students write three word problems of their own. Provide them (if necessary) with some guidance by providing helpers such as: "Write 3 word problems using these words and numbers:

The Zoologist



1. The zoologist works $8\frac{1}{2}$ h each day.
He works $5\frac{1}{2}$ d a week.
How many hours does he work each week? $46\frac{3}{4}$ h
2. There are 60 monkeys in the zoo.
There are $\frac{3}{4}$ as many snakes.
How many snakes are there? 45
3. An adult seal eats 3 kg of fish each day.
An adult polar bear eats $5\frac{3}{4}$ times as much. $17\frac{1}{4}$ kg
How much fish does an adult polar bear eat?
4. A Thomson's gazelle can run $1\frac{2}{3}$ times as fast as a grizzly bear.
A grizzly bear can run 48 km/h.
How fast can a Thomson's gazelle run? 80 km/h
5. The zoologist told the keeper to put 21 kg of monkey feed in the monkey cage each day.
Each monkey eats $\frac{3}{4}$ kg of feed.
How many monkeys are in the cage? 28
6. A chicken egg develops for 21 d before it hatches.
A duck egg takes $1\frac{3}{4}$ times as long.
How long does it take a duck egg to hatch? 30 d
7. A baby rabbit is born about 36 d after it is conceived.
A baby moose takes $6\frac{2}{3}$ times as long before it is born.
How many days after conception will a moose give birth to a young moose? 240 d



214 Problem solving

(a) $2\frac{1}{2}$, rolls of tape, 20, hockey games.

(b) cases of soup, $8\frac{1}{3}$, weekend sale, 18 cans per case.

(c) pizzas, party, $\frac{1}{4}$, $2\frac{1}{2}$.

3. Prepare an activity card such as:

Animal Research

Choose an animal to learn about. Read reference books, speak to experts, and find out all the information you can about this animal.

Design 3 questions using this information.

Write your questions along with a picture of your animal. Sign your name to your work.

Share your questions with your classmates.

EXTRA PRACTICE

Answer 2 or 3 questions designed by one of your classmates.

Decimal Equivalents

About $\frac{3}{4}$ of a ship is above water

Express this fraction as a decimal.

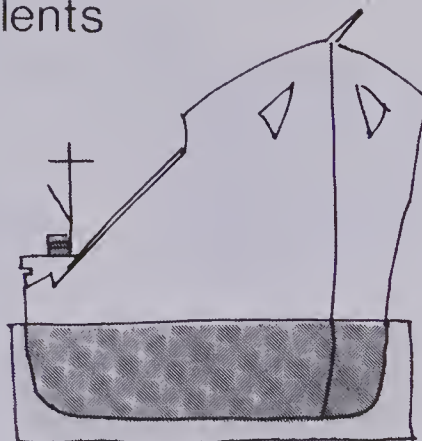
We find the decimal equivalent by dividing:

Think $\frac{3}{4} \rightarrow 4 \overline{)3}$

$$\begin{array}{r} 0.75 \\ 4 \overline{)3.00} \\ \underline{28} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

The decimal equivalent of $\frac{3}{4}$ is 0.75

0.75 of the ship is above water



Exercises

Divide to find the decimal equivalents.

- | | | | |
|---------------------------|----------------------------|----------------------------|----------------------------|
| 1. $\frac{1}{4} = 0.25$ | 2. $\frac{1}{5} = 0.2$ | 3. $\frac{4}{5} = 0.8$ | 4. $\frac{3}{5} = 0.6$ |
| 5. $\frac{1}{25} = 0.04$ | 6. $\frac{1}{50} = 0.02$ | 7. $\frac{7}{20} = 0.35$ | 8. $\frac{9}{25} = 0.36$ |
| 9. $\frac{19}{50} = 0.38$ | 10. $\frac{21}{25} = 0.84$ | 11. $\frac{19}{20} = 0.95$ | 12. $\frac{37}{50} = 0.74$ |

By dividing we know

$$\frac{1}{20} = 0.05$$

Therefore:

$$\begin{aligned} \frac{7}{20} &= 7 \times \frac{1}{20} \\ &= 7 \times 0.05 \\ &= 0.35 \end{aligned}$$

Find these decimal equivalents using multiplication:

- | | | | |
|---------------------------|--------------------------|----------------------------|----------------------------|
| 13. $\frac{3}{20} = 0.15$ | 14. $\frac{6}{20} = 0.4$ | 15. $\frac{11}{20} = 0.55$ | 16. $\frac{19}{20} = 0.95$ |
|---------------------------|--------------------------|----------------------------|----------------------------|

Find these decimal equivalents using multiplication:

- | | | | |
|---------------------------|---------------------------|---------------------------|--------------------------|
| 17. $\frac{3}{8} = 0.375$ | 18. $\frac{5}{8} = 0.625$ | 19. $\frac{7}{8} = 0.875$ | 20. $\frac{6}{8} = 0.75$ |
|---------------------------|---------------------------|---------------------------|--------------------------|

Decimal equivalents 215

SUGGESTIONS

Initial Activity Start with a review of the fraction line being used to say "divide". Point out that this is nothing new and that they have used it before (see Background notes above).

Complete several examples ($\frac{15}{3}$, $\frac{24}{4}$, $\frac{48}{16}$, etc.) to reinforce the idea. Point out that, in these instances, the fractions were improper, that is, *greater than 1*. Therefore, the quotients were also *greater than 1* (i.e., 5, 6, 3).

Extend the idea by informing the students that this same process can be performed on fractions which are *less than 1*. Ask: "What effect should this have on our answers?" Elicit the prediction that the quotients will be also *less than 1*.

Numerically demonstrate the following:

(a) $\frac{1}{2}$ means $1 \div 2 \longrightarrow 2 \overline{)1.0}$

$$\begin{array}{r} 0.5 \\ 2 \overline{)1.0} \\ \underline{10} \\ 0 \end{array}$$

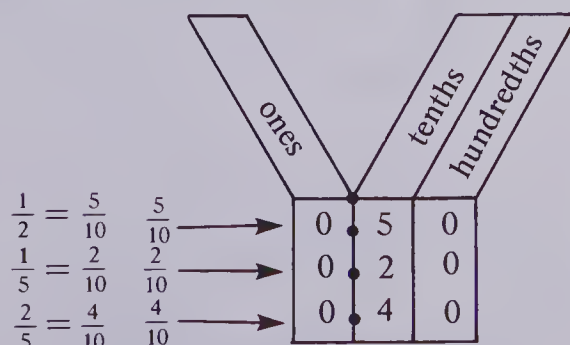
(b) $\frac{1}{5}$ means $1 \div 5 \longrightarrow 5 \overline{)1.0}$

$$\begin{array}{r} 0.2 \\ 5 \overline{)1.0} \\ \underline{10} \\ 0 \end{array}$$

(c) $\frac{2}{5}$ means $2 \div 5 \longrightarrow 5 \overline{)2.0}$

$$\begin{array}{r} 0.4 \\ 5 \overline{)2.0} \\ \underline{20} \\ 0 \end{array}$$

(d) Check using equivalent fractions and place-value charts.



With regard to the above, emphasize that

(a) in these cases, divide until the remainder is zero;

OBJECTIVE

To change fractions to their decimal equivalents

PACING

Level A 1, 2, 4, 5, 7, 8, 11-16

Level B All

Level C All

RELATED AIDS

BFA COMP LAB II — 119.

BACKGROUND

Division is used because fractions imply division. A meaning of $\frac{1}{2}$ is "1 thing divided by 2" or "1 thing divided into 2 equal parts." You may have discussed this in conjunction with page 205. If so, remind the students that, just as $\frac{12}{3}$ can be interpreted as $12 \div 3$, or 4, any numerator can be interpreted as capable of being divided by its denominator without changing the essential value represented.

(b) once we knew that $\frac{1}{5} = 0.2$, we

we could just have easily stated

$$\begin{aligned} \frac{2}{5} &= 2 \times \frac{1}{5} \\ &= 2 \times 0.2 \\ &= 0.4 \end{aligned}$$

without redoing the division process over again for $\frac{2}{5}$;

(c) fractions and their decimal equivalents represent *the same amount*. They are simply different ways of writing it. (See pages 2, 3, and 200.)

USING THE BOOK

Read through the example at the top of the page together, consolidating what was presented during the Initial Activity.

Assign the exercises. Point out the different instructions for some exercises. Exercises 1 to 12 request

division: $\frac{1}{4} \longrightarrow 4 \overline{)1.00}$

$$\begin{array}{r} 0.25 \\ 4 \overline{)1.00} \\ \underline{8} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

while Exercises 13 to 20 suggest using multiplication as shown by the " $\frac{7}{20}$ " example in the box.

(Continued on page 218)

OBJECTIVE

To investigate fractions whose decimal equivalents are repeating decimals

PACING

Level A 1-24
Level B 1-25
Level C 1-25

MATERIALS

a four-function calculator

RELATED AIDS

BFA COMP LAB II — 120.

BACKGROUND

In this exercise encourage the students to look for patterns in the quotients.

SUGGESTIONS

Initial Activity Illustrate $\frac{1}{3}$ as a decimal:

$$\begin{array}{r} 0.333 \dots \\ 3 \overline{) 1.0000} \\ \underline{9} \\ 10 \\ \underline{9} \\ 10 \\ \underline{9} \\ 10 \\ \vdots \end{array}$$

$$\frac{1}{3} = 0.333 \dots$$

Point out that the division does not end. The quotient will continue to be 3's.

Have the children predict what $\frac{2}{3}$ will be.

Write down the predictions.

Show the division.

$$\begin{array}{r} 0.666 \dots \\ 3 \overline{) 2.000} \\ \underline{18} \\ 20 \\ \underline{18} \\ 20 \\ \underline{18} \\ 2 \\ \vdots \end{array}$$

$$\frac{2}{3} = 0.666 \dots$$

Tell the students that some decimal equivalents have digits which repeat.

Repeating Decimals

Dividing to find the decimal equivalents of $\frac{1}{3}$ and $\frac{3}{11}$ produces some interesting patterns

$$\frac{1}{3} = ?$$

$$\begin{array}{r} 0.333 \dots \\ 3 \overline{) 1.000} \\ \underline{9} \\ 10 \\ \underline{9} \\ 10 \\ \underline{9} \\ 10 \\ \vdots \end{array}$$

$$\frac{1}{3} = 0.333 \dots$$

$$\frac{3}{11} = ?$$

$$\begin{array}{r} 0.2727 \dots \\ 11 \overline{) 3.0000} \\ \underline{22} \\ 80 \\ \underline{77} \\ 30 \\ \underline{22} \\ 80 \\ \vdots \end{array}$$

$$\frac{3}{11} = 0.2727 \dots$$

Which numbers repeat?



Exercises

Use division and/or patterns to determine the decimal equivalents.

- | | | | | |
|-----------------------------------|-----------------------------------|------------------------------------|-----------------------------------|-----------------------------------|
| 1. $\frac{1}{6} = 0.1666 \dots$ | 2. $\frac{2}{6} = 0.3333 \dots$ | 3. $\frac{3}{6} = 0.5$ | 4. $\frac{4}{6} = 0.6666 \dots$ | 5. $\frac{5}{6} = 0.8333 \dots$ |
| 6. $\frac{1}{9} = 0.1111 \dots$ | 7. $\frac{2}{9} = 0.2222 \dots$ | 8. $\frac{3}{9} = 0.3333 \dots$ | 9. $\frac{4}{9} = 0.4444 \dots$ | 10. $\frac{5}{9} = 0.5555 \dots$ |
| 11. $\frac{6}{9} = 0.6666 \dots$ | 12. $\frac{7}{9} = 0.7777 \dots$ | 13. $\frac{8}{9} = 0.8888 \dots$ | 14. $\frac{1}{11} = 0.0909 \dots$ | 15. $\frac{2}{11} = 0.1818 \dots$ |
| 16. $\frac{3}{11} = 0.2727 \dots$ | 17. $\frac{4}{11} = 0.3636 \dots$ | 18. $\frac{5}{11} = 0.4545 \dots$ | 19. $\frac{6}{11} = 0.5454 \dots$ | 20. $\frac{7}{11} = 0.6363 \dots$ |
| 21. $\frac{8}{11} = 0.7272 \dots$ | 22. $\frac{9}{11} = 0.8181 \dots$ | 23. $\frac{10}{11} = 0.9090 \dots$ | | |

24. Pick out 2 pairs of equivalent fractions that have identical decimal equivalents. $\frac{2}{6}$ and $\frac{3}{9}$ $\frac{4}{6}$ and $\frac{6}{9}$
- ★25. Investigate fractions whose denominators are 27, 33, 66, 45. What do you notice about their decimal equivalents?

216 Repeating decimals

USING THE BOOK

The decimal equivalent for $\frac{3}{11}$ repeats in a different way.

$$\begin{array}{r} 0.2727 \dots \\ 11 \overline{) 3.0000} \\ \underline{22} \\ 80 \\ \underline{77} \\ 30 \\ \underline{22} \\ 80 \\ \underline{77} \\ 3 \\ \vdots \end{array}$$

$$\frac{3}{11} = 0.2727272727 \dots$$

This time 2 digits repeat in order — 2 then 7.

Assign the exercises. Encourage the students to look for patterns to save time.

Allow some students to use

calculators for Exercise 25.

Remind the students that (a) zero should be written in the one's place in the quotients to help prevent confusion and that (b) "... " means "and so on".

ACTIVITIES

1. Prepare some fraction-decimal statements for completion by students. Have answers expressed in decimals.

Example

$$(a) \frac{1}{4} + \frac{1}{3} = \square \quad (b) \frac{3}{4} + 0.3 = \square$$

$$(c) 0.66 - \frac{1}{3} = \square \quad \text{etc.}$$

2. See the "find-the-pattern" activities noted for pages 198 and 199.

3. Prepare an assignment card such as:

(Continued on page 219)

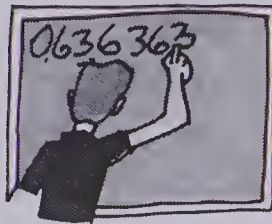
Rounding Repeating Decimals

In order to use repeating decimals such as 0.636 363... in calculations, we round the decimal to the degree of accuracy required.

0.636 363... to the nearest tenth = 0.6

0.636 363... to the nearest hundredth = 0.64

0.636 363... to the nearest thousandth = 0.636



Exercises

Round to the nearest tenth.

1. 0.333 **0.3** 2. 0.1818 **0.2** 3. 0.3939... **0.4**

Round as indicated in the chart.

	Repeating decimal	To nearest tenth	To nearest hundredth	To nearest thousandth
4.	0.833 33...			
5.	0.454 545			
6.	0.575 757...			
7.	0.606 060...			
8.	0.148 148			
9.	0.727 272			
10.	0.740 740			



11. Baseball batting averages are calculated by dividing:

number of times at bat $\overline{)$ number of hits

The quotient is then rounded to 3 decimal places.

Find each player's batting average.

(a) 2 hits in 9 times at bat **0.222** $9 \overline{) 2.000}$ **0.222**

(c) 45 hits in 147 times at bat **0.306**

(b) 6 hits in 11 times at bat **0.545**

(d) 26 hits in 97 times at bat **0.268**

Repeating decimals 217

ANSWERS:

1. 0.8, 0.83, 0.833 5. 0.5, 0.45, 0.455 6. 0.6, 0.58, 0.576
 2. 0.6, 0.61, 0.606 8. 0.1, 0.15, 0.148 9. 0.7, 0.73, 0.727
 3. 0.7, 0.74, 0.741

USING THE BOOK

Assign the exercises.

ACTIVITIES

- See "Eraser" as described in the Activity Reservoir.
- See "Road Rally" as described in the Activity Reservoir. Include Pit Stop cards which review the fraction skills learned to date.
- Play "Bingo" as described in the Activity Reservoir. Have the players write the answers to Exercises 1 to 10 randomly in the 24 available places on their blank Bingo grids. Have the leader call out (again in random fashion) the various questions (i.e., 0.1818 rounded to the nearest

tenth; 0.148 148 rounded to the nearest thousandth; etc.

EXTRA PRACTICE

- "Batting Averages"
 - Create 5 realistic batting averages.
Example
 "____ hits in ____ times at bat."
 - Express each fraction as a decimal rounded to the nearest thousandth.
- "Sevenths"
 - Divide to find the repeating decimal equivalents for $\frac{1}{7}$, $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$, and $\frac{6}{7}$.
 - Round each to the nearest hundredth.

OBJECTIVE

To round repeating decimals to the nearest tenth, hundredth, or thousandth

PACING

Level A All
 Level B All
 Level C All

RELATED AIDS

BFA COMP LAB II — 120.

BACKGROUND

Many students find the repeating digits confusing. Encourage them to cover up the unnecessary digits when rounding.

SUGGESTIONS

Initial Activity Use these examples to explain the rounding of:

(a) 0.636 363 63... to the nearest tenth.

Step 1 Look only at the digits in the tenth's and hundredth's column.

Step 2 Cover up the other digits.

0.63

Step 3 Is the digit in the hundredth's place 5 or more?
 If *yes*, increase the previous digit by 1.
 If *no*, make no change and drop the digits after the tenths.
 \therefore 0.636 363... to the nearest tenth is 0.6.

(b) 0.636 363 63... to the nearest hundredth.

Step 1 Look only at the digits in the tenth's, hundredth's, and thousandth's columns.

Step 2 Cover up the rest of the digits.

0.636

Step 3 Is the digit in the thousandth's place 5 or more?
 If yes, increase the previous digit by *one*.
 If *no*, make no change and drop the digits after the hundredths.
 Since the answer is yes
 0.636 363... to the nearest hundredth is 0.64.

OBJECTIVE
To evaluate achievement of the chapter objectives

PACING

- Level A All
Level B All
Level C All

RELATED AIDS
HMS — DM52.

USING THE BOOK

Each student should do this test independently under supervision. Assistance should be given only when the instructions are not understood. After the work has been corrected, you should provide appropriate remedial work. You may wish to reteach if a large number of students had difficulty with a particular topic or concept.
The following chart will help in this regard. The specific objectives are listed in the Chapter Overview (see page 184).
An alternate Chapter Test can be found in the Holt Mathematics System Duplicating Masters available for use with this grade level.

Table with 3 columns: Test Item, Objective, Text Page Number. Rows include items 2-9, 1, 33-35, 5-12, 10, 11, 13-25, 29-32, 26-28, 36 with corresponding objectives A-H and page ranges.

Chapter Test
1. Complete to form an equivalent fraction! 4/5 = 24/30
Calculate.
2. 8/10 - 3/10 = 5/10 or 1/2
3. 7/10 + 2/10 = 9/10
4. 8/10 + 1/10 = 9/10
5. 4/10 - 1/10 = 3/10
6. 3/4 + 1/4 = 1
7. 5/10 - 4/10 = 1/10
8. 12/12 + 1/12 = 13/12
9. 4/10 - 1/10 = 3/10
10. 3 5/10 + 2 5/10 = 6
11. 6 1/6 - 2 4/6 = 4 1/6
12. Which is larger 7/10 or 4/5?
13. Find 1/3 of 45. 15
14. 1/4 x 16 = 4
15. 1/5 x 25 = 5
16. 3 x 1 1/2 = 4 1/2
17. 1/2 x 3 = 1 1/2
18. 1/2 x 7/25 = 7/50
19. 2 1/4 x 3 1/2 = 7 7/8
20. 3 1/2 x 1 = 3 1/2
21. 4 1/3 - 2 1/2 = 1 2/3
22. 1/2 - 1/5 = 3/10
23. 3 - 1/18 = 53/18
24. 1/2 - 1/2 = 0
25. 1/2 - 2/9 = 1/18
26. 1/5 + n = 7/5, n = 6/5
27. 3/4 - n = 1/2, n = 1/4
28. 6 x n = 3, n = 1/2
29. 0.6666...
30. 0.05
31. 0.6
32. 0.68
33. 3 9/10
34. 2 2/5
35. 3 1/8
36. A class contains 35 students. 2/5 of the class are in the class play.
(a) How many students are in the play? 14
(b) How many students are not in the play? 21

218 Chapter 7 test

(Continued from page 215)

ACTIVITIES

- 1. See "The P.V. Game" as described in the Activity Reservoir. Use both decimal and fraction blanks, i.e., _ _ . _ _ and _ _ / _ _ to build the greatest (or least) values.
2. See "Square It" as described in the Activity Reservoir. Use a mix of both fractions and decimals on the dot paper. Have fractions, when they are squared, changed to decimals when computing cumulative score.
3. Provide a challenge such as:

If 1/4 = 0.25, what are the decimal equivalents of 6/24, 10/40, 8/32, 312/1248?
You should
(a) answer in less than thirty seconds,
(b) not need to divide.

Have the students who try this activity explain why both of the final points are possible. [All four fractions are equivalent to 1/4 and therefore also equal 0.25.]

EXTRA PRACTICE

- A "Equivalent 25ths". Given 1/25 = 0.04. Write the decimal equivalents for:
1. 2/25 2. 6/25 3. 9/25 4. 12/25 5. 13/25
6. 15/25 7. 19/25 8. 21/25 9. 25/25 10. 26/25
B "Equivalent Fortieths". Use a calculator to determine the decimal equivalent for 1/40. Calculate these decimal equivalents.

- 1. 3/40 2. 7/40 3. 9/40 4. 15/40 5. 17/40
6. 20/40 7. 23/40 8. 27/40 9. 33/40 10. 39/40

Cumulative Review

Perform the indicated operations

$$\begin{array}{r} 1. \quad \$ 32.70 \\ 189.16 \\ + 703.09 \\ \hline \$ 924.95 \end{array}$$

$$\begin{array}{r} 2. \quad 0.935 \\ - 0.192 \\ \hline 0.743 \end{array}$$

$$\begin{array}{r} 3. \quad 351 \\ \times 100 \\ \hline 3510 \end{array}$$

$$\begin{array}{r} 4. \quad 2037 \\ \times 5.3 \\ \hline 10796.1 \end{array}$$

$$5. \quad 6 \overline{)384.6} \quad 64.1$$

$$6. \quad 1.3 \overline{)59.67} \quad 45.9$$

$$7. \quad 7 \times 7 \times 7 \quad 343$$

$$\begin{array}{r} 8. \quad \frac{5}{8} \\ + \frac{1}{3} \\ \hline \frac{23}{24} \end{array}$$

$$\begin{array}{r} 9. \quad \frac{11}{12} \\ - \frac{1}{4} \\ \hline \frac{2}{3} \end{array}$$

$$\begin{array}{r} 10. \quad 2\frac{2}{3} \\ + 9\frac{1}{4} \\ \hline 11\frac{11}{12} \end{array}$$

$$11. \quad 20 \times \frac{3}{10} \quad 6$$

$$12. \quad \frac{1}{3} \times \frac{5}{8} \quad \frac{5}{24}$$

$$13. \quad 1\frac{1}{4} \times 2\frac{1}{2} \quad 3\frac{1}{2}$$

$$14. \quad 4 - \frac{1}{3} \quad 12$$

$$15. \quad \frac{5}{9} \div \frac{1}{4} \quad 2\frac{1}{2}$$

16. What is the angle sum of the four angles in this rectangle? 360°



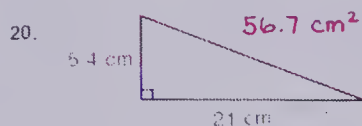
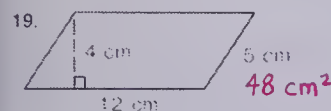
17. Write as a standard numeral

(a) 10^5 $100\,000$

(b) 3^4 $30\,000$

18. What is the time 3 h 15 min after 08 45? $12:00$

Calculate the area.



Which is larger?

21. $\frac{2}{5}$ or $\frac{1}{2}$

22. 2^3 or 3^2

Solve for n .

23. $n + 25 = 37$ 12

24. $7 \times n = 56$ 8

25. $n + \frac{1}{6} = \frac{5}{6}$ $\frac{2}{3}$

26. $\frac{5}{8} - n = \frac{3}{8}$ $\frac{1}{4}$

27. $n + n = \frac{6}{5}$ $\frac{3}{5}, \frac{3}{5}$

28. $n \times n = 16$ $4, 4$

29. 100 record albums have a mass of 17 000 g.
What is the mass of 1 album? 170 g

30. The perimeter of a square is 16.8 cm.
What is the length of each side? 4.2 cm

Chapters 1-7 cumulative review 219

(Continued from page 216)

(a) Using a calculator, determine the decimal equivalents for

$\frac{1}{7}, \frac{1}{8}, \frac{1}{12}, \frac{1}{13}, \frac{1}{14}, \frac{1}{15}, \frac{1}{16}, \frac{1}{18},$ and $\frac{1}{19}$.

(b) Record your answers.

(c) Which decimal equivalents do *not* repeat?

(d) Choose a fraction which has an interesting pattern of repeating digits.

(e) Write the decimal equivalents for all the proper fractions having that denominator.

Example

$\frac{1}{13}$ looks interesting.

$\frac{1}{13} = 0.076\,923\,076\,923\,0\dots$

$\frac{2}{13} = 0.153\,846\,153\,846\dots$

$\frac{3}{13} = 0.230\,769\,230\,769\dots$

$\frac{4}{13} = 0.307\,692\,307\,692\dots$

$\frac{5}{13} = 0.384\,615\,384\,615\dots$

$\frac{12}{13} = 0.923\,076\,923\,076\dots$

(f) Examine the repeating digits carefully. Look for patterns.
Record any interesting patterns.

OBJECTIVE

To review and test selected concepts and skills previously covered

PACING

Level A All

Level B All

Level C All

USING THE BOOK

This page may be used for diagnostic and remedial as well as review purposes. Students should check their work, correct any errors, and review the pages that contain any problems of the type they missed. Some students can do this on their own while others may need help. If a large number of students have a particular problem incorrect, you may want to reteach that topic to the groups, then assign a duplicated worksheet to reinforce that topic or, refer to an appropriate skill card in the BFA Computational Skills Kit II or BFA Problem Solving Kit II.

Test Item	Text Page Number
1, 2	4-8, 201
3	61
4	71
5	102
6	130
7	63
8, 9	191
10	202
11	206
12	208
13	213
14	210
15	211
16	40
17	179
18	146
19	110
20	109
21	190
22	178
23-28	85, 207
29	95
30	105

CHAPTER 8 OVERVIEW

This chapter reviews and extends the concepts of ratio, percent, decimal equivalence of percents and ratios, rounding of percents, and average (arithmetic mean). Some of the applications include purchases, savings and allowances, and using scale ratios to identify actual dimensions and make scale drawings.

OBJECTIVES

- A To write ratios, identify equivalent ratios, and solve problems using rates and ratios
- B To express ratios as percents and decimals and vice versa; to round percents to the nearest whole percent and tenth of a percent
- C To calculate percentages of quantities and amounts of money
- D To calculate and determine the arithmetic mean (average) using given data and/or rates
- E To use and interpret scale ratios when making scale drawings and/or finding real dimensions from scale drawings

BACKGROUND

This chapter begins by presenting the study of ratio as a comparison using numbers. The intent is to provide context for the introduction of percent as a special ratio where the comparison is made to one hundred. This abstract concept is a difficult one to understand. The children will need to be fluent with the use of fractions, decimals, and equivalence before attempting it.

Be sure to introduce the ratio topic thoroughly using concrete examples where possible. Only after the students have had numerous chances to manipulate, observe, and discuss the ratios involved in regular geometric shapes, paper-clip chains, mixed groups of objects, properties of groups (i.e., number of sneaker wearers per class), and any other *relevant* examples you can think of, should an attempt be made to express these relationships using numbers. Even once percent has been presented, return frequently to review, redemonstrate, and have the students explain to you and each other what is meant by ratio. This will help consolidate the conceptual base necessary in order to continue on to express fractions, decimals, and ratios as percent (and vice versa) and to the rounding of decimal and percent amounts.

In order to remind your students of the close relationship between percent and 100 you may wish to inform them that the percent sign may have evolved over the years like this:

$\frac{1}{100} \longrightarrow \square/100 \longrightarrow \square/00 \longrightarrow \square 0/0 \longrightarrow \square \%$

MATERIALS

regular polygons cut out of card stock
paper clips
white chalk
clear plastic metric rule
centimetre graph paper
metric tapes

CAREER AWARENESS

The Architect [238 and 239]

Architects are employed by architectural firms or by large corporations to be involved in the design of office buildings, apartments, stores, homes, and cottages.

All architects are trained in universities and usually “ride the board”, that is, do some drafting in various departments when they first join a large architectural firm.

Most designs of large projects are a result of teamwork as there are many considerations as part of a building design, such as materials, stress, strength, location of services such as elevators, water, heat, hydro, drains, etc.

They must also design a building which reflects the eventual use, so that there is a harmony between design and function.

After a project is designed, a specification sheet which lists the types of materials to be used is also prepared along with projected costs.

Usually an architect is assigned to a project to make sure that the specifications are followed and to answer any queries by the contractors.

Some architects design specialty homes and renovations so that a home can reflect the occupants' way of life more precisely than a mass-produced home.

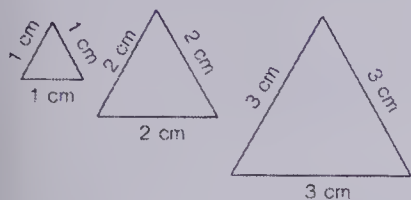
Other architects design homes and frontal elevations for contractors who build subdivisions.

Architects require both engineering and mechanical expertise, and an artistic sensitivity. A judicious blend of both art and skill is required.

Ratios

Ratios are number comparisons.

We can compare the length of each side of each equilateral triangle with its perimeter.



Length of each side	1	2	3	5	7
Perimeter	3	6	9	15	21



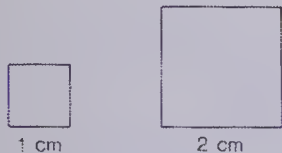
The comparison or *ratio* of length of sides to perimeter is:

1 to 3 or 1:3 or $\frac{1}{3}$

Exercises

Copy and complete these ratio charts.

1.



Squares

Side length	1	2	3	5	7	8	10	12
Perimeter	4					28	40	

2.



Regular hexagons

Side length	1	2	3	4	6	9	12	15
Perimeter	6					36	72	

3. (a)

Side length	1	2	3	4	5
Perimeter		10	15		

(b) What regular polygon is this? *Pentagon*

4. (a)

Side length	1	2	3	4	7
Perimeter			24		

(b) What is this regular polygon called? *Octagon*

Expressing a ratio 221

OBJECTIVE

To write ratios

PACING

Level A All
Level B All
Level C All

MATERIALS

regular polygons made out of Bristol board

BACKGROUND

Whenever possible the ratios should be extracted from *real* situations. A complete understanding of *ratio* is a result of analysing and observing many relationships.

SUGGESTIONS

Initial Activity Demonstrate and record ratios of the following:

- (a) girls:boys (b) $\frac{\text{girls}}{\text{total class}}$
(c) $\frac{\text{boys}}{\text{total class}}$ (d) $\frac{\text{chalkboard erasers}}{\text{chalkboard panels}}$

Extract that ratios are “numerical comparisons” and the order of the ratios statement is significant.

USING THE BOOK

Measure the side lengths and determine the perimeters of the various equilateral triangles.

Note that in all cases the perimeter is 3 times the side length.

Thus the ratio of:

$$\frac{\text{length of sides}}{\text{perimeter of equilateral triangle}} = \frac{1}{3}$$

and

$$\frac{\text{perimeter of equilateral triangle}}{\text{length of sides}} = \frac{3}{1}$$

Point out also that there are three ways of expressing ratios used here: 1 to 3; 1:3; $\frac{1}{3}$.

Define a “regular polygon” as having all sides and angles equal.

Assign the exercises and discuss. Note that for Exercises 1 and 2, some sample drawings of those particular shapes have been provided.

ACTIVITIES

1. Have the children complete a perimeter chart such as:

Perimeters of regular polygons

(1)	Side length	1	2	3	4	5	6	7
	Perimeter		14					49
(2)	Side length	1	3	5	7	9	10	12
	Perimeter		27					
(3)	Side length	1	2	4	6	8	10	12
	Perimeter			48				

Have them name each regular polygon.

- (1) regular septagon or regular heptagon (2) regular nonagon (3) regular dodecagon]

2. Have the children play “Triple Concentration” as described in the Activity Reservoir. Use card sets such as:

1 to 4	1:4	$\frac{1}{4}$
3 to 8	3:8	$\frac{3}{8}$

3. Provide drawings of an equilateral triangle, square, pentagon, hexagon, and heptagon, the measurements of which sides involve decimal amounts (trace, if necessary, those drawn on page 46). Have the children measure and complete ratio charts of the type shown on this page.

OBJECTIVE

To introduce equivalent ratios

PACING

Level A All
Level B All
Level C All

MATERIALS

regular paper clips, an overhead projector, white chalk, a clear plastic metric ruler

BACKGROUND

Be prepared to review equivalent fractions (page 188) with some children.

SUGGESTIONS

Initial Activity Since this exercise is an activity exercise, show the length of a 2-clip chain using the clear plastic ruler and the overhead projector.

Record ratios: $\frac{\text{number of clips}}{\text{length of chain}} = \frac{2}{6}$

Repeat for a 3 clip chain: $\frac{3}{9}$

Note that these ratios reduce to the basic ratio: $\frac{1}{3}$.

USING THE BOOK

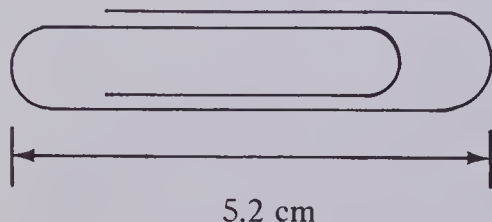
Allow students to use paper clips and new chalk to do the activities suggested.

Discuss the results with groups of students as they are working. For ease of calculation, provide new, equal length pieces of chalk for Activity II.

ACTIVITIES

1. Have the children record chain lengths charts for chains made using (a) very small clips, (b) large paper clips.

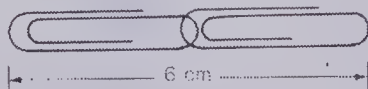
Example



2. Have students bring in various chains, e.g., tow chain, bicycle chain lock, dog leash chain, plastic chain fencing, etc.

Determine ratios for the various lengths of chain.

Paper-clip Chains

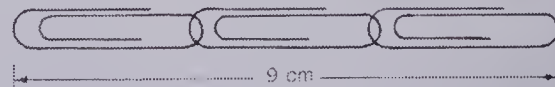


The ratio of the number of clips to the length of the chain

is $\frac{2}{6}$.

This ratio can also be written as

$\frac{2}{6}$ or $\frac{2}{3}$ or $\frac{1}{3}$



The ratio of the number of clips to the length of the chain

is $\frac{3}{9}$

This ratio can also be written as:

$\frac{3}{9}$ or $\frac{1}{3}$ or $\frac{3}{9}$

The ratios are equivalent because $\frac{2}{6} = \frac{1}{3}$ and $\frac{3}{9} = \frac{1}{3}$

Activities

Answers given for Activity I are for a 3 cm paper-clip.

- I
 1. Make paper-clip chains using:
 - (a) 5 clips
 - (b) 7 clips
 - (c) 10 clips
 2. Measure each chain (in centimetres) and record the ratio of $\frac{\text{number of clips}}{\text{length of chain}}$.

(a) $\frac{5}{15}$ (b) $\frac{7}{21}$ (c) $\frac{10}{30}$
 3. Check that all the ratios are equivalent.

All the ratios equal $\frac{1}{3}$.
 4. Copy and complete this chart.

Number of clips	3	5	7	10	12	14
Chain length						
 - ★ 5. Using your ratios, find out how long a chain would be with each number of clips.

(a) 15 clips *45 cm* (b) 21 clips *63 cm* (c) 20 clips *60 cm*
(d) 50 clips *150 cm* (e) 63 clips *189 cm* (f) 80 clips *240 cm*

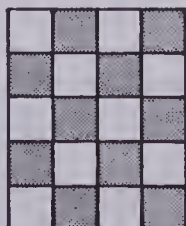
- II Find the lengths of the pieces of chalk as shown below. Copy and complete this chart.

Number of pieces of chalk	1	2	3	4	10	20	25
Total length							

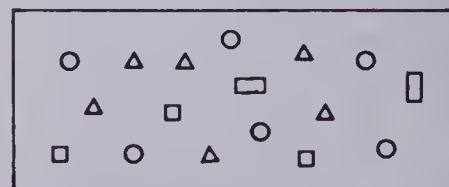
222 Ratio activities

3. Use graph paper to provide drawings as shown. Have the students determine ratios such as:

(a) $\frac{\text{number of coloured squares}}{\text{number of squares in all}}$



(b) $\frac{\text{number of triangles}}{\text{number of shapes in all}}$



Cross Products

These two ratios are equivalent

$$\frac{3}{4} \times \frac{9}{12}$$

$$4 \times 9 = 36$$

$$3 \times 12 = 36$$

$$4 \times 9 = 3 \times 12$$

The cross products are equal.

$$\text{Thus } \frac{3}{4} = \frac{9}{12}$$

These two ratios are *not* equivalent.

$$\frac{2}{5} \times \frac{6}{14}$$

$$5 \times 6 = 30$$

$$2 \times 14 = 28$$

$$5 \times 6 \neq 2 \times 14$$

The cross products are *not* equal.

$$\text{Thus } \frac{2}{5} \neq \frac{6}{14}$$

Exercises

Calculate cross products to find whether these ratios are equivalent. (Use = or \neq .)

$$\frac{3}{2} \neq \frac{9}{5}$$

$$\frac{15}{24} = \frac{5}{8}$$

$$\frac{4}{5} \neq \frac{17}{21}$$

$$\frac{60}{40} = \frac{3}{2}$$

$$\frac{20}{9} \neq \frac{7}{3}$$

$$\frac{6}{7} \neq \frac{12}{15}$$

$$\frac{7}{8} = \frac{21}{24}$$

$$\frac{10}{12} = \frac{5}{6}$$

$$\frac{1.5}{2} = \frac{3}{4}$$

$$\frac{6}{25} = \frac{1.2}{5}$$

$$\frac{3.5}{14} = \frac{1}{4}$$

Use equal cross products to find the missing terms in the equivalent ratios.

$$\frac{1}{2} \times \frac{\blacksquare}{10}$$

$$2 \times \blacksquare = 1 \times 10$$

$$2 \times \blacksquare = 10$$

$$\blacksquare = 5$$

$$\frac{1}{2} = \frac{5}{10}$$

$$\frac{2}{3} = \frac{\blacksquare}{9}$$

$$\frac{5}{6} = \frac{\blacksquare}{24}$$

$$\frac{3}{4} = \frac{12}{\blacksquare}$$

$$\frac{4}{5} = \frac{20}{\blacksquare}$$

$$\frac{7}{10} = \frac{\blacksquare}{40}$$

$$\frac{7}{8} = \frac{35}{\blacksquare}$$

$$\frac{1}{6} = \frac{5}{\blacksquare}$$

$$\frac{\blacksquare}{3} = \frac{4}{12}$$

$$\frac{2}{\blacksquare} = \frac{10}{25}$$

$$\frac{7}{\blacksquare} = \frac{21}{24}$$

Write these ratios in lowest terms. Check your answers, using cross products

$$\frac{16}{20} = \frac{\blacksquare}{\blacksquare}$$

$$\frac{5}{15} = \frac{\blacksquare}{\blacksquare}$$

$$\frac{12}{30} = \frac{\blacksquare}{\blacksquare}$$

$$\star \frac{30}{72} = \frac{\blacksquare}{\blacksquare}$$

- ★ 27. Toothpaste is \$1.98 for 150 mL or \$2.54 for 250 mL.
Which is the better buy?

Equivalent ratios 223

OBJECTIVE

To determine equivalent ratios using cross products

PACING

Level A 1-16

Level B 1-20

Level C All

RELATED AIDS

HMS — DM53.

BACKGROUND

Equivalent fractions were presented on pages 188 to 190.

SUGGESTIONS

Initial Activity Use the clips and the ratios from the Activity on page 222.

Number of clips	3	5	7	10	12	14
Chain length	9	15	21	30	36	42

Redemonstrate, if necessary, that all of the ratios are equivalent. Point out that, when ratios are equivalent, the cross products are equal.

$$\frac{3}{9} = \frac{5}{15}$$

$$3 \times 15 = 45 \text{ and } 9 \times 5 = 45.$$

Allow students to select other pairs of ratios from the chart to show that the cross products are equal.

USING THE BOOK

Using $\frac{2}{5}$ and $\frac{6}{14}$ from the display at the top of the page, show that when the cross products are *not* equal the ratios are not equivalent.

Show how cross products can be used to form equations to find missing terms in ratios.

Example

Exercise 12:

$$\frac{1}{2} = \frac{\blacksquare}{10}$$

$$2 \times \blacksquare = 1 \times 10 \quad \text{Cross products}$$

$$2 \times \blacksquare = 10 \quad \text{Simplify } 1 \times 10 \text{ to } 10$$

$$\blacksquare = 5 \quad \text{Solve}$$

$$\therefore \frac{1}{2} = \frac{5}{10}$$

Assign the exercises.

ACTIVITIES

1. Have the children identify the nonequivalent ratios in groups such as:

(a) $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{10}, \frac{5}{10}$

(b) $\frac{2}{3}, \frac{6}{9}, \frac{9}{12}, \frac{10}{15}, \frac{14}{21}$

(c) $\frac{3}{7}, \frac{6}{16}, \frac{9}{24}, \frac{12}{32}$

2. Provide equivalent ratio Tic Tac Toe sheets such as:

$\frac{1}{2} = \frac{\quad}{12}$	$\frac{1}{3} = \frac{\quad}{15}$	$\frac{1}{4} = \frac{\quad}{12}$
$\frac{5}{6} = \frac{\quad}{18}$	$\frac{3}{4} = \frac{\quad}{20}$	$\frac{2}{5} = \frac{\quad}{40}$
$\frac{12}{20} = \frac{\quad}{10}$	$\frac{9}{15} = \frac{\quad}{5}$	$\frac{8}{12} = \frac{\quad}{3}$

Players take turns filling in the various blanks, attempting to fill in three in a row horizontally, diagonally, or vertically in "Tic Tac Toe" fashion.

3. Have your Level C students help prepare ratio groups and Tic Tac Toe sheets for use in Activities 1 and 2.

OBJECTIVE

To solve problems involving ratios

PACING

Level A 1, 2 (parts (a), (b)); 3, 4 (parts (a), (b), (e), (f))

Level B All

Level C All

RELATED AIDS

HMS — DM54.

BFA PROB. SOLVING LAB II — 146.

SUGGESTIONS

Initial Activity If a car travels on the highway at 1.5 km/min, how far will it travel in (a) 10 min?

(b) 30 min?

(c) 1 h?

Discuss this generally.

Example

Each minute the car travels 1.5 km.

In 2 min, 3.0 km. In 3 min, 4.5 km.

etc.

In 10 min: $10 \times 1.5 = 15$ km

In 30 min: $30 \times 1.5 = 45$ km

In 60 min: $60 \times 1.5 = 90$ km

(1 h)

USING THE BOOK

Answer the above problem using equivalent ratios.

Distance (in kilometres) $\rightarrow \frac{1.5}{1} = \frac{\blacksquare}{10}$ Equivalent ratios

$1 \times \blacksquare = 1.5 \times 10$ Cross products

$1 \times \blacksquare = 15$ Simplify

$\blacksquare = 15$ Solve

In 10 min the car travels 15 km.

Repeat for 30 min and 60 min to show that the car travels 45 km and 90 km respectively.

Discuss each problem and determine the given $\frac{\text{time}}{\text{distance}}$ or $\frac{\text{distance}}{\text{time}}$ ratio.

Assign the exercises. Remind students to follow steps carefully.

ACTIVITIES

1. Have the students time each other as they complete various activities such as answering 10 basic division fact questions, performing 10 pushups (or 5 chin ups, 20 bent-knee situps, etc.), and walking or running 100 m. Have them keep a record of their performances.

224 Ratios problem solving

Track and Pool Ratios

Sandy can walk 6 km in 60 min.

At this rate, how far would Sandy walk in 35 min?

Step 1 Set up equivalent ratios:

Ratio: $\frac{\text{Distance}}{\text{Time}} \rightarrow \frac{6}{60} = \frac{\blacksquare}{35}$

Step 2 Cross products:

$60 \times \blacksquare = 6 \times 35$

Step 3 Related division statement:

$\blacksquare \times 60 = 210$

Step 4 Divide:

$210 \div 60 = \blacksquare$

$3.5 = \blacksquare$

	35
60	210.0
	180
	300
	300
	0



Sandy could walk 3.5 km in 35 min.

Exercises

Use the 4 steps to help you.

● Sandy's younger brother, Michael, can walk 5 km in 60 min.

At this rate, how far could Michael walk in:

(a) 120 min? **10 km** (b) 30 min? **2.5 km** (c) 48 min? **4 km** (d) 4 h? **20 km**

2. Sandy and Mike can run 100 m in the three-legged race in 40 s.

At this rate, how far could they run in:

(a) 20 s? **50 m** (b) 60 s? **150 m** (c) 2 min? **300 m**

Remember
60 min = 1 h
60 s = 1 min

3. Karen can swim 200 m in 4 min. At this rate how far could she swim in:

(a) 1 min? **50 m** (b) 6 min? **300 m** (c) 20 min? **1000 m** (d) 1 h? **3000 m**

At this rate, how long (in minutes) would it take her to swim:

(e) 100 m? **2 min** (f) 400 m? **8 min** (g) 50 m? **1 min** (h) 500 m? **10 min**

4. Karen's brother can run 1500 m in 6 min.

At this rate, how far could he run in:

(a) 3 min? **750 m** (b) 9 min? **2250 m** (c) 12 min? **3000 m** (d) 15 min? **3750 m**

At this rate, how long (in minutes) would it take him to run:

(e) 500 m? **2 min** (f) 2000 m? **8 min** (g) 1000 m? **4 min**

The Shopping Mall

Merchants rent floor space in shopping malls at a yearly rate based on the floor area in square metres.

Bob's Hobby Shop rents 50 m² of floor space at a rate of \$150/m² for each year.

What is the yearly rent?

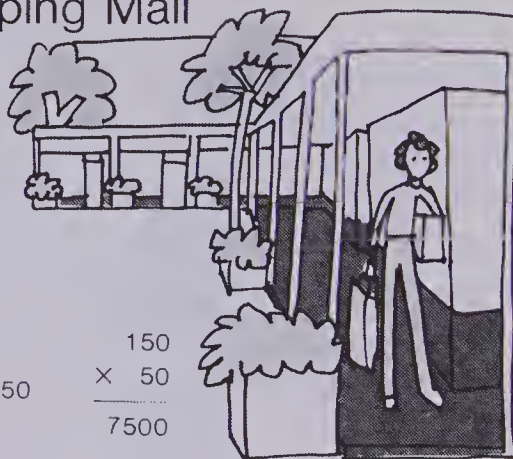
Use the ratios:

$$\frac{\text{Floor Area}}{\text{Yearly Rent}} = \frac{1}{150} = \frac{50}{\blacksquare}$$

$$1 \times \blacksquare = 50 \times 150$$

$$\blacksquare = 7500$$

The rent for one year is \$7500.



Exercises

- Bob needs more room and is considering renting a larger store. Calculate the yearly rents for these larger stores.

Rate						
Floor area (m ²)	1	80	100	140	200	210
Yearly rent	\$150					

\$12 000 \$15 000 \$21 000 \$30 000 \$31 500

- A large food store can rent space for \$130/m² for one year. Calculate the yearly rent for the following floor areas.

(a) 600 m² *\$78 000* (b) 850 m² *\$110 500* (c) 1000 m² *\$130 000* (d) 1800 m² *\$234 000*

- Roasts of beef sell for \$7.40/kg in the food store. Calculate the cost of the following roasts.

(a) 2 kg *\$14.80* (b) 5 kg *\$37.00* (c) 3.2 kg *\$23.68* (d) 4.5 kg *\$33.30*

- Calculate the sales tax for these purchases at a rate of 7%.

Price	\$2.00	\$1.00	\$5.00	\$12.30	\$45.00	\$100.00
Sales tax	\$0.14					

\$0.07 \$0.35 \$0.86 \$3.15 \$7.00

Rates and ratios 225

OBJECTIVE

To solve problems involving rates and ratios

PACING

Level A All
Level B All
Level C All

RELATED AIDS

HMS — DM55.

SUGGESTIONS

Initial Activity Discuss the placement of stores in plazas and malls — Which are large? Which are small? Why?

Discuss also how rent should be determined. List suggestions.

Explain that one way is by setting a rate of a certain dollar amount for each square metre of floor space in the store.

Present a sample problem:

Bob rents floor space for \$150/m² for a year.

If his store was 10 m², what would his rent be?

Point out that there are two ways to solve it.

Solution A

Rate: \$150/m²

Size: 10 m²

Rent is \$150 × 10
or \$1500.

Solution B

$$\frac{\text{Floor area}}{\text{Rent}} = \frac{1}{150}$$

$$\frac{1}{150} = \frac{10}{\blacksquare}$$

$$1 \times \blacksquare = 150 \times 10$$

$$1 \times \blacksquare = 1500$$

$$\blacksquare = 1500$$

The rent is \$1500.

Discuss and compare these solutions.

USING THE BOOK

Discuss the solution in the display on this page. Review the steps, as on page 224. Remind the students that “\$150/m²” is read “one hundred fifty dollars per square metre”.

Discuss the 4 problems and determine the basic ratio for each.

Example

Exercise 3: $\frac{\text{Selling price}}{\text{Mass of meat}}$ is $\frac{7.40}{1}$.

Exercise 4: $\frac{\text{Purchase}}{\text{Sales tax}}$ is $\frac{100}{7}$ where both are in cents.

Assign the problems.

ACTIVITIES

- Have the students use centimetre graph paper and a scale of 1 cm represents 1 m to draw possible store floor plans for Bob's Hobby Shops which have areas of:

a) 50 m² (b) 80 m² (c) 100 m²
d) 140 m² (e) 200 m² (f) 210 m²

Note: The stores need not be rectangular.

- Have the students (a) measure

and calculate the area of the classroom, (b) write a ratio statement about the amount of floor to the number of persons in the class, (c) determine the floor space to single person ratio.

- Perform the calculations described in Activity 2 for your class group when it uses the gym or library. Compare the space : person ratio of your room to other classrooms in the school.

OBJECTIVE

To express ratios as percents

PACING

Level A All

Level B All

Level C 1-29 (odd), 30

RELATED AIDS

BFA COMP LAB II — 122.

BACKGROUND

Percentage is a special ratio which allows easy comparison because the amounts are all compared to 100. Percent means “per hundred”. Thus, a percent tells “how many out of a hundred” (15% means 15 out of a hundred). Percent can also be thought of as a fraction whose denominator is 100 ($3\% = \frac{3}{100} = 0.03$).

SUGGESTIONS

Initial Activity Review some multiplication facts which produce 100.

$$2 \times \blacksquare = 100 \quad 50 \times \blacksquare = 100$$

$$4 \times \blacksquare = 100 \quad 25 \times \blacksquare = 100$$

$$5 \times \blacksquare = 100 \quad 20 \times \blacksquare = 100$$

$$10 \times \blacksquare = 100$$

Have students record these products for future reference.

Present situations such as:

(a) 27 cars out of 100 cars which passed the school on Monday were blue.

(b) 56 students out of 100 students surveyed in the school on Monday live in apartments.

(c) 3 out of 100 stamps collected by Gail were from Germany.

Emphasizing the “*n* out of 100” form tends to prevent students from making a mistake such as: “50 black cats; 100 orange cats; 50% of the cats were black.”

Actually, $33\frac{1}{3}\%$ of the cats were black.

Discuss these examples, eliciting the various ratios involved (i.e., the ratio of blue cars to all of the cars counted was 27 to 100 or 27:100 or $\frac{27}{100}$; the ratio of apartment dwellers to children surveyed was 56 to 100 or 56:100 or $\frac{56}{100}$; etc.). Tell the students that, whenever a ratio is compared to 100, another way of expressing it is as a percent, i.e., $\frac{27}{100}$ or 27%, $\frac{56}{100}$ or 56%, $\frac{3}{100}$ or 3%. Elicit percent expressions which describe the situations originally

The Special Ratio

A ratio compared to 100 can be expressed as a percent.

Philip achieved 72 marks out of a possible 100 on a math test.

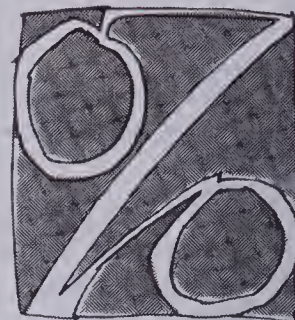
The ratio is $\frac{72}{100}$ or 72%.

Philip's mark is 72%.

Tanya achieved 19 marks out of a possible 25 marks on her math test.

The ratio is $\frac{19}{25}$ or $\frac{19 \times 4}{25 \times 4} = \frac{76}{100}$

Tanya's mark is 76%.



Exercises

Write the following ratios as percents.

1. 60 to 100 **60%** 2. 17 to 100 **17%** 3. 43 to 100 **43%** 4. 97 to 100 **97%**

5. 85:100 **85%** 6. 12:100 **12%** 7. 9:100 **9%** 8. 1:100 **1%**

9. $\frac{25}{100}$ **25%** 10. $\frac{50}{100}$ **50%** 11. $\frac{3}{100}$ **3%** 12. $\frac{7}{100}$ **7%**

Express each ratio as a ratio compared to 100, and then as a percent

$$\frac{7 \times 10}{10 \times 10} = \frac{70}{100} = \blacksquare\% \quad \frac{4 \times 10}{10 \times 10} = \frac{40}{100} = \blacksquare\%$$

$$\frac{9 \times 5}{20 \times 5} = \frac{45}{100} = \blacksquare\% \quad \frac{12 \times 4}{25 \times 4} = \frac{48}{100} = \blacksquare\%$$

$$\frac{4 \times 10}{10 \times 10} = \frac{40}{100} = \blacksquare\%$$

$$\frac{12 \times 4}{25 \times 4} = \frac{48}{100} = \blacksquare\%$$

17. $\frac{20}{25}$ **80%** 18. $\frac{22}{25}$ **88%** 19. $\frac{3}{5}$ **60%** 20. $\frac{4}{5}$ **80%** 21. $\frac{1}{4}$ **25%** 22. $\frac{3}{4}$ **75%**

23. $\frac{19}{50}$ **38%** 24. $\frac{45}{50}$ **90%** 25. $\frac{11}{20}$ **55%** 26. $\frac{7}{20}$ **35%** 27. $\frac{13}{25}$ **52%** 28. $\frac{19}{20}$ **95%**

29. Krista achieved 24 marks out of a possible 25 marks on her math test. What was her percent mark? **96%**

30. Steve achieved 18 out of a possible 20 marks. What was his percent mark? **90%**

226 Expressing ratios as percents

presented —“27% of the cars which passed the school were blue” and so on.

Present 3 more situations like:

- Out of 50 cars which passed the school on Tuesday, 12 were red.
- Out of 25 students surveyed in the school on Tuesday, 8 live in houses.
- Out of 20 stamps collected by Jim, 2 were from Germany.

Discuss these examples, first establishing the direct ratios, then (using procedures used to find equivalent fractions) as compared to 100, and finally as percents (i.e., 24% of Tuesday's cars were red, 32% of the Tuesday sample of children live in houses, 10% of Jim's collection is from Germany).

USING THE BOOK

Read through the display at the top of the page, consolidating the points made during the Initial Activity demonstration and discussion. If you complete Exercises 1 and 13 to 16 orally before assigning the balance of the exercises, it will yield 25 exercises

for individual completion. When corrections have been completed, have the students compute their personal percentage performances.

ACTIVITIES

1. Have the students gather information of the sort mentioned in the Initial Activity. Have them record their data in ratios, percents and present the information to the class.

2. Have the children try their skill at an activity which can be easily scored (i.e., number of beanbags out of 10 thrown into a pail from a distance of 5 m; number of erasers out of 20 which, when dropped from a standing position, will stay within a 30 cm × 30 cm square taped on the floor; number of bottle caps out of 25 that can be knocked from a table edge using only elastics from a distance of 2 m; etc.). Have the participants keep a record of their performances at the various stations. Have these expressed as percents.

(Continued on page 227)

Exams and Sports

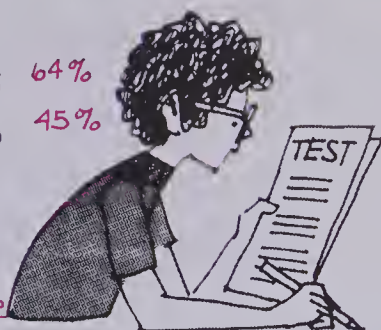
1. John received the following marks on exams.

Language Arts	— $\frac{16}{25}$	64%	Mathematics	— $\frac{32}{50}$	64%
Spelling	— $\frac{42}{50}$	84%	Science	— $\frac{9}{20}$	45%
Social Studies	— $\frac{15}{20}$	75%			

(a) Express each mark as a percent. *See above.*

(b) Which subject is best? *Spelling*

(c) Which subject requires more studying? *Science*



2. Last week Judy was practising her free-throw shots in basketball.

On Monday she sank 8 out of 20 shots. *40%*

On Tuesday she sank 5 out of 10 shots. *50%*

On Wednesday she sank 13 out of 25 shots. *52%*

On Thursday she sank 9 out of 20 shots. *45%*

On Friday she sank 21 out of 50 shots. *42%*

(a) Express each day's record as a percent. *See above.*

(b) Which day did Judy have the highest free-throw percentage? *Wednesday*



3. Tom plays defence in a minor hockey league.

Here are his statistics for the first four games.

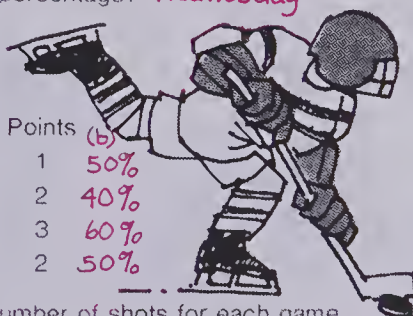
	(a) Shots on Goal	Goals	Assists	Points	(b)
Game 1	<i>0%</i>	2	0	1	<i>50%</i>
Game 2	<i>20%</i>	5	1	2	<i>40%</i>
Game 3	<i>0%</i>	5	0	3	<i>60%</i>
Game 4	<i>25%</i>	4	1	2	<i>50%</i>

(a) Find the goal-scoring percentage based on the number of shots for each game. *See above.*

(b) Find the point percentage based on the number of shots for each game. *See above.*

(c) For which game did Tom have the best goal-scoring percentage? *Game 4*

(d) For which game did Tom have the best point percentage? *Game 3*



Percent problems 227

OBJECTIVE

To solve problems involving percent

PACING

Level A All

Level B All

Level C All

VOCABULARY

percentage, statistics

BACKGROUND

You may wish to mention that percentage is one kind of statistic that can be used for comparing performance. "17 out of 45" is hard to compare to "26 out of 60" whereas 38% is easily recognized as less than 43%.

SUGGESTIONS

Initial Activity Review the products of 100.

$$2 \times 50 = 100 \quad 50 \times 2 = 100$$

$$4 \times 25 = 100 \quad 25 \times 4 = 100$$

$$5 \times 20 = 100 \quad 20 \times 5 = 100$$

$$10 \times 10 = 100$$

Review how ratios can be expressed as ratios compared to 100 and then percents.

Example

John's language arts mark is $\frac{16}{25}$.

$$\frac{16}{25} = \frac{16 \times 4}{25 \times 4}$$

$$= \frac{64}{100}$$

or 64%

John's language arts mark is 64%.

USING THE BOOK

Assign the exercises. Encourage students to record percentages using a short statement or record them on a chart. For some groups, you may wish to read through Exercise 3 completely, discussing the various statistics and which ones are being compared.

ACTIVITIES

1. "Statistics". Have students design a challenging target game using materials from the gymnasium (beanbags, balls, nets, ...). Do the game. Have them record the number of scores, baskets, etc. out of (a) 20 shots (b) 25 shots. Express each statistic as a percent.

2. Collect the games and run an activity "Play Day".

Have each student explain and demonstrate his target event to the class. Students try 20 shots at each and record. Have them change each to a percent.

3. If you have not already done so, see Activity 1 on page 226.

Continued from page 226)

3. Begin a percent file.

Have each student bring in a newspaper or magazine article or advertisement which involves percents.

Allow students to design problems using these resources.

OBJECTIVE

To introduce percent and decimal equivalents

PACING

Level A 1-4
Level B All
Level C All

RELATED AIDS

HMS — DM56.
BFA COMP LAB II — 123, 124.

BACKGROUND

Expressing hundredths as fractions and decimals was presented on pages 2, 3, and 200.

SUGGESTIONS

Initial Activity Review the meaning of percent. Point out that since they mean "out of 100", percents can be expressed as decimals.

Examples

$$(a) 83\% = \frac{83}{100} = 0.83 \quad (b) 7\% = \frac{7}{100} = 0.07$$

$$(c) 70\% = \frac{70}{100} \text{ or } \frac{7}{10} = 0.70 \text{ or } 0.7$$

Similarly decimals can be expressed as percents.

Examples

$$(a) 0.72 = \frac{72}{100} = 72\% \quad (b) 0.9 = \frac{9}{10} = \frac{9 \times 10}{10 \times 10} = \frac{90}{100} = 90\%$$

USING THE BOOK

Discuss each of the examples in the display at the top of the page, drawing particular attention to the fact that the denominators are 100 in every case.

Note: It is customary to write the decimal in its simplest form:

$$70\% = \frac{70}{100} = 0.70 = 0.7$$

$$\frac{70}{100} = \frac{7}{10} = 0.7$$

Assign the exercises. Be certain that the students are familiar with the accepted answer format.

ACTIVITIES

1. Provide an activity assignment such as:

"Percent Scan".

(a) Scan one section of your local

newspaper and clip out all activities or advertisements which contain a percent sign.

- Paste these in your notebook.
- Express all percents as decimals.
- Using 2 pieces of information, write a word problem and solve it.

2. Prepare a challenge card such as:

"Percent Count".

- Select any whole percent between 1% and 100%.
- Write the next 8 whole percents.
- Express each as a decimal in simplest form.

Percents and Decimals

Percent is a comparison to 100. It can be expressed as a decimal.

Express as decimals.

(a) 45%

$$45\% = \frac{45}{100} = 0.45$$

(b) 9%

$$9\% = \frac{9}{100} = 0.09$$

Express as percents.

(a) 0.85

$$0.85 = \frac{85}{100} = 85\%$$

(b) 0.07

$$0.07 = \frac{7}{100} = 7\%$$

Exercises

1. Express as decimals

$$25\% = \frac{25}{100} = 0.25$$

$$3\% = \frac{3}{100} = 0.03$$

$$(c) 50\% = 0.5 \quad (d) 17\% = 0.17 \quad (e) 93\% = 0.93$$

$$(f) 35\% = 0.35 \quad (g) 85\% = 0.85 \quad (h) 8\% = 0.08$$

$$(i) 20\% = 0.2 \quad (j) 1\% = 0.01$$

2. Copy and complete these place-value charts.

	ones	tenths	hundredths
(a) 45%	0	4	5
(b) 9%	0	0	9
85%	0	8	5
7%	0	0	7

	ones	tenths	hundredths
(e) 70%	7	0	0
(f) 17%	1	7	0
(g) 0%	0	0	0
(h) 100%	1	0	0

Express as percents.

$$0.75 = \frac{75}{100} = 75\%$$

$$0.02 = \frac{2}{100} = 2\%$$

$$(c) 0.41 = 41\% \quad (d) 0.89 = 89\%$$

$$(e) 0.05 = 5\% \quad (f) 0.99 = 99\% \quad (g) 0.5 = 50\%$$

$$(h) 0.3 = 30\% \quad (i) 0.01 = 1\%$$

4. Out of every dollar's allowance, Jamie saves \$0.25.
What percent does he save? **25%**

5. Mr. Thompson earns \$1000 per month (after taxes are paid).
He pays \$380 for rent, \$260 for food, and saves \$70.
What percent of his salary does he use for:

(a) rent? **38%**

(b) food? **26%**

(c) savings? **7%**

★(d) other expenses? **29%**



3. See "Concentration" as described in the Activity Reservoir.
Use triple card sets such as:

$$\frac{27}{100}$$

$$27\%$$

$$0.27$$

$$\frac{1}{4}$$

$$\frac{25}{100}$$

$$25\%$$

$$50\%$$

$$0.50$$

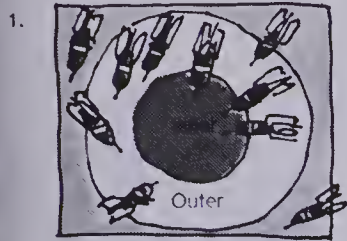
$$0.5$$

$$\frac{1}{2}$$

$$\frac{5}{10}$$

$$50\%$$

Percent Problems



1. 10 darts were thrown at the dart board.

- (a) How many darts hit the inner ring? **3**
- (b) How many darts hit the outer ring? **5**
- (c) How many darts missed the rings? **2**
- (d) What percent hit the inner ring? **30%**
- (e) What percent hit the outer ring? **50%**
- (f) What percent missed the rings? **20%**



2. There are 25 fish in the classroom aquarium. 7 are adult fish, and the rest are young fish.

- (a) How many are young fish? **18%**
- (b) What percent are adults? **28%**
- (c) What percent are young fish? **72%**

3. Sam plays hockey for the Raiders.

Sam has taken 50 shots on goal in 20 games.

He scored 14 goals.

- (a) What is his scoring percentage based on shots on goal? **28%**
- (b) What is his scoring percentage based on the number of games played? **70%**

4. There are 50 Grade 6 students at Howard Avenue Public School.

6 were absent on Wednesday.

On that day, what percent were:

- (a) absent? **12%** (b) present? **88%**

5. 500 students attend Glen Grove School

75 students sing in the senior choir **15%**

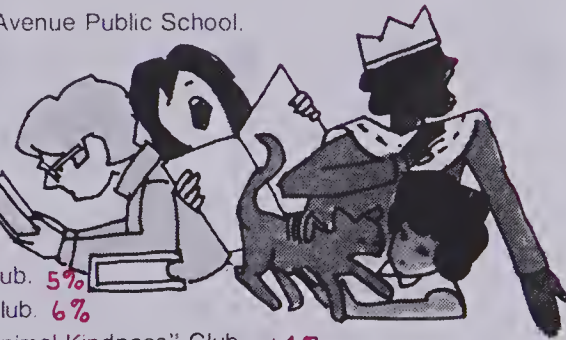
200 students play intramural soccer **40%**

25 students are members of the Drama Club. **5%**

30 students are members of the Library Club. **6%**

70 students are active members of the "Animal-Kindness" Club. **14%**

- (a) What percent of the 500 students participate in each of these school activities? **See above.**
- (b) What percent do not participate? **20%**



Percent problems 229

OBJECTIVE

To solve percent problems

PACING

Level A All

Level B All

Level C All

VOCABULARY

intramural

BACKGROUND

Exercise 5 requires that the fractions be written in lowest terms (i.e., reduced to lowest terms). See page 189.

SUGGESTIONS

Initial Activity Have one or two students demonstrate their skills at a simple activity of the sort described in Activity 2, page 226. Record their performances on the chalkboard. Present problems, based on the data, such as:

- (a) How many beanbags landed in the pail?
- (b) How many beanbags missed the pail?
- (c) What percent of those thrown landed in the pail?
- (d) What percent of those thrown missed the pail?

Solve the problems orally, discussing the steps and strategies involved.

Collect some bits of other classroom trivia such as: "of these 10 people, 4 have blue eyes;" "of these 25 people, 16 are wearing sneakers;" "of these 20 books, 3 have over 250 pages;" etc. Express the ratios as percents.

USING THE BOOK

Read through the problems with those groups or individuals who have reading difficulties. Be certain that the students are familiar with the accepted answer format.

ACTIVITIES

See the "Concentration" idea described in Activity 3, page 228.

2. Using cards similar to those required for Concentration above, see "Rummy" as described in the Activity reservoir.

3. Provide, if necessary, a rounded-to-the-nearest-100 student total for your school. Have some of your Level C students collect data such as that described in Exercise 5. Have them report their percent findings to the class.

EXTRA PRACTICE

At Glen Arbour School there are 200 students.

30 are in the Library Club.

20 are in the primary choir.

50 are in the junior choir.

140 play in school sports.

20 are in the safety patrol.

- (a) What percent of the total student body participate in each of the above school activities?
- (b) 270 students have been counted. Explain briefly why this is possible.

OBJECTIVE

To calculate a percent of a whole number

PACING

Level A 1-12, 15, 18, 20-22

Level B All

Level C All

RELATED AIDS

BFA COMP LAB II — 125.

BFA PROB. SOLVING LAB II — 141.

BACKGROUND

The multiplication of whole numbers by tenths and by hundredths is presented on pages 71 and 74 respectively.

SUGGESTIONS

Initial Activity Before discussing the work on this page:

(a) Review the expression of percents as decimals.

Examples

$$\begin{aligned} 45\% &= \frac{45}{100} & 70\% &= \frac{70}{100} & 7\% &= \frac{7}{100} \\ &= 0.45 & &= \frac{7}{10} & &= 0.07 \\ & & &= 0.7 & & \end{aligned}$$

(b) Review the product of decimals and whole numbers.

Examples

$$\begin{aligned} 50 &\leftarrow \text{no decimal places} \\ \times 0.6 &\leftarrow \text{1 decimal place} \\ \hline 30.0 &\leftarrow \text{1 decimal place in the answer} \end{aligned}$$

$$\begin{aligned} \text{and } 780 &\leftarrow \text{no decimal places} \\ \times 0.35 &\leftarrow \text{2 decimal places} \\ \hline 39\ 00 \\ 234\ 0 \\ \hline 273.00 &\leftarrow \text{2 decimal places in the answer} \end{aligned}$$

USING THE BOOK

Discuss the problems presented in the display at the top of the page.

Justify that 56 students are going camping

(a) using common sense
— 560 students are going?
— 5.6 students going?

(b) $70 \leftarrow$ no decimal places
 $\times 0.8 \leftarrow$ 1 decimal place

$56.0 \leftarrow$ 1 decimal place in the answer

Justify that the sales tax for Nicole's camping equipment is \$2.40.

The Camping Trip

80% of the 70 Grade 6 students are going on the school camping trip. How many are going camping?

To solve:

Step 1 Express the percent as a decimal.

$$\begin{aligned} 80\% &= \frac{80}{100} \\ &= 0.80 \\ &\text{or } 0.8 \end{aligned}$$

Step 2 Calculate 80% of 70 as 0.8×70 .

$$\begin{array}{r} 70 \\ \times 0.8 \\ \hline 56.0 \end{array} \quad \begin{array}{l} \text{56 students} \\ \text{are going camping} \end{array}$$

Nicole's parents bought camping equipment costing \$48. How much money for sales tax did they pay if the sales tax rate was 5%?

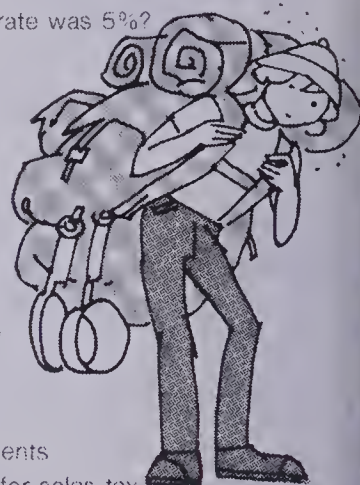
To solve:

Step 1 Express the percent as a decimal.

$$\begin{aligned} 5\% &= \frac{5}{100} \\ &= 0.05 \end{aligned}$$

Step 2 Calculate 5% of \$48 as 0.05×48 .

$$\begin{array}{r} 48 \\ \times 0.05 \\ \hline 2.40 \end{array} \quad \begin{array}{l} \text{Nicole's parents} \\ \text{paid \$2.40 for sales tax.} \end{array}$$



Exercises

Calculate.

- | | | | |
|-----------------------------|-----------------------------|-----------------------------|----------------------------|
| 1. 50% of 18 9 | 2. 70% of 40 28 | 3. 10% of 80 8 | 4. 15% of 200 30 |
| 5. 25% of 60 15 | 6. 42% of 300 126 | 7. 66% of 150 99 | 8. 20% of 85 17 |
| 9. 90% of 500 450 | 10. 100% of 16 16 | 11. 6% of 150 9 | 12. 4% of 50 2 |
| 13. 2% of 300 6 | 14. 8% of 75 6 | 15. 1% of 400 4 | 16. 5% of 700 35 |
| 17. 3% of \$600 \$18 | 18. 7% of \$200 \$14 | 19. 9% of \$400 \$36 | 20. 5% of \$180 \$9 |

21. 65% of the Grade 6 students brought their own sleeping bags. How many students is this? **46**

22. The price of Paul's sleeping bag was \$65. He had to pay 7% more for sales tax. What was the total cost of the bag? **\$69.55**

230 Finding a percent of a whole number

What was the total price paid?

$$\begin{array}{r} \$48.00 \leftarrow \text{sales price} \\ + 2.40 \leftarrow \text{sales tax} \\ \hline \$50.40 \leftarrow \text{total cost} \end{array}$$

Assign the exercises.

ACTIVITIES

1. To review the multiplication of whole numbers by tenths and hundredths, see the "500 Grand" idea in the Activity Reservoir. At each grid location, write questions like:
 8×0.7 , 12×0.5 , 30×0.06 .
Adjust the total points required to win accordingly.

2. Use the "500 Grand" idea described above but use exercises of the sort on this page.
7% of 700, 12% of 60, 95% of 500, etc.

3. Have students, working in groups, use camping or department store catalogues to select necessary camping gear and calculate *total* costs. Have them calculate sales taxes for these purchases (use the provincial rate) and add total costs together.

EXTRA PRACTICE

- 1% of 350 [3.5]
- 10% of \$85 [\$8.50]
- 9% of 200 [18]
- 25% of 640 [160]
- 37% of \$500 [\$185]
- 5% of 450 [22.5]
- 2% of \$170 [\$3.40]
- 100% of \$792 [\$792]
- 70% of 150 [105]
- 15% of 810 [121.5]

Allowances and Savings

Mario receives an allowance of \$5.50 per week. He puts 30% in his savings account. How much does he save each week?

Step 1 Express 30% as a decimal.
 $30\% = 0.30$ or 0.3

Step 2 Calculate:
 $0.3 \times \$5.50$
 $\begin{array}{r} 5.50 \text{ (2 decimal places)} \\ \times 0.3 \text{ (1 decimal place)} \\ \hline 1.650 \text{ (3 decimal places)} \end{array}$
 Mario saves \$1.65 each week.

Jean makes \$12.60 each week from her paper route. She saves 25% of her money. How much does she save each week?

Step 1 Express 25% as a decimal.
 $25\% = 0.25$

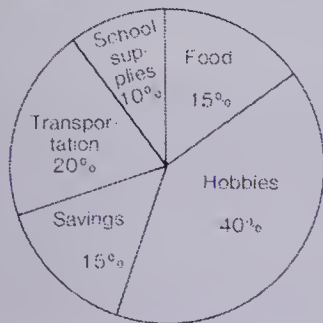
Step 2 Calculate:
 $0.25 \times \$12.60$
 $\begin{array}{r} 12.60 \text{ (2 decimal places)} \\ \times 0.25 \text{ (2 decimal places)} \\ \hline 6.300 \\ 25.200 \\ \hline 3.1500 \text{ (4 decimal places)} \end{array}$
 Jean saves \$3.15 each week.

Exercises

Calculate.

1. 10% of \$32.00
2. 50% of \$79.00
3. 25% of \$11.60
4. 6% of \$45.00
5. 70% of \$54.50
6. 90% of \$35.00
7. 8% of \$9.50
8. 38% of \$60.00
9. 12% of \$4.50
10. 2% of \$83.00

11.



John receives an allowance of \$7.00 per week.

How much does he spend on:

- (a) school supplies? \$0.70
- (b) transportation? \$1.40
- (c) food? \$1.05
- (d) hobbies? \$2.80
- (e) How much does he save each week? \$1.05

Percents and money 231

OBJECTIVE

To find a percent of a decimal (no rounding)

PACING

Level A 1-10
 Level B All
 Level C All

RELATED AIDS

BFA PROB. SOLVING LAB II — 240.
 CALC. ACTIVITY MASTERS — 54, 89.

SUGGESTIONS

Initial Activity Discuss *savings* — Where do people keep savings? How much do you save? How? Where?

Discuss *allowances* — What is the purpose of an allowance? List suggestions on the chalkboard or on chart paper.

Point out that savings could be a percentage saved each week or each month.

USING THE BOOK

Demonstrate the 2 steps shown for solving the problems involving Mario and Jean. Focus attention on the way of positioning the decimal point in the answer. Also discuss the reasonableness of the answer.

Have the students record their solutions in an organized way.

Example

$$\begin{array}{l} 30\% \text{ of } \$15.00 \\ = 0.3 \times 15.00 \\ = \$4.50 \end{array} \quad \begin{array}{l} 15 \\ \times 0.3 \\ \hline 4.5 \end{array} \quad \text{or} \quad \begin{array}{l} 15.00 \\ \times 0.3 \\ \hline 4.500 \end{array}$$

EXTRA PRACTICE

1. 60% of \$83.50 [\$50.10]
2. 4% of \$208.50 [\$8.34]
3. 10% of \$75.60 [\$7.56]
4. 100% of \$16.50 [\$16.50]
5. 1% of \$562.00 [\$5.62]
6. 25% of \$950.00 [\$237.50]

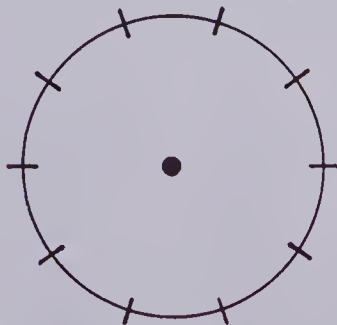
ACTIVITIES

1. See Activities 1 and 2 on page 230. Use appropriate numbers on the grid. 50% of \$18.50, 25% of \$20, etc.
2. See "Bingo" as described in the Activity Reservoir. Modify the game, however, by (a) using a blank 4×4 grid; (b) selecting the answers from 16 exercises and writing these on the chalkboard (use Exercise 8 on page 358, Exercise 7 on page 359, and four more exercises from Extra Practice as a source); (c) have the players write these answers randomly on their grids. Play in the usual manner.

3. Provide circle graph stencils as shown. Have the children graph approximate percentages for time

spent doing various activities on:

- (a) a weekday during the school year
 - (b) a Saturday
 - (c) a Sunday
 - (d) a Special Day.
- Choose 2.



OBJECTIVE

To round percents to the nearest:

- (a) whole percent and (b) tenth of one percent

PACING

Level A 1-3 (parts (a), (b))

Level B 1-3 (parts (a), (b), (c))

Level C All

BACKGROUND

These exercises require that students round amounts to the nearest whole number (see pages 13 and 217).

SUGGESTIONS

Initial Activity Review the rounding of decimal numbers to the nearest whole number.

Example

17.83 to the nearest whole is 18.

17.38 to the nearest whole is 17.

Review rounding to the nearest tenth. (Refer to examples from page 217 of this text.)

Review the division involved to show that $\frac{1}{7} = 0.142857 \dots$

$$\begin{array}{r} 0.142857 \\ 7 \overline{) 1.000000} \\ \underline{7} \\ 30 \\ \underline{28} \\ 20 \\ \underline{14} \\ 60 \\ \underline{56} \\ 40 \\ \underline{35} \\ 50 \\ \underline{49} \\ 1 \end{array}$$

USING THE BOOK

Discuss how $66.66 \dots \%$ is 67% to the nearest whole percent

and 66.7% to the nearest tenth.

Similarly, $14.2857 \dots \%$ is 14% to the nearest whole percent

and $14.2857 \dots \%$ is 14.3% to the nearest tenth.

Remind the students that, if the digit in the next place to the right is 5 or more, increase the previous digit by one.

Show how Exercises 1(a) and 2(a) could be organized.

$$\begin{array}{r} 0.33333 \dots \\ 1(a) \quad 3 \overline{) 1.00000} \end{array}$$

Percents and Rounding

Some ratios produce decimals that need rounding when written as percents.

Ratio	Decimal	Percent	Rounded
$\frac{2}{3}$	$0.666 \dots$	$66.666 \dots \%$	<div> <div>(Whole %)</div> <div>(Nearest tenth %)</div> </div>
$\frac{1}{7}$	$0.142857 \dots$	$14.2857 \dots \%$	

Exercises

Express each ratio as a decimal rounded to two decimal places.

1. $\frac{1}{3} = 0.33$ (a) $\frac{1}{8} = 0.13$ (b) $\frac{5}{8} = 0.63$ (c) $\frac{8}{9} = 0.89$ (d) $\frac{1}{11} = 0.09$ (e) $\frac{10}{11} = 0.91$

2. Express each of the decimals in Exercise 1 as a percent rounded to the nearest whole percent. (a) 33% (b) 13% (c) 63% (d) 89% (e) 9% (f) 91%

3. Express each ratio as a percent rounded to the nearest tenth of a percent.

$\frac{1}{12} = 8.3\%$ (b) $\frac{5}{12} = 41.7\%$ (c) $\frac{7}{11} = 63.6\%$ (d) $\frac{14}{15} = 93.3\%$ (e) $\frac{1}{14} = 7.1\%$

4. Calculate the percent correct to the nearest whole percent:

- (a) Jill got 12 questions correct out of 13. 92%
(b) Allan got 49 questions correct out of 65. 75%

BRAINTICKLER

$\frac{1}{7} = 0.142857 \dots$ The line means that 142857 repeats.
 $\frac{1}{7} = 0.142857142857 \dots$



(a) Express all the sevenths from $\frac{1}{7}$ to $\frac{6}{7}$ as decimals.

(b) Express each seventh as a percent rounded to the nearest whole percent.

232 Rounding percents

ANSWERS:

Braintickler (a) $\frac{1}{7} = 0.142857$ $\frac{2}{7} = 0.285714$ $\frac{3}{7} = 0.428571$ $\frac{4}{7} = 0.571428$
 $\frac{5}{7} = 0.714285$ $\frac{6}{7} = 0.857142$ (b) $\frac{1}{7} = 14\%$ $\frac{2}{7} = 29\%$ $\frac{3}{7} = 43\%$ $\frac{4}{7} = 57\%$
 $\frac{5}{7} = 71\%$ $\frac{6}{7} = 86\%$

$0.333 \dots \rightarrow 0.33$ to 2 decimal places.

2(a) $0.3333 \dots = 33.333 \dots \%$

$33.333 \dots \% \rightarrow 33\%$ to the nearest whole percent.

Assign the exercises.

each of the "elevenths" from $\frac{1}{11}$

through $\frac{10}{11}$ as a repeating decimal;

(b) express each repeating decimal as a percentage rounded to the nearest whole percent.

3. Prepare 10 to 15 cards which show various whole and tenth percent amounts.

66%	37.4%	11%	66.7%
-----	-------	-----	-------

Have the students write two percent amounts for each card, one below and one above, which when correctly rounded, would yield the amounts shown on the original card. When complete and correct, use decks of cards made up of the originals and the student responses rewritten on cards. Shuffle the decks well. Have them exchanged for re-sorting.

ACTIVITIES

1. Have the children play "Triple Concentration" (see the Activity Reservoir). Use card sets which reinforce some of the more common percent amounts.

$\frac{1}{2}$	50%	0.5
$\frac{3}{4}$	0.75	75%
1	1.0	100%

2. Have the students (a) express

Stamp Collecting

40% of all the entries in the stamp-collectors' exhibition received prizes.
What ratio of the entries received prizes?

- Step 1** Write the percent as a fraction. $40\% = \frac{40}{100}$
- Step 2** Reduce, using division. $\frac{40 \div 10}{100 \div 10} = \frac{4}{10}$
- Step 3** Reduce again (if necessary). $\frac{4 \div 2}{10 \div 2} = \frac{2}{5}$

Two out of every 5 entries received prizes.



Exercises

Express these percents as reduced ratios.

- 30% = $\frac{30}{100} = \frac{3}{10}$ 60% = $\frac{60}{100} = \frac{6}{10} = \frac{3}{5}$ 25% = $\frac{25}{100} = \frac{1}{4}$
4. 10% = $\frac{1}{10}$ 5. 75% = $\frac{3}{4}$ 6. 80% = $\frac{4}{5}$ 7. 50% = $\frac{1}{2}$
8. 12% = $\frac{3}{25}$ 9. 32% = $\frac{8}{25}$ 10. 48% = $\frac{12}{25}$ 11. 44% = $\frac{11}{25}$
12. 56% = $\frac{14}{25}$ 13. 72% = $\frac{18}{25}$ 14. 84% = $\frac{21}{25}$ 15. 96% = $\frac{24}{25}$
16. 20% of Josh's stamps are Canadian stamps.
What ratio of Josh's stamps are Canadian stamps? $\frac{1}{5}$
17. 8% of his stamps are Dutch stamps.
What ratio of his stamps are Dutch stamps? $\frac{2}{25}$
18. 36% of his stamps are from African countries.
What ratio of his stamps are African stamps? $\frac{9}{25}$
19. 15% of his stamps are from the U.S.A.
What ratio of his stamps are American? $\frac{3}{20}$



Percent as a reduced ratio 233

OBJECTIVE

To express percents as a reduced ratio

PACING

Level A All
Level B All
Level C All

BACKGROUND

It is assumed that students have completed the section on equivalent fractions and fractions in lowest terms (pages 188 and 189).

SUGGESTIONS

Initial Activity Review the reduction of fractions.

Example

$$\frac{16}{100} = \frac{16 \div 4}{100 \div 4} = \frac{4}{25}$$

Remind the students that: (a) both numerator and denominator are divided by the same amount; (b) though the numbers have changed, the amount expressed remains the same.

Review also, if necessary, the meaning of simple ratios as presented on pages 221 and 222.

Example

Two out of every five people surveyed preferred chocolate ice cream \rightarrow 2 to 5, 2:5, or $\frac{2}{5}$.

USING THE BOOK

Explain the steps in writing a percent as a reduced ratio:

36% of the people in the class today are wearing jeans.

Example

$$36\% = \frac{36}{100} \quad (1) \% \text{ as a fraction.}$$

$$\frac{36 \div 2}{100 \div 2} = \frac{18}{50} \quad (2) \text{ Reduce, using division.}$$

$$\frac{18 \div 2}{50 \div 2} = \frac{9}{25} \quad (3) \text{ Reduce again (if necessary).}$$

$\therefore 36\% = \frac{9}{25}$ (4) Repeat 3 if necessary.
9 out of 25 people in class today are wearing jeans.

Emphasize that, in the examples above, at the top of the pupil page and in all of the exercises on this page, though percents have been or will be rewritten as reduced ratios, the amounts expressed remain the same.

Assign the exercises.

ACTIVITIES

1. Prepare some "fill-in-the-blank" puzzles as shown for completion by the students.

a) $60\% = \frac{60}{100} = \frac{30}{25} = \frac{6}{5} \rightarrow 3:5$

b) $36\% = \frac{36}{100} = \frac{9}{25} \rightarrow 9:25$

c) $7\% = \frac{7}{100} = \frac{7}{100} \rightarrow 7:100$

d) $1\% = \frac{1}{100} = \frac{1}{100} \rightarrow 1:100$
etc.

2. Have students who have stamp collections bring them to school for this ratio activity (or sketch a demonstration-size page or two on experience chart paper. Be sure to include (a) 10 to 20 "stamps" on each page, (b) different colours of stamps, (c) different values showing on each stamp).

Instruct the children as follows. Using 1 or 2 pages of stamps, write these ratios:

a) green stamps ... and other colours
all stamps

b) stamps with less than 20 on the face
all the stamps

(c) some other ratio.

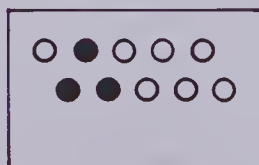
Divide to form the decimal equivalent for the above ratios.

Express each decimal as a percent rounded to the nearest whole percent.

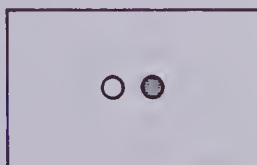
3. Have some students use water-colour paints to show percent amounts. They might like to use thumbprints, vegetable-piece stamps or brush-dab marks to show patterns which are:

- (a) 25% red
(b) 50% blue
(c) 60% black
etc.

Example



30% black



50% shaded

(Continued on page 235)

OBJECTIVE
Ratio, decimals, and percent
equivalents

PACING
Level A All
Level B All
Level C All

RELATED AIDS
HMS — DM57 and DM58.
CALC. ACTIVITY MASTERS — 88.

SUGGESTIONS
Initial Activity Show the format
meant for each.
Example
Ratio Fraction Decimal Percent
Ratio
1:50 1/50 0.02 2%

Review how division is performed
to change ratios to decimal
equivalents (see pages 215 to 217).
Example
7 out of 18 or 7/18
0.3888
18) 7.0000
54
160
144
160
144
16
7/18 = 0.3888...
0.3888... -> 39%

USING THE BOOK
Assign all the exercises 1 through 15.
Students can mark by exchanging
books. Note that the evaluation
comments at the right side of the page
for Exercises 1 to 10 is separate from
Exercises 11 to 15.

ACTIVITIES
1. Have the students write the various
data from the charts on the pupil
page onto cards.
Example

7:10 7/10 0.70 70%

Use this 60-card deck to play
“Rummy” as described in the Activity
Reservoir.
2. Have the students make up
their own charts showing data
collected from the class, school, or
home, such as:

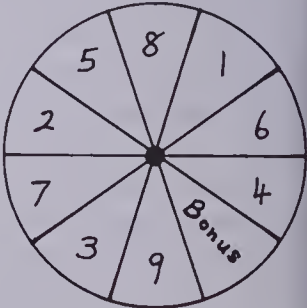
Tune Up
Copy and complete this chart of equivalents in your notebook.
Table with 4 columns: Ratio, Fraction ratio, Decimal, Percent. Rows 1-10.
Score: 25-30 Top shape, 20-24 Good shape, Less than 20 More exercise required.
Copy and complete this chart of equivalents.
(Round all decimals to 2 decimal places.)
(Round all percents to the nearest whole percent.)
Table with 4 columns: Ratio, Fraction ratio, Decimal, Percent. Rows 11-15.
Score: 12-15 Super shape, 8-11 Good shape, Less than 8 More training required.

234 Practice ratios decimals and percents

Table with 5 columns: Ratio, Fraction ratio, Decimal, Percent. Rows: Number of boys in class, Number of females in family, Number of brown-eyed people in school.

3. Prepare a chart and spinner as
shown. Players (from 2 to 4), each
with their own colour of pencil, take
turns twirling the spinner twice to
generate an ordered pair. If the
ordered pair indicates an empty place
on the grid, players write the
appropriate answer. If, on one of the
twirls, the spinner arrow indicates
“Bonus”, the player may fill in any
blank, available cell of the grid. Player
to have supplied the most answers
when the grid is full wins.

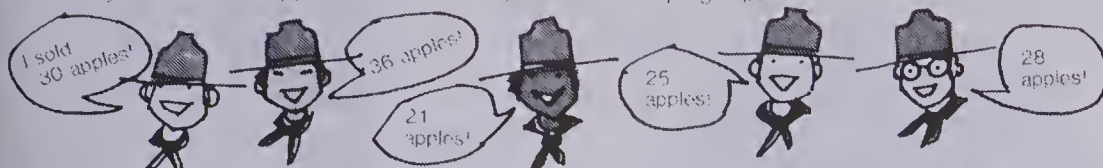
Table with 4 columns: Ratio, Fraction ratio, Decimal, Percent. Rows 1-9.



Example
Twirl three, then four. Locate and fill
either (3,4) or (4,3) with 0.76 or 35%
respectively.
Twirl two, then four — no response
allowed because both (2,4) and (4,2)
are occupied.

Boy Scout Day

Five Boy Scouts sold apples to raise money for their camping trip.



What was the average number of apples sold by each Scout?

To find average:

Step 1 Add the total number
of apples.

$$30 + 36 + 21 + 25 + 28 = 140$$

Step 2 Divide by the number
of Scouts

$$\begin{array}{r} 28 \\ 5 \overline{)140} \\ \underline{10} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

Each Scout sold an average of 28 apples.

Exercises

Find the average of these sets of numbers.

1. 7, 12, 11 **10** 2. 10, 10, 7, 13 **10** 3. 40, 50, 55, 20, 35 **40**
4. 125, 75, 100, 100 **100** 5. 2, 4, 6, 1, 5 **3.6** 6. 187, 99, 142, 160, 157 **149**

Find the average of these sets of numbers. (Calculate all averages to the nearest tenth.)

7. 8, 12, 20, 18 **14.5** 8. 38, 47, 54, 73, 94 **61.2**
9. 327, 209, 252 **262.7** 10. 1725, 1620 **1672.5**
11. 12, 17, 22, 6, 10, 15, 25, 17 **15.5** 12. 68, 55, 40, 81, 77, 72 **65.5**

13. Ten Girl Guides sold the following number of boxes of cookies to raise money for their activities

12, 17, 28, 32, 9, 51, 19, 24, 15, 23

What was the average number of boxes sold by each Girl Guide? **23**

Averages 235

OBJECTIVE

To calculate the arithmetic mean

PACING

Level A All
Level B All
Level C All

MATERIALS

metre tapes (marked in centimetres)

RELATED AIDS

CALC. ACTIVITY MASTERS — 10.

BACKGROUND

By average is meant the calculation of the arithmetic mean.

Example

$$\frac{\text{sum of the data}}{\text{number of pieces of data}}$$

SUGGESTIONS

Initial Activity To review the concept of average, divide the class into groups of from 4 to 8 students. Distribute the metre tapes and have the students measure each other's height (to the nearest centimetre) and record their findings at a place on the chalkboard reserved for their particular team.

When all measurements are complete, reassemble the class and, using the various data, demonstrate the calculation of the average height for each particular team. Discuss the steps involved in the process, emphasizing that the average simply gives us a general idea of about how tall each member of a particular team tends to be. Point out that, more often than not, the *actual* measurement of a team member is different (though usually close) from the *average* measurement for the team.

USING THE BOOK

Discuss the problem in the display to show the 2 steps required to determine the averages. Ask: "Which Boy Scout sold closest to the average number of apples?" [The one wearing glasses]

Exercises 1 to 6 could be done with the class. Assign teams of students to work on Exercises 7 through 13. You may wish to record the starting and finishing times for the exercises (see Activity 1).

Continued from page 233)

EXTRA PRACTICE

Express these percents as reduced ratios.

1. 20% 2. 22% 3. 24%
4. 26% 5. 28% 6. 30%
7. 32% 8. 34% 9. 36%
10. 38% 11. 40% 12. 42%
13. 44% 14. 46% 15. 48%
16. 50%

ACTIVITIES

1. Average Time

If all groups began the exercises at the same time, have them record the time when they finished. Calculate the elapsed time required to do the exercises. Record each group's elapsed time. Calculate the average time taken to the nearest minute.

2. Prepare a challenge card such as:

- (a) Write any ten different numbers between 20 and 100. Find the sum of these numbers; then calculate the average rounded to the nearest *tenth*.
(b) Repeat (a) using 6 numbers.
(c) Repeat (a) using 13 numbers.

3. Provide a metric weigh scale or bathroom scale. Have some of your Level C students calculate the average mass (to the nearest kilogram) of various groups in the class. Have them report their findings.

OBJECTIVE

To solve word problems involving averages

PACING

Level A 1-4
Level B 1-5
Level C All

RELATED AIDS

BFA PROB. SOLVING LAB II — 145.
CALC. ACTIVITY MASTERS — 41, 65.

SUGGESTIONS

Initial Activity Review rounding of whole and decimal numbers as it was presented on pages 12 and 13.

Present a problem such as:
Rob is training for a cross-country 10 km run.

His practice record for the course was:
Monday 57 min, Tuesday 50 min,
Wednesday 47 min, Thursday 48 min,
Friday 45 min.

What was his average time?

Demonstrate the solution as:

$$\begin{aligned} \text{Average} &= \frac{\text{total number of minutes}}{\text{number of practice runs}} \\ &= \frac{57 + 50 + 47 + 48 + 45}{5} \\ &= \frac{247}{5} \rightarrow 49.4 \\ &\quad \downarrow \\ &49.4 \rightarrow 49.0 \\ &= 49 \text{ min rounded} \\ &\quad \text{to the nearest minute} \end{aligned}$$

Rob's average time was 49 min.

USING THE BOOK

Discuss the method displayed at the top of the page for finding the report-card average when all the marks are given in percent.

Assign the problems in the exercises.

Encourage the students to organize their solution similar to the display.

Write a brief statement as a solution for each problem.

ACTIVITIES

1. Divide the class into groups of 5 or 6. Have them record their performances at some of the following tasks (by setting up activity stations, you may be able to have the groups rotate, thereby trying their hand at all 5 stations):

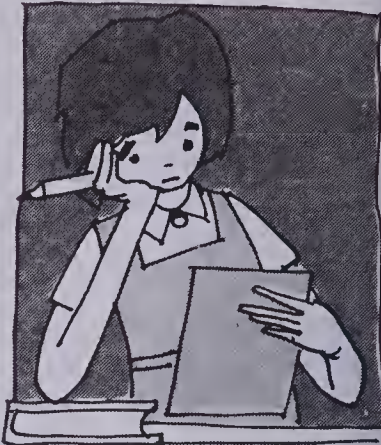
Averages

Tobi received the following marks on her report card:
66%, 72%, 58%, 90%, 77%, 79%, 82%, and 74%.

What was her average mark?

$$\begin{aligned} \text{Average} &= \frac{\text{total of marks}}{\text{actual number of marks}} \\ &= \frac{66 + 72 + 58 + 90 + 77 + 79 + 82 + 74}{8} \\ &= \frac{598}{8} \rightarrow 8 \overline{)598.00} \rightarrow (74.75) \rightarrow (75.00) \\ &= 75 \text{ rounded to the nearest whole percent} \end{aligned}$$

Tobi's average mark was 75%.



Exercises

1. Gary received the following marks on his second-term report card: 69%, 50%, 72%, 85%, 58%, 63%, 79%, and 75%. What was his average mark? (Round to the nearest whole percent.) **69%**
2. Shots on goal during the six championship hockey games were 60, 82, 45, 58, 70, and 51. What was the average number of shots on goal per game? **61**
3. Steven worked for four nights on his school project. He worked for 85 min, 60 min, 45 min, and 90 min. What was the average length of time he worked per night? **70 min**
4. On a summer trip the Cargill family travelled 2448 km in six days. What was the average distance they travelled each day? **408 km**
5. County-fair attendance for seven days was 18 044 people. What was the average daily attendance? (Round to the nearest whole number.) **2578**
6. Wendy received the following scores in figure skating: 5.8, 5.7, 5.5, 5.6, 5.7, and 5.9. What was her average score? **5.7**

- (i) record the number of bent-knee situps that each group member can complete in one minute;
- (ii) provide a large jar (or small bucket) containing small items such as marbles, counters, bottle caps, popcorn, elastics, centimetre cubes, paper clips, erasers, etc. or a mix of all of these. Record the number of items each group member can grab in one handful;
- (iii) record the number of correct multiplication and/or division facts each group member can complete correctly in 30 s (use a stencilled sheet or flash cards);

- (iv) record the amount of money each group member has with them today;
- (v) record the greatest number covered when six pennies are dropped from a standing position onto a numbered grid (use a blank wall calendar or the grid on text page 143).

Have the children in groups calculate the average performance for the team for the activities tried. Make up a bulletin-board display with the results.

2. To review and practise rounding, see the Activities listed on pages 12, 15, and 84.

Hot-Air Balloons

Hot-air balloons use propane burners to keep the air warm inside the balloon.

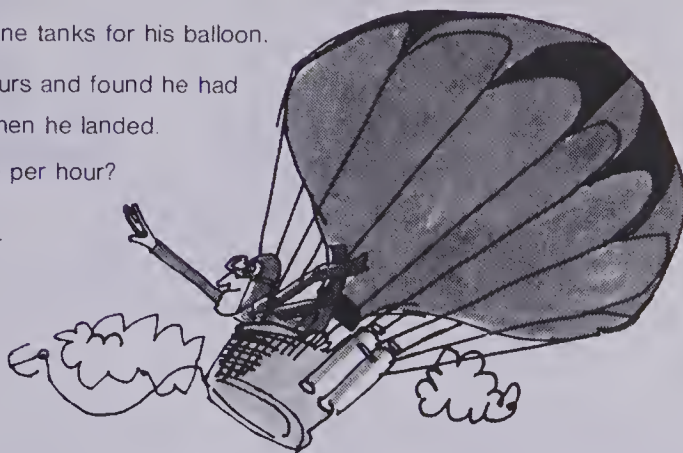
Mr. Russell has two full 90 L propane tanks for his balloon.

One Saturday, he flew for three hours and found he had 60 L of propane left in his tanks when he landed.

What was his average propane use per hour?

$$\begin{aligned}\frac{\text{Propane used}}{\text{Time in hours}} &= \frac{(2 \times 90) - 60}{3} \\ &= \frac{180 - 60}{3} \\ &= \frac{120}{3} \\ &= 40\end{aligned}$$

He used 40 L of propane per hour.



Exercises

- On a cold winter day, Mr. Russell used 108 L of propane for a four-hour flight. What was the average amount burned in each hour? **27 L**
- One morning Mr. Russell soared to a height of 1500 m in ten minutes. What was his average rate of ascent in metres per second? **2.5 m/s**
- Once in the sky, balloons travel at the speed of the prevailing winds. If the wind is blowing at an average of 32 km/h, how far would the balloon travel in 2.5 h? **80 km**
- Mr. Russell had to soar to 3000 m and then return to the ground to pass a test. He took 6 min to reach 3000 m, and 24 min to descend and land.
 - What was his average rate of ascent in metres per second? **8.3 m/s**
 - What was his average rate of descent in metres per second? **2.1 m/s**
- ★(c) The prevailing winds were blowing at an average speed of 50 km/h. How far from his take-off spot did he land? **25 km**

Word problems: averages 237

OBJECTIVE

To solve word problems involving averages

PACING

Level A 1-3
Level B 1-4(a)
Level C All

VOCABULARY

propane, ascent, prevailing, descend, descent

RELATED AIDS

HMS — DM59.

BFA PROB. SOLVING LAB II — 145.

SUGGESTIONS

Initial Activity Discuss how *hot-air balloons* work. A large gas-powered fan is used to fill the balloon with air while it is on the ground. One or two propane burners are mounted on a frame above the basket. When the cord is pulled, the burners heat the air in the balloon and the warm air expands. This hot air is lighter than the surrounding air and thus the balloon with its passengers rises. Once aloft, the balloon drifts with the prevailing wind.

Balloonists fly in the early morning or late afternoon when the winds are calmer. Landing when the winds are blowing is extremely dangerous.

The basket is made of wicker which is not only light but pliable in the event of a hard landing.

USING THE BOOK

These problems could be assigned in conjunction with the problems on the previous page.

Discuss the context of each problem with the students.

Assign the exercises.

Note: In Exercise 4, the students might question the safety of trying to land a balloon if the prevailing winds were 50 km/h. Explain that the wind at 1000 m might be 50 km/h and the wind at ground level might be 30 km/h. However, many balloonists would experience a rough landing at even 20 km/h.

ACTIVITIES

1. See "Square It" as described in the Activity Reservoir. Adjust the numbers on the dot paper so that they are suitable for your class. Play the game in the manner described with this twist: when totals are established, each player counts the number of squares that he/she completed; players calculate their average score.

2. Provide the students with this challenge:

If A has a value of 1, B = 2, C = 3, ..., Z = 26, what is the average value of each letter of the alphabet? [13.5]

3. Using an alphabet value system such as in Activity 2 above, have the students find (a) the average value of the letters of their name, (b) two words which have an average value less than 5, (c) two words which have an average value greater than 18.

OBJECTIVE

To determine the actual dimensions given a scale ratio and a plan

PACING

- Level A All
- Level B All
- Level C All

MATERIALS

centimetre graph paper and centimetre rulers

RELATED AIDS

HMS — DM60.

SUGGESTIONS

Initial Activity Discuss the duties of an architect. See the Career Awareness notes in the Chapter Overview on page 220.

Review, if necessary, the concept of ratio as it pertains to measurement as presented on pages 221 and 222.

USING THE BOOK

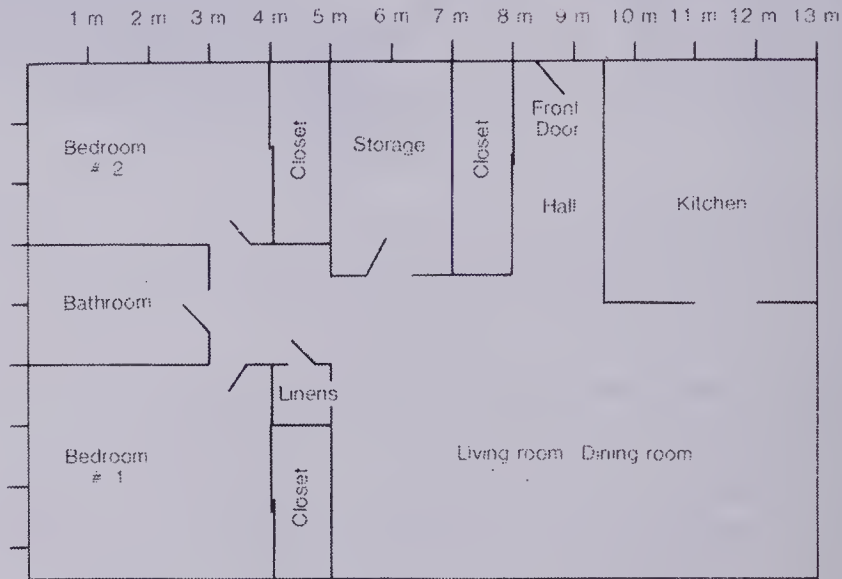
Read through the information presented on the page, discussing the need for scale drawings, the legend shown (i.e., $1\text{ cm} : 1\text{ m}$), and the various rooms and dimensions shown in the apartment floor plan. You may wish to mention things which are not shown in this “view from above” drawing (i.e., windows, furnishings, sinks, cupboards, etc.). Emphasize the meaning of the scale ratio — the scale drawing here ($13\text{ cm} \times 8.5\text{ cm}$) represents an actual apartment which is $13\text{ m} \times 8.5\text{ m}$.

The Architect

An architect drew some plans for an apartment.
The scale ratio is

$1\text{ cm} : 1\text{ m}$

This means 1 cm on the drawing represents 1 m in the actual room.



The length of the living/dining room is 8 cm on the blueprint. This represents an actual length of 8 m in the full-sized apartment.

Exercises

1. Find the actual dimensions of

- (a) kitchen 3.5 m X 4 m
- (b) bedroom # 1 4 m X 3.5 m
- (c) bedroom # 2 4 m X 3 m
- (d) bathroom 3 m X 2 m
- (e) storage room 2 m X 3.5 m
- (f) closet in bedroom # 1 1 m X 2.5 m
- (g) closet in bedroom # 2 1 m X 3 m
- (h) linen closet 1 m X 1 m



2. Trace bedroom # 1 in your notebook and draw to scale:

- (a) a bed that is 2 m by 1 m.
- (b) a rug that is 2 m by 2 m.
- (c) a dresser that is 1 m by 0.5 m.
- (d) a bookshelf that is 2 m by 0.25 m.



3. Trace another room in your notebook and draw in some furniture, using a scale factor of 1 cm : 1 m.



Scale ratios 239

Complete 1 or 2 exercises orally at the chalkboard before assigning the exercises. You may wish to (a) distribute graph paper for use with Exercises 2 and 3, (b) suggest that the students record the actual dimensions in a chart.

	Room	Length	Width
1(a)	kitchen	4 m	3.5 m
1(b)	bedroom #1		

ACTIVITIES

1. A different scale.

Using the centimetre graph paper, have the students draw the same apartment floor plan as the one on page 238 using a scale ratio or 2 cm:1 m. (That is, 2 cm represents 1 m.)

The drawing of the floor plan would be 26 cm by 17 cm.

Have them locate all the rooms and draw some furniture in using the same ratio.

2. Present the students with a ditto of the outline of the classroom on white paper.

If the room is 11 m long, this would be represented by 11 cm. If the room is 7.5 m wide, this would be represented by 7.5 cm.

Point out that this is the scale ratio 1 cm:1 m, 1 cm on the drawing representing 1 m actual.

Have them locate the door and one other feature on the stencil.

Ask the students to carefully sketch in some major features in the room on the classroom floor plan.

Discuss the results. Discuss any problems encountered.

3. Some of your Level C students might enjoy investigating the scale ratios used on some local municipal or provincial maps. Have them trace the legends used and use their tracings to estimate some "as the crow flies" distances between local towns or cities.

OBJECTIVE

To use ratios to determine actual sizes or scale sizes

PACING

Level A All
Level B All
Level C All

MATERIALS

centimetre graph paper and metric rulers

RELATED AIDS

CALC. ACTIVITY MASTERS — 82.

BACKGROUND

Solving ratio equations and using cross multiplication are assumed skills (see pages 223 to 225).

SUGGESTIONS

Initial Activity Review cross multiplication as it applies to equivalent ratios.

Example

$$\frac{1}{5} = \frac{6}{\blacksquare}$$

$$1 \times \blacksquare = 5 \times 6$$

$$1 \times \blacksquare = 30$$

$$\blacksquare = 30$$

$$\therefore \frac{1}{5} = \frac{6}{30}$$

Point out that, if we had a scale drawing of a rectangle whose width on the drawing is 6 cm and we are told the scale ratio is 1:5, then we could calculate the actual width of the rectangle.

$$\frac{\text{Drawing width}}{\text{Actual width}} \longrightarrow \frac{1}{5} = \frac{6}{\blacksquare}$$

By cross multiplying $1 \times \blacksquare = 5 \times 6$
 $1 \times \blacksquare = 30$
 $\blacksquare = 30$

The actual width is 30 cm.

USING THE BOOK

Discuss the problem in the display at the top of the page.

Have the students measure the drawing to see that the scale dimensions are accurate.

Explain the meaning of the scale ratio 1:4.

Demonstrate both calculations.

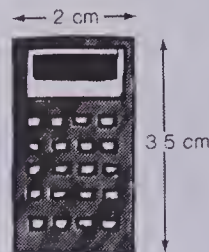
(a) Since the scale width is 2 cm, then the real width is 2×4 or 8 cm.

(b) Since the scale length is 3.5 cm, the real length is 3.5×4 or 14 cm.

Measure a real calculator to see if these dimensions are reasonable.

Scale Drawings I

This is a scale drawing of a calculator.
What is its actual size?



Use the ratio:

$$\frac{\text{Drawing width}}{\text{Actual width}} \longrightarrow \frac{1}{4} = \frac{2}{\blacksquare}$$

$$1 \times \blacksquare = 4 \times 2$$

$$\blacksquare = 8$$

The actual width is 8 cm.

$$\frac{\text{Drawing length}}{\text{Actual length}} \longrightarrow \frac{1}{4} = \frac{3.5}{\blacksquare}$$

$$1 \times \blacksquare = 4 \times 3.5$$

$$\blacksquare = 14$$

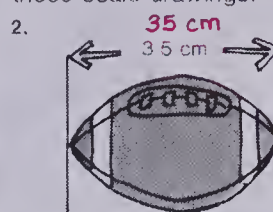
The actual length is 14 cm.

Exercises

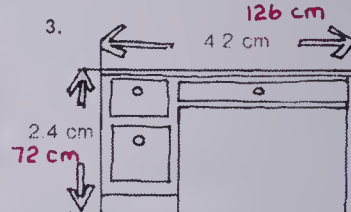
Use ratios to find actual sizes of these scale drawings.



1:50

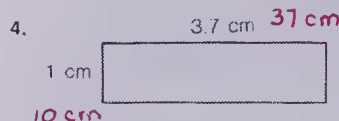


1:10

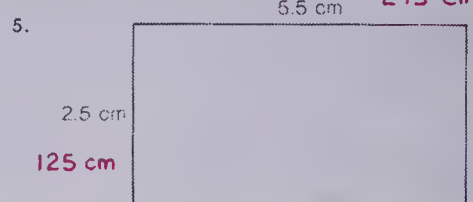


1:30

Measure and calculate the actual dimensions of these rectangles.



1:10



1:50

Scale Drawings II

The dimensions of a standard briefcase are 45 cm × 30 cm.

To make a scale drawing of this using a scale ratio of 1:10, Rob had to calculate the drawing dimensions. He used ratios:

$$\frac{\text{Drawing width}}{\text{Actual width}} = \frac{1}{10} = \frac{\blacksquare}{30}$$

$$10 \times \blacksquare = 1 \times 30$$

$$10 \times \blacksquare = 30$$

$$\blacksquare = 3$$

The drawing width should be 3 cm.

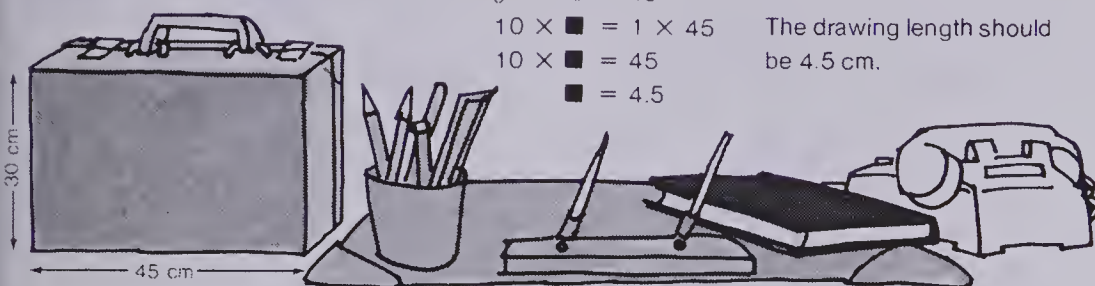
$$\frac{\text{Drawing length}}{\text{Actual length}} = \frac{1}{10} = \frac{\blacksquare}{45}$$

$$10 \times \blacksquare = 1 \times 45$$

$$10 \times \blacksquare = 45$$

$$\blacksquare = 4.5$$

The drawing length should be 4.5 cm.



Exercises

Using a scale ratio of 1:10, make the following scale drawings. *Dimensions for scale drawings are given.*

1. a rectangle 20 cm by 30 cm *2 cm by 3 cm*
2. a square with sides 55 cm *5.5 cm sides*
3. a desk top 70 cm by 50 cm *7 cm by 5 cm*
4. an envelope 25 cm by 10 cm *2.5 cm by 1 cm*
5. a cupboard door 35 cm by 60 cm *3.5 cm by 6 cm*

Using a scale ratio of 1:20, make the following scale drawings. *Dimensions for scale drawings are given.*

6. an office-desk top 140 cm by 80 cm *7 cm by 4 cm*
7. a queen-sized bed 150 cm wide by 200 cm long *7.5 cm by 10 cm*
8. a coffee-table top 70 cm square
9. a poster 80 cm by 50 cm
- ★ 10. a rug 2 m by 3 m

Activity

Make scale drawings of 4 objects in your classroom by using a scale ratio of 1:10 or 1:20.

Drawing to scale 241

Point out that we can measure real things and make scale drawings of them too. Discuss the problem in the display at the top of the page. Point out that, whereas on page 240 we started with a scale drawing and calculated the actual dimensions, here we start with the actual dimensions and try to calculate and make the scale drawing. Note that, since the scale ratio is 1:10, then

(a) 1 cm represents 10 cm.

(b) The real width, 30 cm, would be drawn as $30 \div 10$ or 3 cm and

(c) the real length, 45 cm, would be drawn as $45 \div 10$ or 4.5 cm.

Show the equivalent ratio equations for the above 2 situations (b) and (c).

Assign the exercises on both pages.

ACTIVITIES

1. To provide practice identifying and calculating simple equivalent ratios, play "Concentration" as described in the Activity Reservoir. Use cards such as:

$\frac{1}{4}$	$\frac{3}{12}$	$\frac{1}{10}$	$\frac{20}{200}$	$\frac{1}{3}$	$\frac{5}{15}$
---------------	----------------	----------------	------------------	---------------	----------------

2. See the Activity suggested at the bottom of pupil page 241. Assign it or prepare an activity card such as:

Choose *one* object to make a scale drawing. Measure the actual dimensions and record these on a sketch. Construct your actual scale drawing on centimetre graph paper using a scale ratio of 1:10.

3. See Activity 3, pages 238 and 239. Have them use equations and cross multiplication to calculate distances.

Example

1 cm:8.5 km

$$\frac{1}{8.5} = \frac{6}{\blacksquare}$$

$$1 \times \blacksquare = 8.5 \times 6$$

$$\blacksquare = 51$$

Two towns that are 6 cm apart on the map are actually 51 km apart "as the crow flies".

OBJECTIVE

To evaluate achievement of the chapter objectives

PACING

Level A All
Level B All
Level C All

RELATED AIDS

HMS — DM61.

USING THE BOOK

Each student should do this test independently under supervision. Assistance should be given only when the instructions are not understood. After the work has been corrected, you should provide appropriate remedial work. You may wish to reteach if a large number of students had difficulty with a particular topic or concept.

The following chart will help in this regard. The specific objectives are listed in the Chapter Overview (see page 220).

Given the context of this chapter and assuming that (a) each part of Exercises 1 to 7, 9, and 10 are assigned a value of 1 mark (i.e., Exercise 1 is worth 4 marks, Exercise 3 is worth 5 marks, and so on); (b) Exercise 8 is assigned a value of 5 marks; (c) Exercise 11 is assigned a value of 6 marks; have the students calculate their decimal equivalent and then their percent mark as follows:

Example
 $\frac{44}{50} \rightarrow \frac{88}{100} \rightarrow 0.88 \rightarrow 88\%$

An alternate Chapter Test can be found in the Holt Mathematics System Duplicating Masters available for use with this grade level.

Test Item	Objective	Text Page Number
1-3, 8	A	222, 223, 225
4-7	B	226, 228, 232, 233
10	C	230
9	D	235
11	E	241

Chapter Test

- Calculate cross products and state which pairs of ratios are equivalent. *(b), (c), and (d) are equivalent.*
 (a) $\frac{7}{5}, \frac{20}{15}$ $5 \times 20 = 100$ $7 \times 15 = 105$ (b) $\frac{3}{5}, \frac{15}{25}$ $5 \times 15 = 75$ $3 \times 25 = 75$ (c) $\frac{12}{15}, \frac{4}{5}$ $15 \times 4 = 60$ $12 \times 5 = 60$ (d) $\frac{45}{30}, \frac{3}{2}$ $30 \times 3 = 90$ $45 \times 2 = 90$
- Find the missing term in these equivalent ratios.
 (a) $\frac{1}{3} = \frac{\blacksquare}{12}$ $\blacksquare = 4$ (b) $\frac{5}{8} = \frac{15}{\blacksquare}$ $\blacksquare = 24$
- Copy and complete the following ratio chart

		15	25	
Oranges	12	6	3	100
Total cost	96¢		120¢	200¢

48¢ 24¢
80¢
- Express each as a percent.
 (a) $\frac{7}{20}$ **35%** (b) $\frac{11}{25}$ **44%** (c) $\frac{13}{50}$ **26%** (d) 0.42 **42%** (e) 0.06 **6%** (f) 0.8 **80%**
- Express each as a decimal
 (a) 75% **0.75** (b) 40% **0.4** (c) 4% **0.04** (d) 12% **0.12** (e) 9% **0.09**
- Express the percents in Exercise 5 as reduced ratios.
 (a) $\frac{3}{4}$ (b) $\frac{2}{5}$ (c) $\frac{1}{25}$ (d) $\frac{3}{25}$ (e) $\frac{9}{100}$
- Express each ratio as a percent rounded to the nearest tenth.
 (a) $\frac{1}{3}$ **33.3%** (b) $\frac{3}{8}$ **37.5%** (c) $\frac{2}{7}$ **28.6%**
- Susan can swim 200 m in 4 min. How far can she swim in 6 min? **300 m**
- Find the average of the percents in Exercise 5. **28%**
- Find:
 (a) 40% of 300 **120** (b) 8% of 50 **4** (c) 10% of \$7.20 **\$0.72** (d) 1% of \$320 **\$3.20**
- Use a scale ratio of 1:20 to make a scale drawing of a wall map 50 cm by 80 cm
2.5 cm by 4 cm

Cumulative Review

Write in expanded notation.

1. 72 826

2. 902 400

3. 389.25

Write as a numeral

4. fourteen million, six hundred twenty-three thousand, eight hundred two **14 623 802**

5. seventy-eight decimal two five **78.25**

Perform the indicated operations.

6.
$$\begin{array}{r} 793 \\ \times 85 \\ \hline 67405 \end{array}$$

7.
$$\begin{array}{r} 238 \\ \times 100 \\ \hline 23800 \end{array}$$

8.
$$\begin{array}{r} 321.7 \\ \times 6.4 \\ \hline 2058.88 \end{array}$$

9. $4.83 - 0.001$ **4830**

10. $0.31 \overline{)27652}$ **89.2**

11.
$$\begin{array}{r} 19 \\ 20 \end{array}$$

12.
$$\begin{array}{r} 5 \\ 24 \end{array}$$

13.
$$\begin{array}{r} 6\frac{2}{5} \\ + 2\frac{1}{6} \\ \hline 8\frac{17}{30} \end{array}$$

14. $\frac{3}{5} \times 4$ **$1\frac{1}{2}$**

15. $1\frac{3}{5} \times 3\frac{3}{4}$ **6**

16. $10 - \frac{2}{3}$ **15**

17. $\frac{3}{5} - 2$ **$\frac{3}{10}$**

Solve for n .

18. $n + \frac{3}{4} = 1$ **$\frac{1}{4}$**

19. $\frac{3}{8} = \frac{n}{24}$ **9**

20. $\frac{3}{5} = \frac{12}{20}$

Find

21. 20% of 30 **6**

22. 7% of 500 **35**

23. 50% of \$7.25 **\$3.63**

24. 5% of \$16.80 **\$0.84**

25. Record albums were on sale for 65% of their regular price.

What would the sale price be for an album that normally costs \$9.80? **\$6.37**

Chapters 1-5 cumulative review 243

ANSWERS:

$72\,826 = (7 \times 10\,000) + (2 \times 1000) + (8 \times 100) + (2 \times 10) + (6 \times 1)$
 $= (7 \times 10^4) + (2 \times 10^3) + (8 \times 10^2) + (2 \times 10^1) + (6 \times 1)$

$902\,400 = (9 \times 100\,000) + (2 \times 1000) + (4 \times 100)$
 $= (9 \times 10^5) + (2 \times 10^3) + (4 \times 10^2)$

$389.25 = (3 \times 100) + (8 \times 10) + (9 \times 1) + (2 \times 0.1) + (5 \times 0.01)$

OBJECTIVE

To review and test selected concepts and skills previously covered

PACING

Level A All

Level B All

Level C All

USING THE BOOK

This page may be used for diagnostic and remedial as well as review purposes. Students should check their work, correct any errors, and review the pages that contain any problems of the type they missed. Some students can do this on their own while others may need help. If a large number of students have a particular problem incorrect, you may want to reteach that topic to the groups, then assign a duplicated worksheet to reinforce that topic or, refer to an appropriate skill card in the BFA Computational Skills Kit II or BFA Problem Solving Kit II.

Also, given the context of the preceding chapter, have the students calculate their percent marks. Assign a value of one mark per question.

Test Item	Text Page Number
1-3	177
4	10
5	2
6	65
7	61
8	75
9	135
10	134
11, 12	191
13	202
14	205
15	213
16, 17	210
18	207
19, 20	188, 223
21-24	230
25	231

CHAPTER 9 OVERVIEW

This chapter uses all operations involving whole numbers, decimals, and percents; and applies these operations to the solution of single- and multiple-step word problems. Many of the problems are life-skill oriented or based on making predictions or stating probabilities.

OBJECTIVES

- A To review addition, subtraction, and multiplication involving decimals and percents
- B To balance bank records involving deposits and withdrawals and calculate simple bank and bond interest
- C To select and solve equations; and use appropriate equations when solving word problems
- D To review problem-solving techniques and apply these techniques to the solution of single- and multiple-step problems involving a variety of topics including discounts, sale prices, sales taxes, and making change
- E To use equivalent ratios and cross products to solve time, speed, and distance problems
- F To acquire some insight into probability and prediction techniques by doing probability activities and reflecting on the results
- G To solve problems related to conservation issues

BACKGROUND

1. Much of the numerical skill development in this chapter has been developed in previous chapters. Thus, the focus is to discuss fully the problem-solving contexts and apply the previously learned skills within these contexts.

2. When solving equations many "related" equations can be used. This is based on an inherent, non-rigorous application of inverse statements.

Examples

	Related Equations
(a) $n + 5 = 7$	$7 - 5 = n$ $2 = n$
(b) $n - 8 = 10$	$10 + 8 = n$ $18 = n$
(c) $n \times 7 = 56$	$56 \div 7 = n$ $8 = n$
(d) $n \div 5 = 40$	$40 \times 5 = n$ $200 = n$

These related equations can best be illustrated using simple true number sentences.

Examples

- (a) $8 + 5 = 13$ \rightarrow $13 - 5 = 8$
 \rightarrow $13 - 8 = 5$
- (b) $10 - 6 = 4$ \rightarrow $4 + 6 = 10$
- (c) $5 \times 3 = 15$ \rightarrow $15 \div 3 = 5$
 \rightarrow $15 \div 5 = 3$
- (d) $32 \div 4 = 8$ \rightarrow $8 \times 4 = 32$

MATERIALS

samples of cheques and deposit slips from various banks or trust companies
4-function calculator with a % key
daily or weekly newspapers
centimetre graph paper
30 cm rulers
unsharpened hexagonal pencils
round pencils, sharpened
pennies (20)
Bristol board or card stock, scissors, glue and/or tape
small wooden blocks to make 20 pairs of dice (unused inch cubes, etc.)

CAREER AWARENESS

Insurance Agent [272]

Insurance agents represent companies that protect individuals and companies against financial loss in case of tragedies or accidents.

They might sell life insurance, which pays the survivors upon the death of the policyholder; or they might sell home or car insurance which protects against loss due to fires, theft, or accidents.

They might also sell health or dental insurance, which pays for dentist, hospital, and medical care.

Insurance agents can sell insurance to protect almost any item or event (e.g., a sponsor of an outdoor activity may purchase "rain" insurance in case the event has to be cancelled due to the weather conditions).

If a client recognizes a need, and a company can determine a rate for the risk, then the insurance agent can write a policy.

Agents may be self-employed and represent numerous companies or they might be employed by a large company.

Insurance agents are usually trained initially by the company they work for, but are eventually licenced by the province after having completed specific courses.

Education requirements for agents vary depending on the types of insurance being sold and the complexities involved. However, most companies require university graduates.

Tune Up

Add.

1. $\$6.25 + \9.32 $\$15.57$
2. $\$12.29 + \7.97 $\$20.26$
3. $\$104.09 + \38.75 $\$142.84$
4. $\$0.72 + \$2.53 + \$7.09$ $\$10.34$
5. $\$35.92 + \78.59 $\$114.51$
6. $\$183.79 + \2.89 $\$186.68$

Subtract.

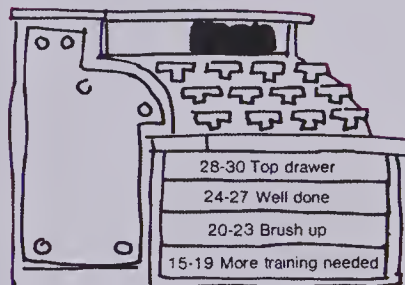
7. $\$1.75 - \1.23 $\$0.52$
8. $\$3.82 - \2.59 $\$1.23$
9. $\$9.12 - \6.58 $\$2.54$
10. $\$24.38 - \19.92 $\$4.46$
11. $\$57.95 - \1.20 $\$56.75$
12. $\$138.85 - \79.09 $\$59.76$
13. $\$10.00 - \7.59 $\$2.41$
14. $\$20.00 - \12.25 $\$7.75$
15. $\$100.00 - \62.99 $\$37.01$

Multiply

16. $\begin{array}{r} \$20.00 \\ \times 0.3 \\ \hline \end{array}$ $\$6.00$
17. $\begin{array}{r} \$70.00 \\ \times 0.5 \\ \hline \end{array}$ $\$35.00$
18. $\begin{array}{r} \$40.00 \\ \times 0.7 \\ \hline \end{array}$ $\$28.00$
19. $\begin{array}{r} \$15.00 \\ \times 0.25 \\ \hline \end{array}$ $\$3.75$
20. $\begin{array}{r} \$37.00 \\ \times 0.35 \\ \hline \end{array}$ $\$12.95$
21. $\begin{array}{r} \$85.00 \\ \times 0.42 \\ \hline \end{array}$ $\$35.70$
22. $\begin{array}{r} \$3.25 \\ \times 0.06 \\ \hline \end{array}$ $\$0.195$
or $\$0.20$
23. $\begin{array}{r} \$5.80 \\ \times 0.07 \\ \hline \end{array}$ $\$0.406$
or $\$0.41$
24. $\begin{array}{r} \$27.50 \\ \times 0.09 \\ \hline \end{array}$ $\$2.475$ or $\$2.48$

Calculate. Round to the nearest cent when necessary.

25. 10% of $\$60.00$ $\$6.00$
26. 15% of $\$20.00$ $\$3.00$
27. 25% of $\$65.00$ $\$16.25$
28. 2% of $\$16.40$ $\$0.33$
29. 6% of $\$9.20$ $\$0.55$
30. 5% of $\$30.00$ $\$1.50$



Practice 245

OBJECTIVE

To practise computations involving decimals and percent

PACING

Level A All
Level B All
Level C 1, 4, 10, 12, 16, 22, 25, 27, 29

BACKGROUND

This series of questions could be assigned as a review or it could be saved for future use at some other time.

USING THE BOOK

Various diagnostic and/or review uses:

- (a) Choose 2 questions of each type. Perform the indicated operations and check. (mark out of 8)
 - (b) Choose 3 questions of each type and all of the percent questions, e.g., Exercises 4, 5, and 6; 12, 14, and 15; 20, 23, and 24; and 25 through 30. (mark out of 15)
 - (c) Do all questions. (mark out of 30)
- If students have unusual difficulty with these problems, you may wish to provide appropriate remedial activities. The following chart shows where the various topics were presented in the text.

Exercise	Page
1-15	4, 5, 201
16-24	70, 77
25-30	230, 232

ACTIVITIES

1. Have the students write 8 questions (2 of each type shown on this page) of their own for exchange with classmates or other groups. Once the 8 questions have been completed, they should be returned to the originator for marking.

2. See the "Coded Riddles" idea in the Activity Reservoir.

3. See "Input-Output" as described in the Activity Reservoir.

OBJECTIVE

To calculate balances of accounts involving deposits and withdrawals

PACING

Level A All
Level B All
Level C All

VOCABULARY

withdrawal, deposit, chequing

MATERIALS

cheques, deposit slips, a simple 4-function calculator

RELATED AIDS

BFA PROB. SOLVING LAB II — 111.

BACKGROUND

Balancing bankbooks or checking bank records is a life skill and simply involves addition and subtraction.

SUGGESTIONS

Initial Activity Discuss banks and banking: What is a bank? How many students have a bank account or trust company account? What do banks do with the money? (Clear up any myths about banking.) You might wish to display and discuss some of the simple forms used for transactions in your local banks (cheques, deposit slips, withdrawal slips, passbook, etc. Consider using the school petty cash book, if one is available.)

USING THE BOOK

Bankbooks or cheque registers are ways of keeping track of banking transactions. In both cases there is a place to record:

- (a) withdrawals or amounts of cheques which are subtracted from the previous balance;
- (b) deposits or amount of deposit which are added to the previous balance;
- (c) the new balance.

Discuss the transactions shown in Kevin's bankbook on page 246.

Point out that the "Balance Forward" is the beginning balance in the account or the balance from the previous page or book.

Since a deposit is added to the previous balance of \$50.72, the new balance is \$53.72.

Kevin's Bank Account

Kevin has a savings account at a local bank. This is a page from his bankbook.

Date	Item	Withdrawals	Deposits	Balance
MAR. 01	Bal. Fwd.			50.72
MAR. 15			3.00	53.72
MAR. 19		7.50		46.22

Deposits are added.

$$\begin{array}{r} \$50.72 \\ + 3.00 \\ \hline \$53.72 \end{array}$$

Withdrawals are subtracted.

$$\begin{array}{r} \$53.72 \\ - 7.50 \\ \hline \$46.22 \end{array}$$

Exercises

Copy and find the missing amounts.

	Date	Item	Withdrawals	Deposits	Balance
	MAR. 10	Bal. Fwd.			72.37
(a)	MAR. 27		3.50		68.87
(b)	APR. 02			4.75	73.62
(c)	APR. 20		10.15		63.47

	Date	Item	Withdrawals	Deposits	Balance
	FEB. 17	Bal. Fwd.			50.00
(a)	MAR. 01			10.00	60.00
(b)	MAR. 15		15.00		45.00
(c)	MAR. 30			12.50	57.50
(d)	APR. 27			20.17	77.67

246 Bank accounts: deposits, withdrawals

Since withdrawals are subtracted from the previous balance of \$53.72, then the new balance is \$53.72

$$\begin{array}{r} \$53.72 \\ - 7.50 \\ \hline \$46.22 \end{array}$$

In Exercise 3, mention that 0901 means Sept. 01.

Assign the exercises. Be sure that the students are familiar with the established answer format used in your class.

3. Yvonne has a chequing account so that she can keep track of the money from her paper route. She keeps track of her own deposits and cheques in her bankbook.
- (a) Calculate the balance after each entry.

Date	Cheque No	Particulars	Amount of Cheque	✓	Amount of Deposit	Balance
09 01		Balance Forward				72.45
09 05	—	Paper Collection			32.00	104.45
09 08	17	Times-Examiner	23.42			81.03
09 12	—	Paper Collection			33.00	114.03
09 15	18	Times-Examiner	24.15			89.88

- (b) What was the total amount collected during the two weeks in September? \$65.00
- (c) What was the total paid to the *Times-Examiner* for papers? \$47.57
- (d) How much did she make delivering papers for two weeks? \$17.43
4. Calculate the balance after each entry.

Hint:
03 15
means
March 15

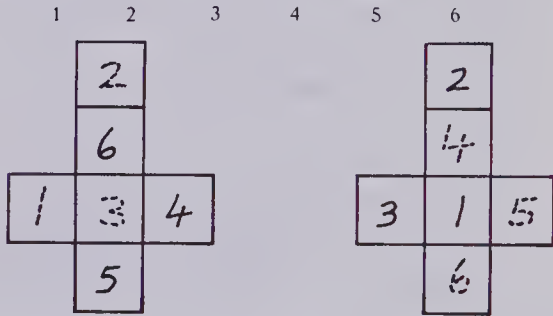
Date	Item	Initials	Withdrawals	Deposits	Balance
	Bal Fwd				72.37
01 17			3.50		68.87
01 30				4.00	72.87
03 15			10.00		62.87
04 05				2.00	64.87
04 19				7.25	72.12
05 20				5.50	77.62
06 19			15.00		62.62
07 03			10.50		52.12
07 30				60.00	112.12
08 10			4.25		107.87
08 30				70.00	177.87
09 07			18.00		159.87
10 15			19.52		140.35

Bank accounts deposits withdrawals 247

ACTIVITIES

1. Provide a grid as shown and two number cubes to play "Bank Balance". Note that some of the numbers on the dice are coloured red (indicated on the cube nets as dotted numerals).

6	\$13.50	\$26.33	\$72.72	\$36	\$3.60	\$1
5	\$10.00	\$100.00	\$8.88	\$6.60	\$0.99	\$55.45
4	\$9.09	\$11.11	\$12.50	\$125.00	\$9.98	\$6.66
3	\$12.95	\$11.88	\$3.85	\$0.01	\$0.10	\$1.00
2	\$38.90	\$0.75	\$15.50	\$6.08	\$21.00	\$0.21
1	\$50.00	\$3.62	\$121.80	\$36.15	\$4.12	\$0.50



Players start with a balance of \$500. Players take turns rolling both dice and locating an amount on the grid which corresponds to the ordered pair rolled. Dice should be rolled one at a time. If both dice show red numerals, or, in the case of a mixed roll, the *second* number is red, the grid amount indicated should be regarded as a withdrawal to be subtracted from the previous balance. Two black numbers rolled or one red *then* one black indicates a deposit. The player with the greatest balance after a predetermined number of turns is the winner. A balance sheet (optional) is also shown.

Deposit	Withdrawal	Balance
		\$500.00

2. A Bank Visit.
Phone local banks to see if a group visit can be organized. If this is not possible, ask if they have a simple booklet or mini-course to explain banking.
3. Introduce a banking game such as "Monopoly" to your games centre. Be sure to explain the rules thoroughly to novices, especially the duties of the player designated as banker.

OBJECTIVE
To calculate interest

PACING

- Level A Page 248 — 1-4
Page 249 — 1-10
Level B Page 248 — All
Page 249 — 1-13
Level C Page 248 — All
Page 249 — 1-14

VOCABULARY

non-chequing, interest, particulars

RELATED AIDS

HMS — DM62.
BFA PROB. SOLVING LAB II — 143.

SUGGESTIONS

Initial Activity Define interest as rent paid for the use of money. Point out also that:
(a) Most interest rates are stated per year.
(b) When money is borrowed the borrower pays interest to the lender.
(c) When a person deposits money in a bank or buys Canada Savings Bonds, the money is in a sense being loaned to the bank or to the company. In return, they pay interest.
Review how to find a percent of a given number.

USING THE BOOK

Read through the information at the top of the page.
Clarify that:
(a) At 9% per year on \$100: \$9.00 interest earned ∴ new balance is \$109.00.
(b) For the second year at 9%:
109.00
x 0.09
9.8100
The interest on \$109.00 at 9% for a year is \$9.81.
Thus the new balance is
\$109.00 + \$9.81 = \$118.81.
This page is closely associated with page 249. You may wish to complete both pages together.

Savings Accounts

When Brenda deposits \$100.00 in a non-chequing savings account that pays 9% interest per year, in a way she is lending her money to the bank. As a reward, the bank pays her interest.
How much interest would Brenda earn in one year?

Step 1 Change 9% to a decimal. 9% = 0.09
Step 2 Find 9% of \$100. 100.00 x 0.09 = 9.0000

In one year, she would earn \$9.00 interest.

Table with 6 columns: Date, Item, Initials, Withdrawals, Deposits, Balance. Rows show transactions for 80, 81, and 82.

Exercises

Calculate the interest earned after one year on the following deposits. State the new balance after one year.

- 1. A \$100.00 deposit that earns 7% interest. \$7.00; \$107.00
2. A \$500.00 deposit that earns 9% interest. \$45.00; \$545.00
3. A deposit of \$100 000.00 from a lottery win that earns 10% interest. \$10 000; \$110 000
4. A \$1 000 000.00 deposit that earns interest at the rate of 11% per year. \$110 000; \$1 110 000

Copy and complete this page from Barry's bankbook. The interest rate is 9%.

Table with 6 columns: Date, Item, Initials, Withdrawals, Deposits, Balance. Rows show transactions for 80, 81, 82, and 83.

Adam's Savings Bond

Buying a Canada Savings Bond is like lending money to the Canadian government. As a reward, the government pays interest.

Adam has a \$100 Canada Savings Bond that pays interest at an annual rate of 12%.

How much interest did he receive in one year?

Step 1 Change 12% to a decimal.

$$12\% = 0.12$$

Step 2 Find 12% of \$100.00.

$$\begin{array}{r} 100.00 \\ \times 0.12 \\ \hline 12.0000 \end{array}$$



He earned \$12.00 interest

Exercises

Calculate the interest for one year.

$$\begin{array}{r} \$500.00 \text{ at } 12\% \quad 500.00 \\ \$60.00 \times 0.12 \\ \hline \end{array}$$

$$\begin{array}{r} \$200.00 \text{ at } 10\% \quad 200.00 \\ \$20.00 \times 0.10 \\ \hline \end{array}$$

3. \$150.00 at 11% **\$16.50** 4. \$800.00 at 10% **\$80.00** 5. \$50.00 at 12% **\$6.00**
6. \$350.00 at 11% **\$38.50** 7. \$1000.00 at 10% **\$100.00** 8. \$1000.00 at 14% **\$140.00**

Calculate the interest earned or owed for one year.

9. Mr. Smith borrowed \$500.00 at 18% for one year. **\$90.00**
10. Steven deposited \$300.00 in a 6% bank account for one year. **\$18.00**
11. The Pender family borrowed \$16 000.00 at 15% for one year to buy a cottage. **\$2400.00**
12. The Hogon family borrowed \$6000.00 at 14% to purchase a new car. **\$840.00**
13. George Bell purchased a \$200.00 Canada Savings Bond that pays interest at the rate of 12% per year. **\$24.00**
★14. Kevin's sister earned \$11.00 on a \$100.00 bond last year. What was the interest rate? **11%**

Simple interest 249

For all the exercises on this page, only the interest need be calculated.

Since all the dollar amounts do not include any cents, then the solutions could be calculated this way:

$$\begin{array}{r} 100 \\ \times 0.12 \\ \hline 200 \\ 100 \\ \hline 12.00 \end{array} \quad \text{or} \quad \begin{array}{r} 500 \\ \times 0.12 \\ \hline 1000 \\ 500 \\ \hline 60.00 \end{array}$$

Assign the exercises.

ACTIVITIES

1. To review decimal and percent equivalence, play "Concentration" as described in the Activity Reservoir. Use cards such as:

11%	0.11	55%	0.55
99%	0.99		

2. Prepare an Activity Card such as:

Double Your Money.

Assume you have \$100 in a savings bond.

How long would it take for you to double your money to \$200.00 or more if

- (a) the interest rate is 10% per year?
(b) the interest rate is 15% per year?
(c) the interest rate is 19% per year?

Note: You may wish to let students use a calculator for this.

3. "Canada Savings Bonds". At the present interest rate on a compounding bond, how many years before a \$100 bond is worth \$200 or more?

OBJECTIVE

To calculate provincial sales tax
(finding a percent of a number)

PACING

Level A 1-3
Level B 1-3
Level C All

BACKGROUND

Provincial sales taxes were (at first) a result of the 1930s depression which saw reduced provincial revenues, yet greatly increased need for funds to support various social welfare programs — hospitalization, medicare, etc. They were introduced usually at 2 or 3% and against much opposition, in this order: Montreal City Tax, 1935; Alberta, 1936 — discontinued in 1937; Saskatchewan, 1937; Quebec, 1940 (replaced Montreal Tax); British Columbia, 1948; New Brunswick, 1950; Newfoundland, 1950; Nova Scotia, 1959; Ontario, 1962.

Food and other essentials are generally tax exempt in an effort to ease the burden on people at lower income levels.

SUGGESTIONS

Initial Activity Discuss sales taxes, mentioning why sales taxes are levied by Provincial Governments. List items on the board that are sales tax exempt. Investigate: (a) Why are these items not taxed? (b) What is the sales tax rate in your province?

Review finding a percent of a number (see page 230).

USING THE BOOK

Discuss the steps involved in determining the total cost of an item.

Step 1

Calculate the sales tax.

$$\begin{array}{r} 82 \text{ ————— no decimal places} \\ \times 0.07 \text{ ————— 2 decimal places} \\ \hline 5.74 \text{ ————— 2 decimal places} \\ \text{or} \\ 82.00 \\ \times 0.07 \\ \hline 5.7400 \end{array}$$

The sales tax is \$5.74.

Step 2

Add the sales tax to the selling price.

Selling price	\$82.00
Sales tax	\$ 5.74
Total cost	\$87.74

Assign the problems.

Provincial Sales Tax

Most provinces (and territories) charge **sales tax** on items purchased. Each province sets its own rate.

Mr. Gordon purchased a new coat for \$82.00.

The provincial sales tax rate was 7%.

(a) Calculate the sales tax.

(b) What was the total cost of the coat?

Step 1 Calculate sales tax

$$7\% = 0.07$$

$$82.00$$

$$\times 0.07$$

$$5.7400$$

Step 2 Calculate total cost of the coat.

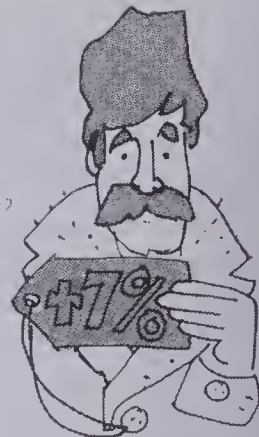
$$82.00$$

$$+ 5.74$$

$$87.74$$

The sales tax was \$5.74

The total cost of the coat was \$87.74



Exercises

- What is the sales-tax rate in your province? *Answers will vary.*
- Calculate the sales tax and total cost for each of these purchases.

	Purchase Price	Sales-Tax Rate	Sales Tax	Total Cost
(a)	\$ 1.00	7%	\$ 0.07	\$ 1.07
(b)	\$ 5.00	7%	0.35	5.35
(c)	\$20.50	8%	1.64	22.14
(d)	\$ 3.75	4%	0.15	3.90
(e)	\$18.95	8%	1.52	20.47

- Tom bought the following clothing: a shirt for \$12.95, jeans for \$21.50, and a sweater for \$17.95. What was the cost of the clothing? How much did he pay if the sales-tax rate was 8%? *\$56.59 \$52.40*
- ★ The regular price of a new 10-speed bicycle is \$200.00. Calculate the total cost if the bicycle is on sale at a discount of 20% and the sales-tax rate is 6%. *\$169.60*

250 Calculating sales tax

ACTIVITIES

1. "Catalogue Orders".

Have each student:

- Bring a catalogue to school to order 5 items.
- Describe the 5 items and list their individual prices on an *order form* chart.
- Calculate the total selling price.
- Calculate the sales tax for this total.
- Find the total cost of the 5 items.

Ask: "How long would you have to save money in order to purchase those 5 items?"

2. Prepare a research card such as:

"Research".

By interviewing a parent, relative, and/or store owners, compile a list of 20 to 30 items that are "sales tax exempt".

Write possible reasons why no sales tax is charged for these items.

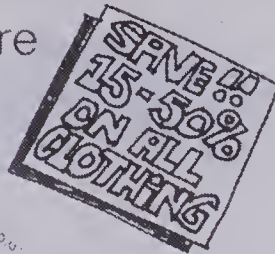
Discuss your findings and add to your list.

3. Some of your Level C students might enjoy researching the sales tax rates of the various provinces. Have them report their findings to the class. Have them calculate the various total cost prices in 3 provinces of an appliance that sells for \$79.95.



Jenner's Department Store

Stores usually have sales to help bring in more business and compete with other stores. During sales, there are discounts off regular prices of some items.



The regular price of a calculator is \$13.00, and its discount rate is 20%. How much is the discount? What is the sale price?

Step 1 Calculate discount

$$\begin{array}{r} 20\% = 0.20 \quad 13.00 \\ = 0.2 \quad \times \quad 0.2 \\ \hline 2.600 \end{array}$$

The discount is \$2.60.

Step 2 Subtract

$$\begin{array}{r} \$13.00 \\ - \quad 2.60 \\ \hline \$10.40 \end{array}$$

The sale price is \$10.40.

Exercises

Calculate the discount and sale prices for the following items.

1. Colour TV Regular \$500.00 Save 20% \$100.00; \$400.00	2. Jeans Regular \$19.00 Save 30% \$5.70; \$13.30	3. Posters Regular \$2.50 Save 40% \$1.00; \$1.50
4. Skates Regular \$55.00 Save 15% \$8.25; \$46.75	5. Track Shoes Regular \$18.00 Save 10% \$1.80; \$16.20	6. Round Trip by Air HALIFAX TO VANCOUVER Regular \$460.00 Save 30% \$138.00; \$322.00
7. Christmas Cards (a) Box of 12 Regular \$2.50 (b) Box of 20 Regular \$4.50 (c) Jumbo Deluxe Box of 50 Regular \$6.50 Save 60% \$2.10; \$4.40	8. Steak Dinner Regular \$7.50 Monday Special Save 20% \$1.50; \$6.00	9. Winter Coats Regular \$85.00 Save 30% \$25.50; \$59.50

ANSWERS:

7(a) \$1.50, \$1.00 (b) \$2.70, \$1.80 (c) \$3.90, \$2.60.

ACTIVITIES

1. "Discount Fever"

Using newspaper advertisements of "door-to-door flyers", cut and paste problems on file cards.

(a) Solve the problems in your workbook.

(b) Check with your teacher.

(c) Sign your name to your problem cards. Exchange and mark the cards.

2. Prepare an instruction card such as:

"One-step Sale Prices"

If all that is required is the final sale price of an item, then this can be determined in *one* step.

Example

(1) If the discount rate is 20%, the sale price is 80% of the regular price.

(2) If the discount rate is 30%, the sale price is 70% of the regular price.

Why?

Since the regular price is 100%
and the discount rate is 30%
the sale price is 70%
of the regular price.

Use this technique to find the sale prices of any three exercises from page 251.

3. See "Scramble" as described in the Activity Reservoir. Use cards such as:

A	$\begin{array}{r} \$460.00 \\ \times \quad 0.2 \\ \hline \end{array}$	E	$\begin{array}{r} 50 \\ \times 0.10 \\ \hline \end{array}$
---	---	---	--

etc.

OBJECTIVE

To calculate discounts and determine the sale price of goods

PACING

Level A All

Level B All

Level C All

MATERIALS

4-function calculator, newspapers

RELATED AIDS

HMS — DM63.

BFA PROB. SOLVING LAB II — 117, 119, 142.

SUGGESTIONS

Initial Activity Using some bold newspaper advertisements discuss the statements, e.g., "Save up to 50%."

The key words are "up to". Only one item need be half price and this ad is true.

List reasons why stores would have sales.

Point out that discount rates are a percent. A discount is an amount of money to be deducted from the regular price to determine the sale price.

USING THE BOOK

Define discounts and discount rates.

Calculate the amount of discount first. Subtract this amount from the regular price to determine the sale price.

Assign the exercises. Remind students to make two statements for each:

1. (a) The discount is _____.

1. (b) The sale price is _____.

OBJECTIVE

To review addition and subtraction of decimals

PACING

Level A All
Level B All
Level C All

RELATED AIDS

BFA PROB. SOLVING LAB II — 109, 110.

SUGGESTIONS

Initial Activity Discuss "making change". Investigate why so many errors are made. (Usually because the amount owed is in dollars and cents, but the amount of money tendered is \$10.00 or \$80.00. There is subtraction with a lot of regrouping.)

Review subtraction with regrouping using these examples.

(a) $\begin{array}{r} \$1.00 \\ - 0.59 \\ \hline \$0.41 \end{array}$ (b) $\begin{array}{r} \$10.00 \\ - 7.28 \\ \hline \$2.72 \end{array}$ (c) $\begin{array}{r} \$30.00 \\ - 26.09 \\ \hline \$3.91 \end{array}$

USING THE BOOK

Discuss how the question in the display at the top of the page is done.

State how the change could be returned to the customer.

Example

\$68.39 → \$68.40 → \$68.50 → \$68.75 → \$69.00

START → 1¢ → 10¢ → 25¢ → 25¢
→ \$70.00 → \$80.00
→ \$1.00 → \$10.00

This is based on counting from the amount owed to the amount tendered.

In Exercise 4, explain that the customer could have given \$40.00 but for some reason gave the cashier \$40.54.

In Exercises 5 and 8 there was a good reason why the customer gave the cashier the additional 9 cents and 2 cents respectively. Ask the children to see if they can find out why as they work on the exercises.

ACTIVITIES

1. "Coins and Bills".

Using toy money count out the coins and bills that would be returned as change for any 5 transactions.

Record as in this example.

Exercise 2:

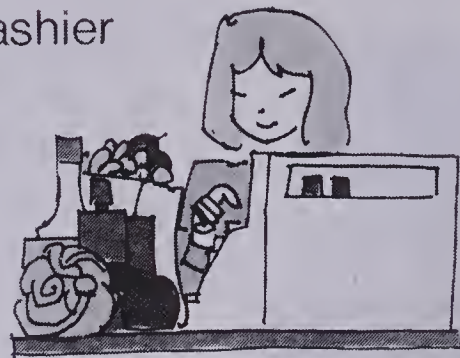
\$11.98 → \$12.00 → \$13.00 → \$15.00

START → 1¢ → 1¢ → \$1.00 → \$2.00

The Cashier

Angela Kapsalis is a cashier in a grocery store. She totals purchases, collects money, and returns the proper change to the customer.

Cash collected → \$80.00
Grocery bill → - 68.39
Change → \$11.61



Exercises

Calculate the change.

1. $\begin{array}{r} \$1.00 \\ - 0.79 \\ \hline \$0.21 \end{array}$	2. $\begin{array}{r} \$20.00 \\ - 11.98 \\ \hline \$8.02 \end{array}$	3. $\begin{array}{r} \$20.75 \\ - 13.75 \\ \hline \$7.00 \end{array}$	4. $\begin{array}{r} \$40.54 \\ - 13.75 \\ \hline \$26.79 \end{array}$	5. $\begin{array}{r} \$60.09 \\ - 42.84 \\ \hline \$17.25 \end{array}$
6. $\begin{array}{r} \$100.00 \\ - 86.75 \\ \hline \$13.25 \end{array}$	7. $\begin{array}{r} \$80.00 \\ - 72.20 \\ \hline \$7.80 \end{array}$	8. $\begin{array}{r} \$40.02 \\ - 32.52 \\ \hline \$7.50 \end{array}$	9. $\begin{array}{r} \$35.00 \\ - 34.71 \\ \hline \$0.29 \end{array}$	10. $\begin{array}{r} \$27.00 \\ - 26.07 \\ \hline \$0.93 \end{array}$

11. Mrs. Jakes buys meats for \$13.23, dairy products for \$4.39, vegetables for \$7.30, canned goods for \$16.83, and household supplies for \$9.82.

What change would she receive if she gave the cashier three \$20 bills? **\$8.43**

12. Lee picked up 6 L of milk and two loaves of bread on the way home from school. The milk cost \$0.62/L and the bread cost \$0.97 a loaf.

What change should he receive if he gives the cashier a \$10 bill? **\$4.34**

13. Brian purchased vegetables for \$6.30, dairy products for \$5.89, and frozen foods for \$9.63. He gave two \$20 bills to the cashier.

What coins and bills might he receive in change? **\$18.18 change — 1 \$10 bill, 1 \$5 bill, 1 \$2 bill, 1 \$1 bill, 1 dime, 1 nickel, 3 pennies. Other answers are possible.**

→ \$20.00

→ \$5.00

2. Use "The P.V. Game" as described in the Activity Reservoir to practise the computations in making change. Use blanks such as:

_____.
_____.

The player with the most change when all digits have been placed wins.

3. Have the students cut out an item from a department store catalogue. Have them compute the (a) sales tax, (b) total price, (c) change from \$50.00. Display some of the best on a bulletin board.

Choosing Equations



Two classes sold 180 spring-fair tickets. One class sold 105 tickets. How many did the other class sell?

Select the correct equation for the problem.

$$180 + 105 = n \quad 180 - 105 = n \quad 180 \times 105 = n \quad 180 \div 105 = n$$

Choose this operation.

$$180 - 105 = n$$

$$75 = n$$

The other class sold 75 tickets.

Check for reasonableness:
 Tickets of one class + tickets of other class = 180
 105 + 75 = 180
 Checks!

Exercises

Select the correct equations, solve, and check for reasonableness.

1. Susan has \$0.35 and Jennifer has \$0.95. How much do they have together?

$$\textcircled{\$0.35 + \$0.95 = n \quad \$1.30}$$

$$\$0.35 \times \$0.95 = n$$

$$\$0.35 - \$0.95 = n$$

$$\$0.35 \div \$0.95 = n$$

2. Jackie purchased school supplies for \$4.65. What change would she get from a \$10 bill?

$$\textcircled{\$10.00 + \$4.65 = n}$$

$$\$10.00 \times \$4.65 = n$$

$$\textcircled{\$10.00 - \$4.65 = n \quad \$5.35}$$

$$\$10.00 \div \$4.65 = n$$

3. \$300 was collected from the sale of tickets. Each ticket cost \$1.50. How many tickets were sold?

$$\$300 + \$1.50 = n$$

$$\$300 \times \$1.50 = n$$

$$\$300 - \$1.50 = n$$

$$\textcircled{\$300 \div \$1.50 = n \quad 200}$$

4. Cindy and Roger are an ice-skating team. They train 5 d a week for a total of 20 h. How long do they train each day?

$$20 + 5 = n$$

$$20 \times 5 = n$$

$$20 - 5 = n$$

$$\textcircled{20 \div 5 = n \quad 4h}$$

Word problems: choosing the appropriate equation 253

OBJECTIVE

To select the appropriate equation to solve problems

PACING

Level A All
 Level B All
 Level C All

BACKGROUND

This page is really an introduction to using equations for problem solving (the work on pages 254 and 255). It also recaps equations and word problems as presented on pages 86 and 136.

USING THE BOOK

Discuss the problem and the four equations listed. Most students can see that subtraction is required. Explain that “ n ” represents the amount that the “other class” sold. Solve the equation by simplifying. Make a final statement. Check for “reasonableness”.

Assign the problems. Allow 10 min for selections and solutions only. Have each student check for reasonableness.

Discuss each problem with the class to answer any queries.

Have students record their final statements for each problem.

ACTIVITIES

- See Activity 1 on page 22.
- Have the students (a) copy any word problem up to this point in the text, (b) write an equation to solve it, (c) write 3 or 4 more “camouflage” equations, (d) copy all of this onto an index card in similar fashion to the exercises on this page, (e) exchange their cards with classmates or contribute them to a problem file or box.
- If you have not already done so, see the closely related activities suggested on pages 19, 86, 136, and 207.

OBJECTIVE

To solve equations

PACING

Level A All
Level B All
Level C All

BACKGROUND

There is no attempt to introduce formal solutions to equations at this time. Occasionally inverse statements could be used (Related Sentences). Most equations can be solved by *inspection*.

SUGGESTIONS

Initial Activity Let's investigate these true number sentences.

Number Sentence	Related True Sentences
-----------------	------------------------

- (a) $6 + 8 = 14 \rightarrow 14 - 8 = 6$ or $14 - 6 = 8$
 (b) $25 - 10 = 15 \rightarrow 15 + 10 = 25$
 (c) $3 \times 4 = 12 \rightarrow 12 \div 4 = 3$ or $12 \div 3 = 4$
 (d) $20 \div 4 = 5 \rightarrow 5 \times 4 = 20$

Point out that these related sentences can be useful when solving equations.

USING THE BOOK

All of the examples in the display at the top of the page can be solved using *inspection*.

Example

$$n + 95 = 120 \quad \text{What?} + 95 = 120$$

$$n = 25$$

or

by using a related sentence

$$n + 95 = 120$$

$$120 - 95 = n$$

$$25 = n$$

Discuss the use of the true related statements in the display.

Assign the exercises. Use *inspection* wherever possible. Use related statements when necessary.

ACTIVITIES

1. Choose an equation from page 254. Write a problem which fits that equation.

2. "Problem Draw".

Write the following on small pieces of paper.

25	50	75	100	200
300	400	500	1000	

Solving Equations

$$\begin{array}{r} 20 + 60 = n \\ 80 = n \end{array}$$

$$\begin{array}{r} n + 95 = 120 \\ 120 - 95 = n \\ 25 = n \end{array}$$

$$\begin{array}{r} n \times 3 = 21 \\ 21 \div 3 = n \\ 7 = n \end{array}$$

$$\begin{array}{r} 20 \\ + 60 \\ \hline 80 \end{array}$$

$$\begin{array}{r} 120 \\ - 95 \\ \hline 25 \end{array}$$

$$\begin{array}{r} 7 \overline{) 21} \end{array}$$

$$\begin{array}{r} 0.75 - 0.35 = n \\ 0.40 = n \end{array}$$

$$\begin{array}{r} n - 80 = 100 \\ 100 + 80 = n \\ 180 = n \end{array}$$

$$\begin{array}{r} n \div 3 = 20 \\ 20 \times 3 = n \\ 60 = n \end{array}$$

Write related sentences.

Exercises
Solve.

1. $12 + 8 = n$
 $20 = n$

2. $50 - 20 = n$
 $30 = n$

3. $n + 20 = 50$
 $50 - 20 = n$
 $? = n$

4. $n - 10 = 60$
 $70 = n$

5. $n + 25 = 100$
 $75 = n$

6. $n - 100 = 5$
 $105 = n$

7. $n + 83 = 142$
 $59 = n$

8. $n + 73 = 195$
 $122 = n$

9. $n - 452 = 82$
 $534 = n$

10. $n + n = 20$
 $10 = n$

11. $n \times 8 = 72$
 $9 = n$

12. $n \div 5 = 8$
 $40 = n$

13. $n \times 12 = 60$
 $5 = n$

14. $n - 3.9 = 12.1$
 $16 = n$

15. $n \times 9 = 108$
 $12 = n$

16. $n \div 2 = 23$
 $46 = n$

17. $n \times 20 = 240$
 $12 = n$

18. $n - 2.7 = 9.2$
 $11.9 = n$

19. $n \div 15 = 20$
 $300 = n$

20. $n + 5.6 = 9.9$
 $4.3 = n$

21. $n \times 0.5 = 2.5$
 $5 = n$

22. $n \times 1.5 = 13.5$
 $9 = n$

Write equations and solve.

28. Sara has saved \$22.50.
New skates cost \$45.90.
How much more money must Sara save in order to buy the skates? **\$23.40**

29. Russell saved \$3.50 each month for a year. He has enough money to buy new roller skates.
What do the roller skates cost? **\$42.00**

The Derby

In a soapbox derby, Elliot's time was 27.2 s.
Lauren's time was 1.9 s longer.
How long was Lauren's run?

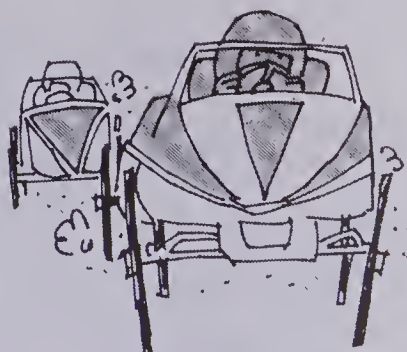
Think:

Lauren's time = Elliot's time + extra time

Write an equation. $n = 27.2 + 1.9$

Solve. $n = 29.1$

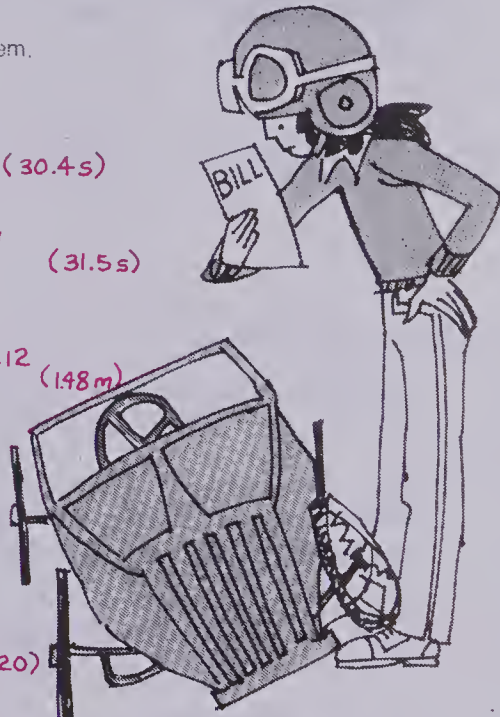
Write a sentence. Lauren's run was 29.1 s.



Exercises

Write an equation for each problem. Solve the problem.

- Betty's time for the race was 29.9 s.
Zella's time was 0.5 s longer.
How long was Zella's run? $n = 29.9 + 0.5$
 $n = 30.4$ (30.4 s)
- Helen's race time was 30.6 s.
Jackie's time was 0.9 s longer.
How long was Jackie's run? $n = 30.6 + 0.9$
 $n = 31.5$ (31.5 s)
- Bruno's car is 1.60 m long.
Kathleen's car is 0.12 m shorter.
How long is Kathleen's car? $n = 1.60 - 0.12$
 $n = 1.48$ (1.48 m)
- Fred's car is 1.8 m long.
Irvine's car is 0.01 m shorter.
How long is Irvine's car? $n = 1.8 - 0.01$
 $n = 1.79$ (1.79 m)
- Bea spent \$2.40 for repairs to her car.
Irene spent 3 times as much to repair her car.
How much did Irene spend? $n = 3 \times 2.40$
 $n = 7.20$ (\$7.20)
- Jim spent 2.5 h painting his car.
Gladys took twice as long to paint her car.
How much time did Gladys spend painting? $n = 2 \times 2.5$
 $n = 5.0$ (5 h)



Solving problems using equations 255

OBJECTIVE

To solve problems using equations

PACING

Level A All

Level B All

Level C All

RELATED AIDS

HMS — DM64.

BACKGROUND

All of the problems on this page are one-step problems.

The recording suggested in the display includes:

the equation,
the solution, and
a final statement.

The authors feel that this is the *minimal* amount of recording. Some groups or individuals may require the guidance offered by a formal problem-solving sequence (see page 22).

SUGGESTIONS

Initial Activity Review the solving of equations using the following examples:

- (a) $n = 17.3 + 9.1$ (b) $3.75 \times 2 = n$
(c) $27.9 - 2.3 = n$ (d) $17.5 \div 2 = n$

USING THE BOOK

Read the problem in the display at the top of the page to the class. Record the necessary information on the chalkboard. Decide what operation will lead to the answer. Write the equation. Solve. Write a sentence. Check the answer for reasonableness.

Assign the exercises.

ACTIVITIES

1. "Soap Box Racer".

Design a simple wooden "soap box" racer. List all the component parts required, e.g., 4 wheels, 2 axles, 5 m wood. Using catalogues, determine the cost of these parts. Calculate the total cost.

2. If you have not already done so, see the Activities suggested on pages 22 and 86.

3. See "Itza Fact!" as described in the Activity Reservoir for a challenging activity designed to

reinforce the recall of related number facts.

EXTRA PRACTICE

- Wheels for Sharon's car cost \$8.95 each and the axles cost \$1.80. How much did she pay for 4 wheels and two axles?
- Greg and his father spent 16 d building his soap box racer. They worked a total of seventy-two hours on the project. What was the average amount of time spent per day?

OBJECTIVE
To provide practice involving
problem-solving techniques

PACING
Level A 1-6
Level B 1-7
Level C All

SUGGESTIONS
Initial Activity Review “Professor Q’s” 4 questions before beginning any problem solutions.
(a) Main Idea— Read for the context.
(b) Question — What do we have to find?
(c) Facts — What do we have to work with?
(d) Operation— +, −, ×, or ÷ or combinations of these?

USING THE BOOK
Discuss the “zoo fund” problem with the class to illustrate:
(a) Professor Q’s 4 questions and
(b) The Four Steps— questions, equation, solve, statement.
Write a minimum solution (model) on the chalkboard for the students.
Example

\$477 collected by 9 classes.
 $477 \div 9 = n$
 $53 = n$

$$\begin{array}{r} 53 \\ 9 \overline{)477} \\ \underline{45} \\ 27 \\ \underline{27} \\ 0 \end{array}$$

Each class collected an
average of \$53 for the zoo fund.

Assign the exercises.
Remind the students to read to answer Professor Q’s 4 questions before they begin the solution. If they don’t have an answer to these 4 questions, have them read the problem carefully again. Also remind them to be on guard for 2-step problems and hidden information.

Professor Q Visits Glendale School

Professor Q noted that the 9 classes at Glendale School collected \$477 for the zoo fund. What was the average amount collected by each class?

To find the solution, use Professor Q’s four steps.

- Step 1 Find:

(a) main idea

(b) the question

(c) the facts

(d) an operation

Collecting money for zoo fund

What was the average amount collected by each class?

\$477 collected by 9 classes

division

Step 2 Write an equation. $477 \div 9 = n$

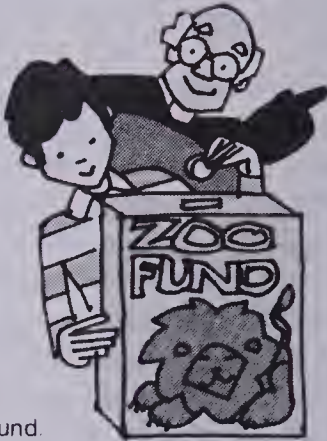
Step 3 Solve the equation.

$$\begin{array}{r} 53 \\ 9 \overline{)477} \\ \underline{45} \\ 27 \\ \underline{27} \\ 0 \end{array}$$

$477 \div 9 = 53$

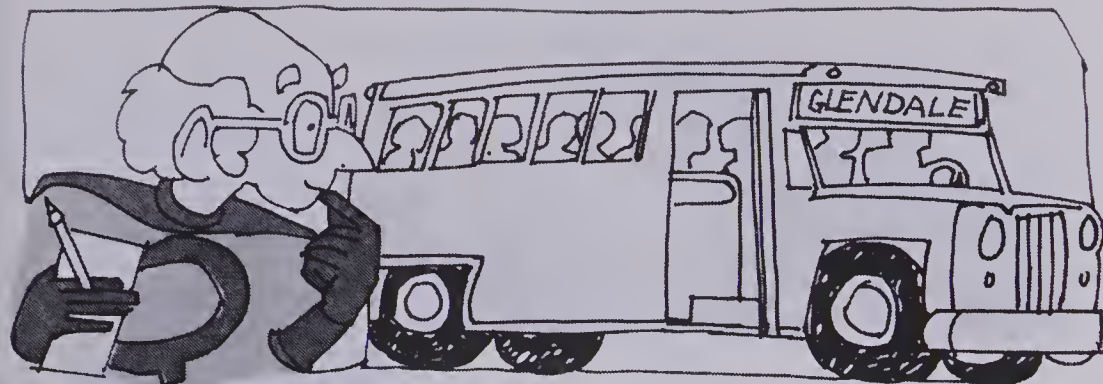
Step 4 Write a statement.

Each class collected an average of \$53 for the zoo fund.



Exercises

- The library has 104 full shelves of books, and each shelf holds 36 books. How many library books are in the Glendale library? 3744
- The storeroom in the gymnasium contains 96 balls. A quarter of the balls are basketballs. How many basketballs are in the storeroom? 24
- The school photographer spent 3 h taking class pictures of the 9 classes of students. How many minutes did he spend photographing each class? 20 min



4. The Glendale School baseball team won 16 games and lost 4 games to win the county championship.
What percent of all the games did the team win? **80%**
5. The girls' volleyball team played 10 games and won 70% of them to finish second in the county.
How many games did they win? **7**
How many games did they lose? **3**
6. There are 280 students at Glendale School, and the average attendance is 95%.
On an average day, how many students are absent? **14**
7. There are 153 students in the senior classes at Glendale School, and each student requires 12 notebooks per year.
If the principal orders 2000 notebooks, will there be enough to last for the year for the senior classes? **Yes**
- ★ 8. Glendale School has 38 students in the senior choir and 32 students in the junior choir.
There are 280 students in the school.
What percent of the students are members of choirs? **25%**
- ★ 9. How many buses are required to take the 280 students on field trips? **6**
A bus holds exactly 52 people.
How many seats are available for teachers and parents? **32**

ACTIVITIES

1. "More Glendale School Problems".
Write 2 problems about the activities of Glendale School.

Write complete solutions to these problems in your notebook.
Exchange problems with a friend and solve each other's problems.
Check your work.

2. File all the problems in the "Glendale School" problem box.
Each student may solve 4 problems during the rest of the week.
Problems should be marked by the creator of the problem.

OBJECTIVE

To solve multi-step problems

PACING

- Level A Page 258 — 1-3
 Page 259 — 1, 3, 4
 Level B Page 258 — All
 Page 259 — 1, 3, 4, 6
 Level C Page 258 — All
 Page 259 — 1, 3-6

BACKGROUND

The methods suggested are to solve 2, 3, or 4 mini-problems rather than designing one equation to solve the whole problem.

This is a reasonable approach based on the average problem-solving maturity for Grade 6 students.

SUGGESTIONS

Initial Activity Review Professor Q's 4 questions (see page 256).

In multi-step problems there may be many questions. For each of these questions there are facts and an operation required. Review problems with insufficient information (see pages 152 and 165). Record the main question and the other supporting questions when doing the solutions.

USING THE BOOK

Here is Professor Q's analysis of Hans' stereo system problem.

Main Idea — Purchasing a stereo system

Question — What was the total cost?

What was the cost of 4 speakers?	What was the cost of 8 m of wire?	Cost of set is \$399.
Facts — 4 speakers at \$140 each	8 m of wire at \$0.49 a metre.	\$399.00
Operations — $n = 4 \times 140$	$w = 8 \times 0.49$	

Once this analysis is discussed, then tie the various operations to the small sub-problems within the total problem. Eventually all the amounts are added to determine the total cost. A *minimum* record of the solution is shown in the display at the top of the page.

Provide lots of scrap paper so that students can investigate the problems and experiment with various approaches.

Hans' Stereo System

Hans decided to put a stereo system in his room

He bought a stereo set, four speakers, and 8 m of speaker wire

The stereo set cost \$399. Each speaker cost \$140, and a metre of wire cost \$0.49

How much did Hans spend in all?

Step 1 Cost of speakers: $4 \times \$140.00 = \560.00

Step 2 Cost of 8 m of wire: $8 \times \$0.49 = \3.92

Step 3 Cost of all items:

\$560.00

3.92

+ 399.00

\$962.92

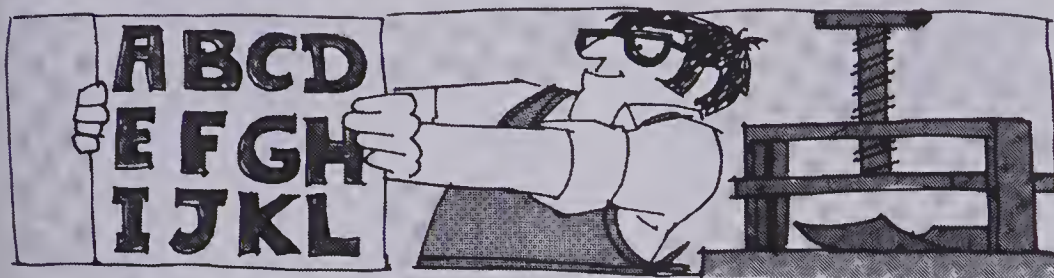
The total amount Hans spent was \$962.92



Exercises

- Hans needed 24 connectors, 1 tape deck, and 6 cassettes.
 The connectors cost \$0.40 each. The tape deck cost \$156.00.
 The cassettes cost \$4.25 each. How much did all the items cost?
 Step 1 Cost of connectors **\$9.60**
 Step 2 Cost of 6 cassettes **\$25.50**
 Step 3 Total cost of connectors, cassettes, and tape deck **\$191.10**
- Hans' brother bought Hans 3 cassettes and 2 record albums.
 The cassettes cost \$3.95 each. The record albums cost \$7.50 each.
 How much did Hans' brother spend altogether? **\$26.85**
- Hans' mother bought 10 records. They were 2 for \$12.00
 She also bought a cassette for \$5.60. How much did she spend altogether?
 Step 1 Number of sets of 2 she bought (■) **5**
 Step 2 Cost of ■⁵ sets of records **\$60.00**
 Step 3 Cost of records and cassette **\$65.60**
- Hans bought 4 albums at \$10.19 each and 1 cassette at \$6.19. He gave the clerk \$60.
 How much change did he get back? **\$13.05**

Typesetter



- Mr. Neilson set 21 pages of type in 3 h.
He worked at a steady rate.
How many pages did he set in 1 h? **7**
- Ms. Carlos had 17 pages to set for one book and 46 pages to set for another book.
She has done 19 pages.
How many more pages does she still have to set? **44**
- Mrs. O'Hare has worked 2 h overtime 3 times this week and 3 h overtime 4 times last week.
How many overtime hours has she worked in two weeks? **18h**
- Mr. Elliott has 190 pages to proofread in 5 h.
How many pages each hour must he check in order to finish on time? **38**
- Mr. Elliott found 5 errors on each of 8 pages, 4 errors on each of 6 pages, and 2 errors on one page.
What was the total number of errors? **66**
- There were 43 lines of type to each page of a paperback book.
Each line contained an average of 8 words.
Approximately how many words on:
(a) 18 pages? **6192** (b) 30 pages? **10320** (c) $21\frac{1}{2}$ pages? **7396**

Problems: multi-step 259

Discuss problem 2, and record the analysis and solution on the board.

Example

Main Idea — Typesetting pages

Questions — (a) Total to be set
(b) How many pages remain to be set?

Facts — (a) 17 pages and 46 pages
(b) 19 pages already done

Operation — (a) Addition
(b) Subtraction

Possible Solution:

Step 1 Number of pages to be set:

$$17 + 46 = n$$

$$63 = n$$

Step 2 Number of pages she still has to set:

$$63 - 19 = x$$

$$44 = x$$

Ms. Carlos has to set 44 more pages.

Assign the problems on pages 258 and 259. Be available for clarification of the students' approaches.

ACTIVITIES

1. To review and maintain "choosing-the-correct-operation" procedures in problem-solving situations, see Activity 1 on page 22.

2. See "Number Sentence" as described in the Activity Reservoir.

3. "Hobbies".

Have each student list some of his/her hobbies, and other hobbies on a worksheet.

My Hobbies	Other Hobbies

Design 2 *multi-step* problems about your hobbies or other people's hobbies. Write full solutions in your notebook. Share your problems with classmates.

EXTRA PRACTICE

Solve 2 or 3 problems designed by your classmates.

OBJECTIVE

To solve problems involving time, speed, and distance

PACING

Level A 1; 2; 3(a), (b); 4(a)-(d); 5
Level B All
Level C All

SUGGESTIONS

Initial Activity Discuss the concept of speed as a distance covered in a given time. Thus speed is a ratio of:

$$\frac{\text{Distance}}{\text{Time}}$$

Using this ratio we can solve time, speed, distance problems.

USING THE BOOK

Review the solving of equations involving ratios, using (a) cross products, (b) equivalent fractions.

Example

(a) $\frac{7}{5} = \frac{\blacksquare}{20}$

Cross multiply $5 \times \blacksquare = 7 \times 20$

Simplify $5 \times \blacksquare = 140$

Related division $140 \div 5 = \blacksquare$

Solution $28 = \blacksquare$

or

(b) $\frac{7}{5} = \frac{\blacksquare}{20}$

Using equivalent fractions:

$$\frac{7 \times 4}{5 \times 4} = \frac{\blacksquare}{20}$$

$$\blacksquare = 28$$

Discuss the problem in the display. You might want to show 2 solutions and the diagram for clarification:

A $\frac{\text{Distance}}{\text{Time}} \longrightarrow \frac{15}{1} = \frac{45}{\blacksquare}$

Using cross products $15 \times \blacksquare = 1 \times 45$

Simplify $15 \times \blacksquare = 45$

Solve $\blacksquare = 3$

It will take 3 h for Tom and Cathy to cycle to the conservation area.

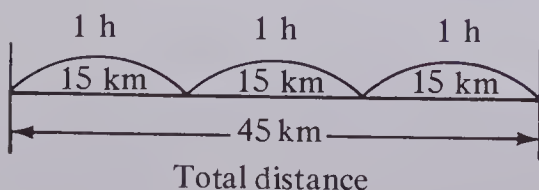
B $\frac{\text{Distance}}{\text{Time}} \longrightarrow \frac{15}{1} = \frac{45}{\blacksquare}$

Using equivalent fractions $\frac{15 \times 3}{1 \times 3} = \frac{45}{\blacksquare}$

Solve $\frac{45}{3} = \frac{45}{\blacksquare}$

$$\blacksquare = 3$$

It will take 3 h for the trip.



Bike Hikes

Tom and Cathy can travel at 15 km/h on their bicycles.
How long would it take to travel 45 km to the conservation area?

km/h
means
kilometres
per hour

Set up equivalent ratios.

$$\frac{15}{1} = \frac{45}{\blacksquare}$$

Cross products

$$15 \times \blacksquare = 1 \times 45$$

$$\blacksquare \times 15 = 45$$

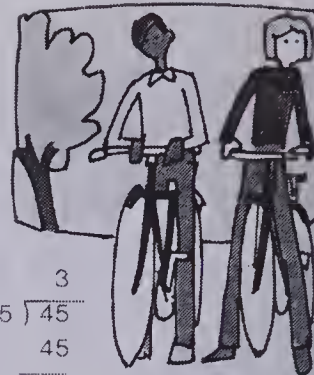
Related division sentence

$$45 \div 15 = \blacksquare$$

$$3 = \blacksquare$$

$$\begin{array}{r} 3 \\ 15 \overline{)45} \\ \underline{45} \\ 0 \end{array}$$

It would take 3 h.



Exercises

- At 15 km/h, how long would it take Tom and Cathy to cycle to Tom's cottage, which is 30 km from town? **2 h**
- At 15 km/h, how many minutes would it take to travel to Cathy's cousin's home, which is 6 km away? **24 min**
- Tom's father can average 80 km/h travelling by car.
How long would it take to drive (round to 1 decimal place):
(a) 400 km? **5.0 h** (b) 200 km? **2.5 h**
(c) 120 km? **1.5 h** (d) 100 km? **1.3 h**
- At an average speed of 14 km/h, how far could Tom and Cathy cycle in:
(a) 2 h? **28 km** (b) 5 h? **70 km**
(c) 3.5 h? **49 km** (d) 2.5 h? **35 km**
(e) 4 h and 30 min? **63 km** (f) 15 min? **3.5 km**
- In a recent "bike-a-thon" to raise money for charity, Tom completed the 30 km course in 1.5 h.
What was his speed in kilometres per hour? **20 km/h**
- Cathy completed the same 30 km course in 1 h 15 min.
What was her speed in kilometres per hour? **24 km/h**

260 Time, speed, distance problems

Remind students to write the

$\frac{\text{Distance}}{\text{Time}}$ ratio for each problem.

Assign the problems.

You may wish to mention that

- (a) All speeds are in kilometres per hour.
(b) 30 min = 0.5 h.
(c) 15 min = 0.25 h.

ACTIVITIES

- Have the students complete rate charts such as:

Distance (km)	30	60	90	120	
Time	__min	__min	1 h		

- Provide a metric distance chart of the sort shown on provincial road maps. Supplying an average speed such as 70 km/h, have the students compute average travelling times between various locations on the map.

- Have the students use the library to research some of our solar system distances (e.g., Earth is 149 731 000 km from the sun, Mercury is about 57 960 000 km from the sun, Jupiter is about 779 240 000 km from Earth, etc.).

The speed of light is about 299 274 km. Have the students compute and report on the amount of time it would take a beam of light to travel between various points.

Running

A cheetah can run at 110 km/h over short distances.
At this rate, how long would it take a cheetah to run 11 km?

Set up equivalent ratios. $\frac{\text{Distance}}{\text{Time}} \rightarrow \frac{110}{60} = \frac{11}{\blacksquare}$

Cross products

$$110 \times \blacksquare = 60 \times 11$$

$$110 \times \blacksquare = 660$$

Related division sentence

$$\blacksquare = 660 \div 110$$

$$\blacksquare = 6$$

The cheetah would take 6 min to run 11 km.

Remember.
1 h = 60 min



Exercises

- Squirrels can travel at 18 km/h.
At this rate, how long (in minutes) would it take to travel 9 km? **30 min**
- Grizzly bears can travel at 48 km/h.
At this rate, how long (in minutes) would a grizzly bear take to run:
(a) 16 km? **20 min** (b) 24 km? **30 min** (c) 4 km? **5 min**
- Lions can travel at 80 km/h.
At this rate, how long (in minutes) would a lion take to run:
(a) 40 km? **30 min** (b) 20 km? **15 min** (c) 16 km? **12 min**
- Elephants can travel at 40 km/h.
At this rate, how long would an elephant take to run:
(a) 80 km? **2 h** (b) 10 km? **15 min** (c) 2 km? **3 min**
- A good marathon runner can travel at 16 km/h.
At this rate, how long would it take to run:
(a) 32 km? **2 h** (b) 40 km? **2.5 h** (c) 24 km? **1.5 h** (d) 8 km? **0.5 h** or **30 min** (e) 4 km? **15 min**
- ★ A garden snail can move at a rate of 0.05 km/h.
At this rate, how many hours would it take a snail to cover one kilometre? **20 h**



Time speed, distance problems 261

OBJECTIVE

To determine the time required to travel a given distance

PACING

Level A 1; 2(a), (b); 3(a), (b); 4(a), (b)

Level B 2-5

Level C 2-6

VOCABULARY

cheetah, squirrel, grizzly bear, marathon runner

BACKGROUND

These speeds are based on maximum speeds that animals can travel over relatively short distances (i.e., 100 m). Thus the problems on this page assume that the animals display incredible endurance. Make this clear to the students.

SUGGESTIONS

Initial Activity Discuss speeds of animals. Which are fast animals? Which are slow? Which can run faster than man? Slower than man? The cheetah is the fastest running animal.

Discuss units of time.

$$1 \text{ h} = 60 \text{ min}$$

$$0.5 \text{ h} = 30 \text{ min}$$

$$0.25 \text{ h} = 15 \text{ min}$$

$$0.2 \text{ h} = 12 \text{ min}$$

$$0.1 \text{ h} = 6 \text{ min}$$

For the problems on this page, we want to use the same ratio but express time in minutes, i.e., 1 h = 60 min.

$$\frac{\text{Distance}}{\text{Time}}$$

USING THE BOOK

Discuss the problem in the display at the top of the page. Show 2 possible solutions.

$$\frac{\text{Distance}}{\text{Time (min)}} \rightarrow \frac{110}{60} = \frac{11}{\blacksquare}$$

Cross products $110 \times \blacksquare = 60 \times 11$

Simplify $110 \times \blacksquare = 660$

Related division $\blacksquare = 660 \div 110$

Solution $\blacksquare = 6$

Statement The cheetah runs 11 km in 6 min.

$$110 \div 10 = 11$$

$$\frac{\text{Distance}}{\text{Time}} \rightarrow \frac{110}{60} = \frac{11}{\blacksquare}$$

Reduced equivalent fractions $\frac{110 \div 10}{60 \div 10} = \frac{11}{\blacksquare}$

Simplify $\frac{11}{6} = \frac{11}{\blacksquare}$

Solution $6 = \blacksquare$

The cheetah runs 11 km in 6 min.

ACTIVITIES

- Measure a 1 km course in the playground.

How long does it take you to run that 1 km course (to nearest 0.5 min)?

Determine your running speed at this rate in kilometres per hour.

$$\frac{\text{Distance}}{\text{Time}} \rightarrow \frac{1}{(\text{your time})} = \frac{\blacksquare}{60}$$

- "Running Two Kilometres?" Referring to speeds on page 261, calculate how long (in minutes) it would take each animal to run 2 km. Round each answer to the nearest tenth of a minute. (squirrel, bear, lion, elephant, marathoner, you)

- If you have not already done so, see Activity 2 on page 260.

OBJECTIVE

To determine speeds given distance and time

PACING

Level A 1-7
Level B 1-8
Level C 1-9

RELATED AIDS

HMS — DM65.

SUGGESTIONS

Initial Activity Review some products of 60.

Example

$$\begin{aligned} 60 &= 30 \times 2 \\ 60 &= 20 \times 3 \\ 60 &= 15 \times 4 \\ 60 &= 12 \times 5 \\ 60 &= 10 \times 6 \end{aligned}$$

These products might prove to be useful when solving the speed problems on this page.

USING THE BOOK

Discuss the sprinter problem, clarifying why our new ratio has "60" at the bottom. Show both the cross product and the equivalent fraction solutions.

$$\begin{array}{c} \text{Distance} \\ \text{Time} \end{array} \longrightarrow \frac{15}{20} = \frac{\blacksquare}{60}$$

$$20 \times 3 = 60$$

Equivalent fractions

$$\frac{15 \times 3}{20 \times 3} = \frac{\blacksquare}{60}$$

$$\frac{45}{60} = \frac{\blacksquare}{60}$$

Solve $45 = \blacksquare$
The sprinter's speed is 45 km/h.
Assign the problems.

ACTIVITIES

1. "Animal Speed Chart".

Using the speeds stated on page 261 and the speeds calculated on page 262, arrange the animals in order from fastest to slowest on the chart.

Speed Chart

	Name	Speed in km/h
Fastest	Cheetah	
Slowest		

Still Running

A sprinter can run at a speed equivalent to 15 km in 20 min over short distances. What is the sprinter's speed in kilometres per hour?

Set up equivalent ratios for

$$\frac{\text{Distance}}{\text{Time}} \longrightarrow \frac{15}{20} = \frac{\blacksquare}{60}$$

Cross products

$$20 \times \blacksquare = 15 \times 60$$

$$20 \times \blacksquare = 900$$

Related division sentence

$$\blacksquare = 900 \div 20$$

$$\blacksquare = 45$$



$$\frac{15}{20} = \frac{45}{60}$$



The sprinter's speed is 45 km/h.

Exercises

(The data in these problems are based on speeds reached in runs less than 0.5 km.)

Calculate the speed of each animal in kilometres per hour.

1. A quarter horse can run 5 km in 15 min. *20 km/h*
2. A zebra can travel 16 km in 15 min. *64 km/h*
3. A wild turkey can run 2 km in 5 min. *24 km/h*
4. A pig can cover 4 km in 15 min. *16 km/h*
5. A wildebeest can cover 4 km in 3 min. *80 km/h*
6. An elephant can run 20 km in 30 min. *40 km/h*
7. A cat can run 8 km in 10 min. *48 km/h*
- ★ 8. A giant tortoise can cover 1 km in 4 h. *0.25 km/h*
- ★ 9. A three-toed sloth can cover 1 km in 6 h. *0.17 km/h*



Include these animals on your chart: cheetah, squirrel, bear, lion, elephant, marathon runner, quarter horse, zebra, wild turkey, pig, wildebeest, cat, giant tortoise, snail, and three-toed sloth.

2. "Animal Research".

Select an animal from the above list in Activity 1 and do some research in the library. Write down all the *new* (i.e., new to you) information you found about that animal. Share this information.

Predictions

- Vern and Joyce counted the number of vehicles passing in front of their school in 45 min. This is their graph.

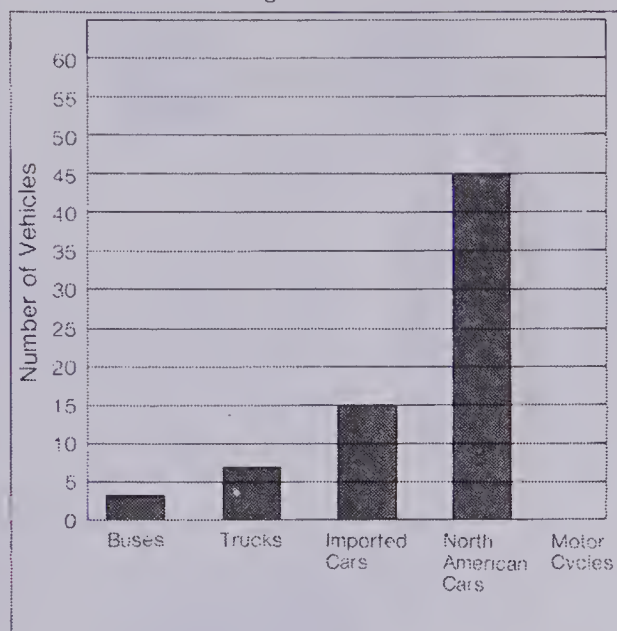
(a) What information does this graph show?

(b) If you went to the front of the school, which type of vehicle would probably pass by first? Explain why you think so. *A North American car.*

(c) What season was it when Vern and Joyce did their survey? Explain why you think this.

(d) Describe the street in front of the school.

Vehicles Passing in Front of the School



Type of Vehicle

- The city hockey league kept statistics on its teams. A team earns 2 points for a win and 1 point for a tie.

County Hockey League

	Wins	Ties	Losses	Points
Trojans	17	2	11	36
Warriors	12	0	18	24
Bears	15	1	14	31
Saints	21	3	6	45
Lions	10	2	18	22
Hurricanes	8	6	16	22

(a) Copy this chart in your notebook and calculate each team's points. *see chart.*

(b) Make a bar graph of your results.

(c) Rank the teams. *see chart.*

(d) If these teams played, predict the winner.

(i) the Warriors and the Saints

(ii) the Bears and the Trojans

(iii) the Lions and the Hurricanes

Explain each prediction.

Predicting outcomes 263

OBJECTIVE

To make predictions based on data

PACING

Level A All

Level B All

Level C All

MATERIALS

centimetre graph paper and metric rulers

BACKGROUND

The next 5 pages of exercises are about predictions, trends, and elementary probability theory.

It is not the intent that students have a deep understanding, but rather look at some situations and get a flavour of predictions.

SUGGESTIONS

Initial Activity Discuss bar graphs. Why are graphs used? (visual, easy to compare, etc.)

USING THE BOOK

Using the graph entitled "Vehicles Passing in Front of the School", discuss the information displayed.

Note the characteristics of a clear bar graph.

- title
- label for each bar
- scale: number of vehicles
- equal width bars

Discuss the questions in Exercise 1 with the class after they have spent 5 min formulating some possible answers. (Note: The questions in Exercise 1(b), (c), and (d) are open ended, but some types of answers are more credible than others.)

Discuss the point system for "hockey statistics" and assign Exercise 2.

Discuss the answers for Exercise 2(c) and (d) with the class.

Note: Accept all reasonable explanations for predictions.

ACTIVITIES

"Traffic Graphs".

Have students work in teams of 2 or 3.

Count the number of types of vehicles which pass a certain spot during a half-hour period.

Make a bar graph similar to that in Exercise 1 on page 263.

Note: This could be a homework assignment. Teacher records *where* students are working and *when*.

When graphs are completed, discuss the results focussing on the following factors: time of day, location, season, weather, etc.

2. "Questions".

Have each team write 2 or 3 questions which could be answered using their graph. Post the graphs and the

questions.

All students are responsible for answering 2 questions about the posted graphs.

3. Write the numerals 0 to 9 on index cards, shuffle them well, and place them face down on a table. You will need 1 deck of cards (i.e., 10 cards) per player. Players take turns selecting 6 cards randomly from their decks and using the cards to make a 6-digit number. Once a card has been placed in a place-value location (i.e., the thousand's place) it must remain. The object of the game is for players to use their powers of prediction (i.e., "I have already chosen an 8, a 6, and a 9; the next card will probably be 5 or lower...") to make the greatest number possible.

OBJECTIVE
To perform some simple experiments to illustrate probability

PACING
Level A 1-7
Level B 1-7
Level C 1-8

MATERIALS
twenty pennies; twenty new hexagonal pencils, unsharpened; some round pencils, sharpened

BACKGROUND
The formal definition of a probability ratio is:
$$\frac{\text{Number of desired outcomes}}{\text{Total number of outcomes}}$$

The three introductory experiments on this page will allow students to see some trends in their results, but not necessarily extrapolate a ratio for each.

SUGGESTIONS
Initial Activity Flip a coin.
How could the coin land? Heads
Tails
On edge
Which results are not likely to occur? Explain why.


USING THE BOOK
Pair students to work on these activities. One student acts as the performer and the other as the recorder. Each series of questions can be done twice so each person performs the experiment and also records the experiment. (Record in workbooks.)
Exercises 1 to 3 refer to coin flipping.
Exercises 4 to 7 refer to rolling hexagonal pencils.
Exercise 8 is an extended activity.
When the students have completed their experiments and recorded their results, gather class data on large charts and make some predictions as a group.

ACTIVITIES
1. "Flipping 2 Coins".
Have teams of students flip a penny and a nickel together 40 times. Record the results on the following tally chart.


Coins and Pencils

A coin has two sides:

heads



and tails



- Toss a coin 20 times and record the results.
- Explain your results.
- Compare your results with the results of someone else in your class.

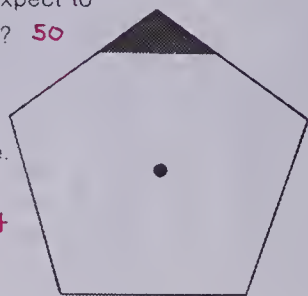
A hexagonal-sided pencil or pen usually has printing on one face and none on the other faces.

- Roll the pencil across your desk 30 times and record the results.
- How many sides have printing? 1
How many sides do not have printing? 5
How many sides are there altogether? 6
- What is the ratio of:

(a) $\frac{\text{sides with printing}}{\text{total number of sides}}$? $\frac{1}{6}$

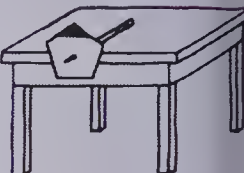
(b) $\frac{\text{sides without printing}}{\text{total number of sides}}$? $\frac{5}{6}$
- (a) How many rolls out of 60 would you expect to have printing on the top side? 10

(b) How many rolls out of 60 would you expect to have a side without printing on the top? 50


- Make this pentagon out of paper.
Colour one vertex.
Put a round pencil point through the middle.

(a) Predict the number of times the red vertex will be up out of 20 rolls. 4

(b) Now roll the shape 20 times. Record your findings on a chart.



Result	Tally	Number
Heads		
Tails		
Total		20





Results	Tally	Number
Printing on top side		
Printing not on top side		
Total		30






Result	Tally	Total
2 heads		
1 head, 1 tail		
2 tails		






Have each group explain their results. Record total class results and make predictions. Discuss.

2. See Activity 3 on page 9 for an activity which uses probability to help maintain computational skills.

3. Have the children test their powers of observation, analysis, and prediction by having them identify the patterns in sequences such as:

(a)  ,  ,  ,  .

(b)  ,   ,   .

(c)  ,   ,   .

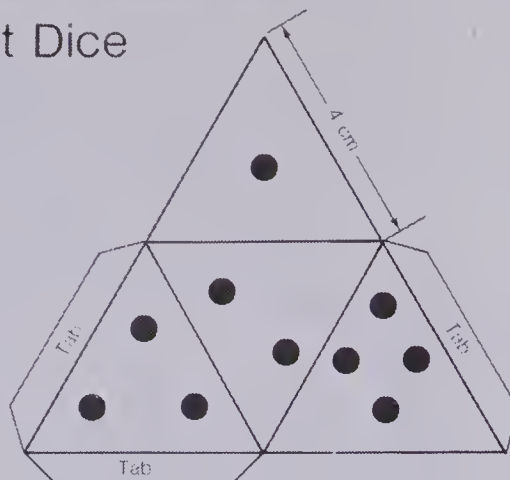
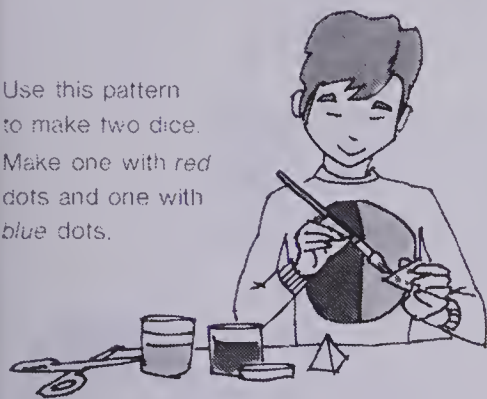
(d) 10^0 , 10^1 , 10^2 , 10^3 .

(e) 1 , 4 , 9 , 16 .

Different Dice

Use this pattern to make two dice.

Make one with red dots and one with blue dots.



Exercises

When rolling and listing results for tetrahedron dice, record the number for the side that lands face down.

- When one die is rolled there are **4** possible results.

List the results.



- When a pair of dice are rolled there are 16 possible results.

Copy this chart and record the possible results.

+				

- What is the smallest sum you can roll? **2**
- What is the largest sum you can roll? **8**
- How many ways can you roll each of the sums? Record on a chart.
- Which sum occurs most often? **5**
- Which sums would occur least often? **2 and 8**

2 - 1 way; 3 - 2 ways
4 - 3 ways; 5 - 4 ways
6 - 3 ways; 7 - 2 ways
8 - 1 way

OBJECTIVE

To analyse totals produced by tetrahedron dice

PACING

Level A All
Level B All
Level C All

MATERIALS

card stock, scissors, glue or tape

BACKGROUND

When reading these dice record the amounts on the bottom. Some students might record the amount facing them but this can lead to some confusion if the die lands with a choice of 2 numbers showing.

USING THE BOOK

Provide students with the materials to make the dice. (They can colour the dots with red and blue crayons, pencils, or felt pens.)

Remind them to read the amount from the face that is down.

Assign the exercises. Discuss the answers thoroughly.

Probability 265

ACTIVITIES

- Have students roll these pairs of dice 32 times and record the sums on a chart like this.

Sum	Tally	Total
2		
3		
4		
5		
6		
7		
8		

32

Compare group results and record class results.

2. "New Games".

Play any popular board game using the tetrahedron dice, e.g., Sorry, Parcheesi, etc. Record how the game is different. Discuss the results with the class. Summarize these.

3. Use a regular deck of playing cards (face cards = 10) to play "31". Players (from 2 to 4) take turns drawing 1 card from the well-shuffled, face-down deck and placing their card face up in front of them. Object is to use memory, observation, power of prediction, and chance to "build" the closest number to 31 without going over. Players score 1 point for each round won. The first player to reach 5 points wins. (In case of a tie, each player draws 1 card from the deck. The player with the lowest card wins the tie breaker and the round.)

Note: Used cards should not immediately be returned to the deck. Rather, have players wait till the deck has been exhausted.

OBJECTIVE

To analyse totals produced by
2 normal dice

PACING

Level A All
Level B All
Level C All

MATERIALS

twenty pairs of dice (These can be made using small blocks, unused inch cubes, etc.)

RELATED AIDS

HMS — DM66.








BACKGROUND

“Dice” is a plural form. The single item is called a “die”. Thus 1 die, 2 dice, 3 dice,

SUGGESTIONS

Initial Activity Have students write down as many games which use dice as they can in two minutes. Make a class list of games which use dice.

USING THE BOOK

Discuss the chart on page 266. When the one die (yellow) is , the other could be  or  or  or  or  or . Thus there are 6×6 or 36 possible combinations.

Assign the exercises and the activity on these two pages. Discuss the results with the class and point out that they remember the answers in Exercise 10 when playing capture games.

Dice

When a single die is thrown, there are 6 possible results. They can each be thrown only one way.



When a pair of dice are rolled, there are 36 possible combinations as shown on the chart listing the sums.

		Yellow Die					
White Die	+	•	••	•••	••••	•••••	••••••
	•	2	3	4	5	6	7
	••	3	4	5	6	7	8
	•••	4	5	6	7	8	9
	••••	5	6	7	8	9	10
	•••••	6	7	8	9	10	11
	••••••	7	8	9	10	11	12



Exercises

Refer to the chart above to answer these questions.

- What is the smallest sum you can roll? **2**
- How many ways can you roll the smallest sum? **1 way**
- What is the largest sum you can roll? **12**
- How many ways can you roll the largest sum? **1 way**



5. How many ways can you roll each of the following sums?

- (a) 2 **1** (b) 3 **2** (c) 4 **3** (d) 5 **4**
 (e) 6 **5** (f) 7 **6** (g) 8 **5** (h) 9 **4**
 (i) 10 **3** (j) 11 **2** (k) 12 **1**

6. Which sum occurs the most often? **7**

7. Which three sums are easiest to roll, using a pair of dice? **6, 7, and 8**

8. Which two sums are hardest to roll? **2 and 12**

9. If you were playing a board game using dice, how far away from the jackpot would you prefer to be? **7** Why? **There are more chances to roll a 7 than any other number.**

10. (a) When playing board games using dice in which one player captures another player, how far away from your opponent would you prefer to be? (3 or 4 answers)
 (b) At what distances would you be in most danger of being captured? (3 answers)

Activity

Roll a pair of dice 36 times and record the sums on this chart.

Compare this with your answers to Exercise 5.

Sum	Tally
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	

BRAINTICKLER

20 black socks and 20 brown socks are in a pile. The room is dark.

How many socks must you pick to be sure you have a matched pair? **3**



ACTIVITIES

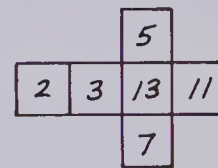
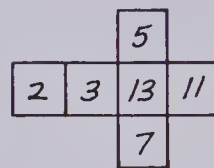
1. "Some Sums".

Perform the activity on page 267 with a partner.

Gather class data and compare this data with the answers in Exercise 5.

2. "Prime Dice".

Make a pair of dice with these prime amounts on the faces.



Make a chart for these dice and record the sums.

Answer Exercises 1 to 4 and 6 to 8 in the textbook based on the sums produced using these prime dice.

3. Play a board game using "prime dice". How is the game different?

OBJECTIVE

To provide an activity involving probability

PACING

- Level A All
- Level B All
- Level C All

MATERIALS

blocks or nets of cubes for making the shape dice, coins for board pieces

SUGGESTIONS

Initial Activity Give the students blocks or nets of cubes on card stock to make the die.

USING THE BOOK

Have the students draw the shapes on the faces of the die. Review their names — equivalent triangle, hexagon, square, rectangle, and star (pentagram).

Review the rules. Have players play: in pairs; in threes; using variation (b) — if a player is “hit”, he/she goes back to the previous space with the same shape.

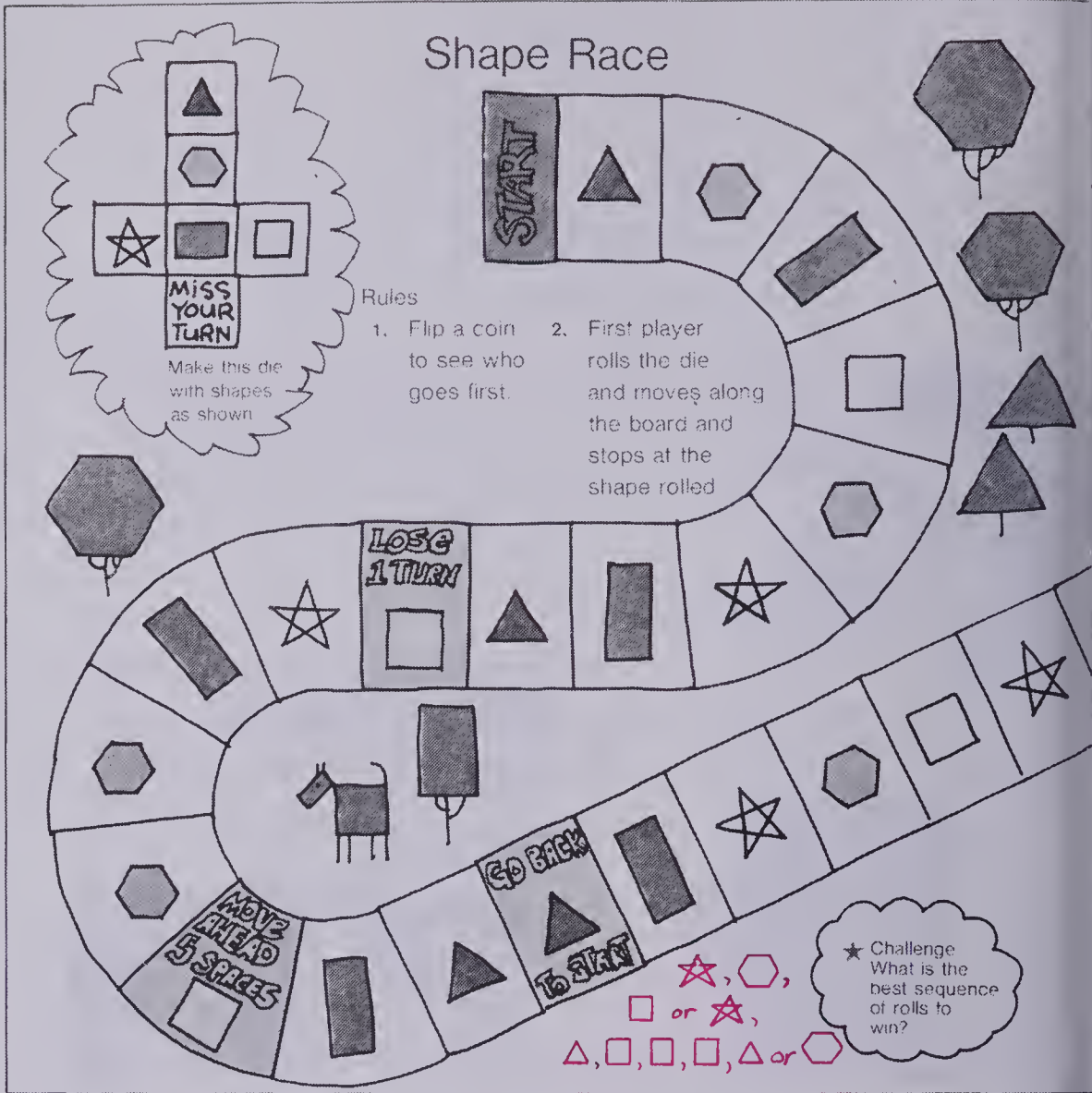
Assign each student to play the variation “Shape Solitaire” twice. Have them record the number of rolls required to play the game.

Have students record the best sequence of rolls to reach “Shape Country” in as few rolls as possible.

They could record their rolls like this.

Roll	Shape
1	
2	
3	
4	
5	
	etc.

Answer: Encourage students to keep trying sequences until they have it in 8 rolls.



ACTIVITIES

1. "The Average Game".

Have students record the number of rolls used to complete the "Shape Solitaire" game.

Find the average number of rolls for the class.

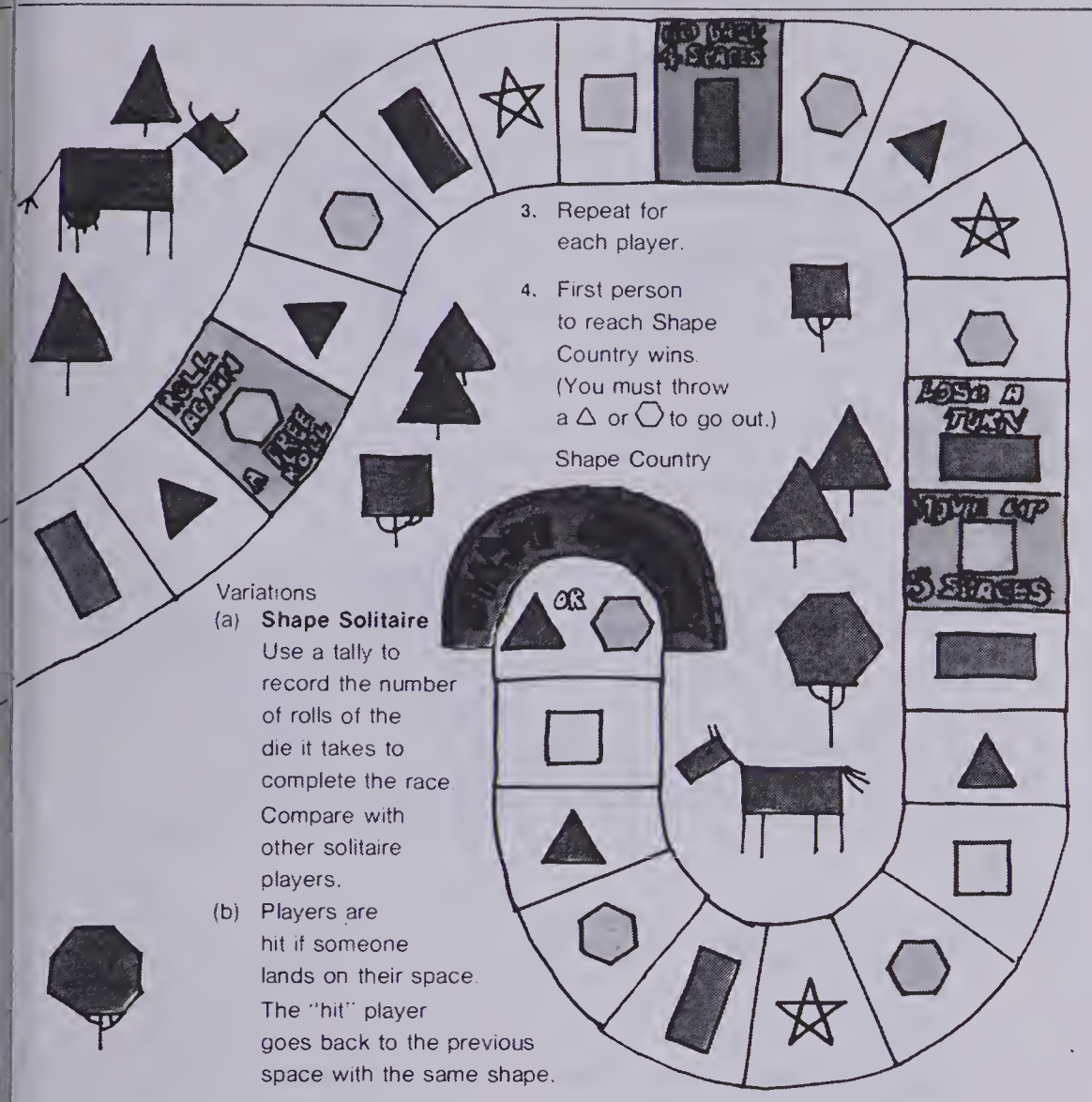
2. "The Average Three-Player Game".

Have students play a game where 3 are competing. Record the number of rolls each player takes to complete the game.

Calculate the average number of rolls for "Three-Player Shape Race".

Why are the two averages different? Discuss.

3. See "Road Rally" as described in the Activity Reservoir for another game involving "shape dice".



OBJECTIVE

To solve conservation problems

PACING

Level A All

Level B All

Level C All

VOCABULARY

conservation, leaking, well-insulated, thermostat

BACKGROUND

These problems relate well with a conservation topic in environmental studies.

SUGGESTIONS

Initial Activity Discuss conservation touching on points such as:

- What does conservation mean?
- What are some things man is going to have to conserve?
- How can man conserve?

USING THE BOOK

Review Professor Q's 4 steps in solving problems (see page 256).

Have the students read the problems and ask any questions regarding the meaning or context.

Do a sample problem together.

Example

Exercise 1(a):

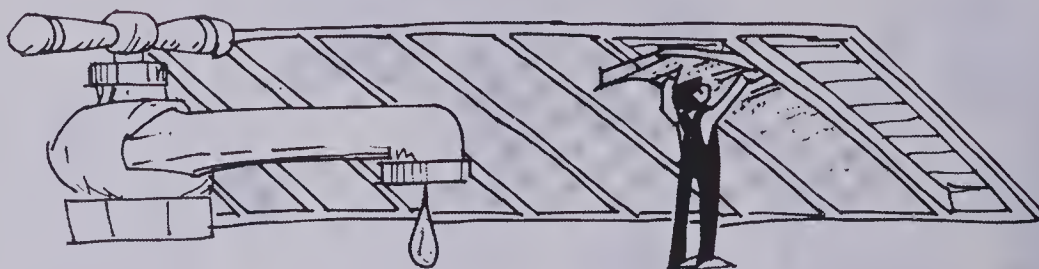
Tap leaks 0.6 L each hour.

24

$\times 0.6$ 14.4 L of water drips away
14.4 in a day.

Assign the exercises. (Remind students to number parts of questions carefully.)

Conservation at Home



Water

- A leaking water tap may drip 0.6 L each hour.
 - How much water would drip away in a day? **14.4 L**
 - How much water would be saved in a year (365 d) if this leaky tap were repaired? **5256 L**
- A tub bath uses 3 times as much water as a shower.
 - If an average family used 27 000 L of water for tub baths each year, how much water would the family use if they all took showers instead? **9000 L**
 - How much water would they save in a year by taking showers? **18 000 L**
- A toilet uses 15 L of water each time it is flushed. An average family flushes the toilets 20 times each day.
 - How much water is used each day? **300 L**
 - How much water is used each year? **109 500 L**
 - Some toilets can be adjusted to use 70% of the normal flush. How much water could be saved in a year by adjusting the toilet? **32 850 L**

Energy

- A well-insulated house saves 40% on heat-energy charges as compared to a poorly insulated house. How much money would a family save by better insulation if their yearly heat-energy charges are:
 - \$800/a? **\$320**
 - \$1100/a? **\$440**
 - \$1550/a? **\$620**
 - \$1960/a? **\$784**

270 Word problems: multi-step

5. Setting the thermostat at 18°C instead of 21°C at night and when no one is at home will save a further 9% in heat-energy charges.

What would be the saving in a well-insulated house if the heat-energy charges are:

- (a) \$500/a? **\$45** (b) \$620/a? **\$55.80** (c) \$870/a? **\$78.30**

6. Families can save 20% on the electrical bill by turning off lights when leaving rooms, deciding what is required from the refrigerator before opening the door, and turning the TV off when no one is watching it

How much would a family save by following these three tips if their yearly electricity bill is:

- (a) \$350? **\$70** (b) \$410? **\$82** (c) \$530? **\$106**

7. The family car will use 18% less gasoline if it is well tuned, the driver does not exceed 90 km/h, and the driver speeds up slowly.

How much would be saved each year by following these tips if the annual gas cost is:

- (a) \$700? **\$126** (b) \$950? **\$171** (c) \$1230? **\$221.40**

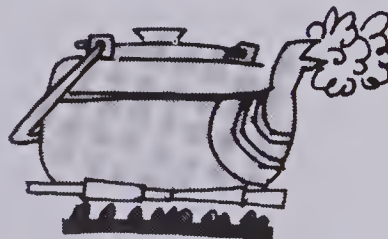


Activity

Research energy conservation under one of these topics:

- (a) heating and insulation
(b) electricity consumption
(c) transportation and gas consumption

List 6 ways of conserving energy.



ACTIVITIES

1. "A Conservation Activity".

By reading reference books and/or government pamphlets, list at least six ways you can conserve a type of energy or natural resource.

2. "Conservation Poster".

Plan and draw or paint a conservation poster which would remind people to conserve. If there is writing on the poster, use no more than seven words.

3. If you have not already done so, challenge the students with the capacity puzzler described on page 117, Activity 3.

OBJECTIVE

To solve word problems

PACING

Level A All

Level B All

Level C All

VOCABULARY

insurance, policy, policies, accident claims

RELATED AIDS

HMS — DM67.

BFA PROB. SOLVING LAB II — 113.

SUGGESTIONS

Initial Activity Discuss the insurance agent's career (see Chapter Overview, page 244), as well as some of the terms and ideas involved in insurance (i.e., policy, accident claim, monthly cost, insured value, etc.).

USING THE BOOK

Encourage the students to read over the problems and ask any questions regarding the terms or context.

Review the steps in problem solving.

Assign the problems.

ACTIVITIES

1. To review and maintain "choosing-the-correct-operation" skills, see Activity 1 on page 22.

2. "Insurance List".

Have each student interview 2 adults about insurance.

List all the different types of insurance a person, family, or company can have. Have them briefly describe each type of insurance (one sentence each).

Insurance Agent



1. Last year Mr. Yellowfeather's auto insurance cost \$450. Since he has a safe driving record he will save 10% this year.
(a) How much will he save? **\$45**
(b) How much will his insurance cost this year? **\$405**
2. Mrs. Trevor sold the Angelo family a health-insurance policy. The monthly cost is \$7.20. What is the annual cost? **\$86.40**
3. Ms. Kim insures her house with the same company that carries her car insurance. The special rate for the house is \$2.50 per \$1000 of its value. Her house is valued at \$60 000. What is the cost of the house insurance? **\$150**
4. An insurance company paid 0.86 of the insured value of a house following a fire. The house was insured for \$55 000. How much was paid to the owner? **\$47 300**
5. Mrs. Trevor sold 156 policies in 1979. In 1981 she sold 2.6 times as many. How many new policies did she sell in 1981? **250**
6. In a certain city there were 32 000 accident claims in one year. The next year there were 1.26 times as many. How many claims were there in the second year? **40 320**

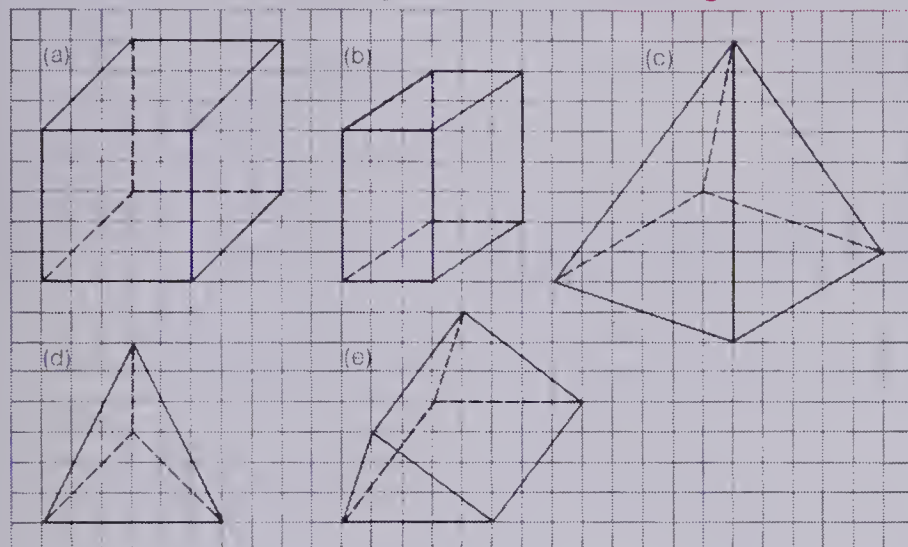
Drawing on Graph Paper

- We can use graph paper to help us draw three-dimensional shapes on two-dimensional surfaces.

Copy each shape on graph paper. Name the shapes.

The dotted lines show hidden edges.

- (a) Cube (b) Rectangular prism
(c) Square-based pyramid
(d) Tetrahedron
(e) Triangular prism



- Draw 4 three-dimensional shapes of your own design on graph paper.

Activity

Is the inside ever the outside?

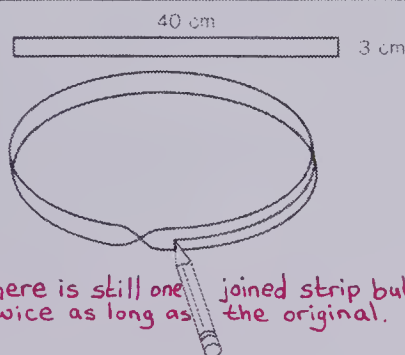
Make a strip of paper as shown.

Twist the strip once and glue.

- Mark the inside of the strip as shown. Keep marking until you get back where you started.

Is there an inside and an outside? **No**

- Use scissors to cut down the middle of the strip. What do you notice?



Drawn 3D shapes 273

OBJECTIVE

Drawing 3D figures using graph paper

PACING

Level A All

Level B All

Level C All

MATERIALS

centimetre or 0.5 cm graph paper,
metric rulers

VOCABULARY

three dimensional, two dimensional

SUGGESTIONS

Initial Activity Demonstrate how certain box shapes can be drawn using graph paper, differentiating between two-dimensional (width and height only) and three-dimensional shapes (width, height, depth).

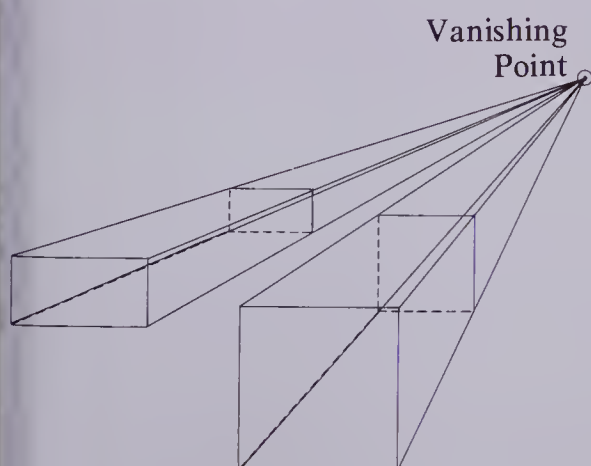
USING THE BOOK

Using graph paper, have the students draw all the shapes in Exercise 1. Remind them to show hidden edges as *dotted lines*.

Assign the exercises, pointing out that all vertical and horizontal lines remain parallel.

ACTIVITIES

- "Perspective Drawings of Boxes". Using centimetre graph paper and a "vanishing point" at the top right corner of the page, have the students draw some box shapes.

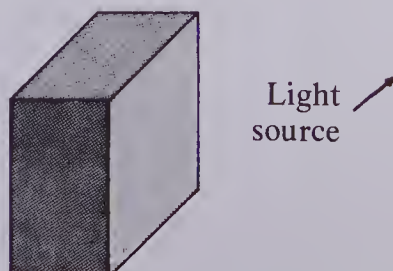


Have students copy these and draw some more.

- Have the students try the "Moebius Strip" activity shown at the bottom of the pupil page.

- Have the students shade in some of their 30 drawings to add depth. Have them (a) choose a shape that shows 3 faces; (b) use pencil to shade the 3 faces using light, medium, and heavy pressure respectively on the 3 faces; (c) identify where the "light source" seems to be.

Example



OBJECTIVE

To evaluate achievement of the chapter objectives

PACING

- Level A All
- Level B All
- Level C All

RELATED AIDS

HMS — DM68.

USING THE BOOK

Each student should do this test independently under supervision. Assistance should be given only when the instructions are not understood. After the work has been corrected, you should provide appropriate remedial work. You may wish to reteach if a large number of students had difficulty with a particular topic or concept.

The following chart will help in this regard. The specific objectives are listed in the Chapter Overview (see page 244).

Also, you may wish to use the following marking scheme (total = 30) when marking the test. Have each student calculate his/her percent mark.

Test Item	Number of Marks
1-10, 16	1
11, 12, 14, 15, 17	2
13	3
18	6
	(1 for each sales tax, 1 for each total cost)

An alternate Chapter Test can be found in the Holt Mathematics System Duplicating Masters available for use with this grade level.

Test Item	Objective	Text Page Number
1-10	A	245
12, 14	B	246-249
11	C	254
13, 18	D	256, 250
15	E	260
16	F	264
17	G	270

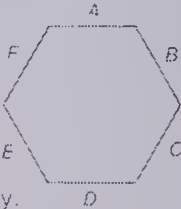
Chapter Test

Perform the indicated operations.

1. $\$12.95 + \103.58 **\$116.53**
2. $\$10.57 - \8.72 **\$1.85**
3. $\$20.00 - \16.83 **\$3.17**
4. $\$3.91 + \$0.59 + \$10.05$ **\$14.55**
5. $\$42.50 \times 0.2$ **\$8.50**
6. $\$19.00 \times 0.32$ **\$6.08**
7. $\$5.75 \times 0.08$ **\$0.46**
8. $\$325.75 \times 0.06$ **\$19.55**
9. 20% of \$83.50 **\$16.70**
10. 4% of \$200.50 **\$8.02**

Solve.

11. (a) $n \times 3 = 36$ **12**
(b) $n \div 6 = 72$ **432**
12. Tony had \$27.83 in his savings account. On April 10 he deposited \$8.50, and on April 24 he withdrew \$2.75. What was his balance on April 24? **\$33.58**
13. Glendale School has 280 students. If each student uses 18 pencils per year, how many boxes containing 100 pencils should be ordered? **51**
14. Katrina received a \$200 bond for her birthday. How much interest did she receive per year if the bond paid interest at a rate of 9%? **\$18**
15. Jeremy ran 2 km in 8 min. What was his speed in kilometres per hour? **15 km**
16. The hexagon as shown is rolled. About how many times out of 30 would A be up? **5**
17. A large city requires 600 L of water per person per day. (Much of this water is used for manufacturing.) How many litres are required per person for a year? **219 000 L**
(Write an equation to solve.) **$365 \times 600 = n$**
18. Calculate the sales tax and total cost paid for the following items if the sales-tax rate is 8%.
(a) a desk lamp costing \$20.00 (b) a desk costing \$162.00 (c) a chair costing \$47.50



274 Chapter 9 test

ANSWERS:

18. (a) \$1.60; \$21.60 (b) \$12.96; \$174.96 (c) \$3.80; \$51.30

Cumulative Review

Perform the indicated operations

1. $16.9 \times 10 = 169$
2. $\$18.73 + \$0.91 + \$103.25 + \$3.99 = \$126.88$
3. $\$206.17 - \$59.69 = \$146.48$
4. $8^2 = 64$
5. $4 - 2^3 = 1$
6. $8 \times 7 \times 6 \times 5 = 1680$
7. $\frac{1}{3} \div \frac{1}{2} = \frac{2}{3}$
8. $3 \div \frac{1}{2} = \frac{6}{1} = 6$
9. $24 \times \frac{1}{4} = 6$
10. $30 \div \frac{3}{4} = 40$

Find the missing terms

11. $\frac{1}{2} = \frac{15}{30}$
12. $\frac{1}{5} = \frac{4}{20}$
13. $\frac{1}{4} = \frac{20}{80}$
14. $\frac{1}{3} = \frac{10}{30}$

15. What is the angle sum of the 3 angles in any triangle? 180°

Calculate the perimeter of these shapes

16.  34.7 cm
17.  53.6 m
18.  58.2 m

- Solve
19. $n + 3.5 = 10$ $n = 6.5$
20. $n \times 8 = 56$ $n = 7$
21. $n - 12.7 = 21.8$ $n = 34.5$

22. Glendale School has 280 students.
35% of these students stay at the school for lunch.
How many students eat lunch at school? 98

23. Toothpaste sells for \$1.98 for 100 mL and \$2.98 for 150 mL. Which is the better buy?

24. Calculate the monthly payments on a new car that costs \$9000 if the payments are 3% of the selling price per month. \$270

25. Calculate the sales tax on a new refrigerator that costs \$870.00 if the sales-tax rate is 6%. \$52.20

Chapter 1-9 Cumulative Review 275

OBJECTIVE

To review and test selected concepts and skills previously covered

PACING

Level A All
Level B All
Level C All

USING THE BOOK

This page may be used for diagnostic and remedial as well as review purposes. Students should check their work, correct any errors, and review the pages that contain any problems of the type they missed. Some students can do this on their own while others may need help. If a large number of students have a particular problem incorrect, you may want to reteach that topic to the groups, then assign a duplicated worksheet to reinforce that topic or, refer to an appropriate skill card in the BFA Computational Skills Kit II or BFA Problem Solving Kit II.

When completed and marked, have the students calculate the percent they got correct.

Test Item	Text Page Number
1	60
2	6
3	8
4, 5	179
6	63
7	191
8	204
9	206
10	210
11-14	188, 223
15	40
16-18	105
19-21	254
22, 24	230
23	225
25	250

CHAPTER 10 OVERVIEW

In this chapter, basic algebra is approached through the solution of simple equations using related sentences and through the graphing of solutions to simple equations. Students generate and graph the ordered pairs for simple relations.

Integers are introduced and used both in graphing and in addition and subtraction.

Statistics is developed further through reading, graphing, and drawing pictographs, bar graphs, broken line graphs, and circle graphs.

OBJECTIVES

- A To use related sentences to solve simple equations
- B To graph on a number line, solutions to simple equations
- C To generate ordered pairs for given simple relations
- D To graph ordered pairs in four quadrants
- E To read, interpret, and draw pictographs, bar graphs, broken line graphs, and to read and interpret circle graphs
- F To add and subtract integers

MATERIALS

whole number line
decimal number line
integer number line
graph paper
Bristol board
model thermometer and Celsius thermometer showing minus readings
index cards
map of world with Mercator's projection
globe

CAREER AWARENESS

Mathematicians [301]

Men and women trained in mathematics have a wide range of employment opportunities. Mathematics is the basis of many vocations, often with more specialized training and/or experience required. Mathematics-trained university or college graduates are employed by companies engaged in engineering, insurance, finance, computer, oil, energy, stocks and bonds, brokerage, banking, and almost any large business.

While few people spend their full time in pure research in mathematics, there are some companies and universities that provide ample opportunity for near full-time research often with a real or practical bent.

Mathematicians can expect very good salaries, excellent working conditions, and good job opportunities. They must expect to work long and hard, since competition for top jobs is stiff.

Graphing Solutions

Joanne solved this equation.

She graphed her solution on a number line.

(a) $2 + N = 7$
 $N = 5$

(b) She drew a number line.



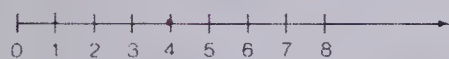
She graphed the solution with a •.



Exercises

Solve Then graph the solution on a number line

1. $2 + N = 6$
 $N = \blacksquare 4$



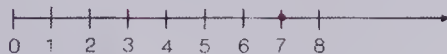
2. $N - 3 = 8$
 $N = \blacksquare 11$



3. $N + 4 = 6$
 $N = \blacksquare 2$



4. $M - 4 = 3$
 $M = \blacksquare 7$



5. $M + 2 = 5$
 $M = \blacksquare 3$

6. $A + 2 = 9$
 $A = \blacksquare 7$

7. $B + 3 = 10$
 $B = \blacksquare 7$

8. $S - 3 = 7$
 $S = \blacksquare 10$

9. $T - 4 = 2$
 $T = \blacksquare 6$

10. $W - 5 = 4$
 $W = \blacksquare 9$

11. $9 + N = 16$
 $N = \blacksquare 7$

12. $12 + N = 18$
 $N = \blacksquare 6$

13. $N + 0 = 4$
 $N = \blacksquare 4$

14. $N - 4 = 3$
 $N = \blacksquare 7$

15. $S - 6 = 0$
 $S = \blacksquare 6$

16. $N + 7 = 13$
 $N = \blacksquare 6$

Graphing solutions 277

OBJECTIVE

To graph a solution set on a whole number line (for an addition or subtraction equation)

PACING

Level A All

Level B All

Level C All

MATERIALS

a page of whole number lines showing 0 to 12 (DM69)

RELATED AIDS

HMS — DM69.

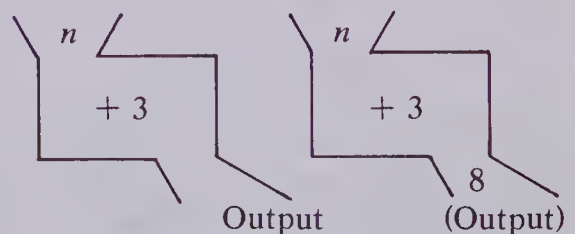
BACKGROUND

Equations have been presented in this book on pages 18, 19, 24, 85, 86, 136, 207, 209, 253, and 254. You may wish to review some of the points presented there, especially the two methods of solving equations — inspection (trial, error, and check) and inverse operations (using related sentences).

The graph of a number is a point. For example, the graph of 4 is the point on the number line represented by 4. The number 4 is the coordinate of the point. When we say “the graph of the equation $2 + n = 7$ ” we mean “the graph of its solution 5”.

SUGGESTIONS

Initial Activity Review, if necessary, the concept of equations as presented so far (see Background notes above). Review a function machine of the sort used in **Book 5** (see pages 1, 184, 185). Start with the straightforward function machine. Provide a value for n and have a child tell you the output. Then proceed to the second type. Provide the child with an output and ask what the input was (n). The child may use the inverse operation without knowing it: $8 - 3 = n$.



3. See “Input-Output” as described in the Activity Reservoir. Use equations like the ones in Exercises 1 to 16.

USING THE BOOK

Let the students use whatever method they want to solve these exercises. You may ask different students to explain how they did them. This will bring out various methods.

Provide the students with number lines on which they can graph the equations.

Provide a means of checking. Write the original equation on the chalkboard.

Have a student erase the “N”.

Have the student write the answer where N was.

Ask: “Is the sentence True or False?”

If it’s true, the answer is correct. Work through the display at the top of the page.

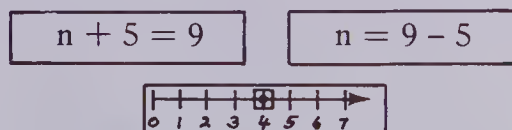
Pass out the sheet of number lines so that the students may graph the solutions.

Assign Exercise 1. Check each student’s work. Then assign the balance.

ACTIVITIES

1. Have students work in pairs at the chalkboard. The first student writes a simple addition or subtraction equation with a variable such as N. The second student erases the variable and writes in the number that would make the equation true. After the game has been played awhile, the second player may, instead of erasing the variable, simply write the solution, e.g., $N = 5$. The players change roles after each equation.

2. Prepare a set of 42 cards made of “3 of a kind”.



Deal 5 to each of 2 to 5 players. Players in turn draw from the “deck” or “discard” pile. 3 cards with equivalent numbers form a book and are laid down. Player with the most books is the winner. (See “Rummy” as described in the Activity Reservoir.)

OBJECTIVE

To use related sentences to solve addition or subtraction equations

PACING

- Level A 1-12 on each page
- Level B All
- Level C 1, 4, 7-18 on each page

VOCABULARY

related sentences

RELATED AIDS

HMS — DM70.

BACKGROUND

Addition and subtraction are opposite operations. This can be found by using a function machine. If we know the input and the operation rule, we can find the output ($7 + 3 = 10$). If we know the output, and the operation rule, we can find the input ($10 - 3 = 7$). Note that we restrict the form of the sentences at this time to $N + 3 = 9$ and $N - 4 = 3$ and do not use $7 - N = 2$.

SUGGESTIONS

Initial Activity Draw on the chalkboard or overhead projector, a picture of a function machine. First use the "Add 5" rule. Let the students give the input number (write it on the machine), say 11. After they agree the output is 16, ask what they would do to get 11 back [subtract 5 from 16]. Show the two related sentences.

$11 + 5 = 16$
 $16 - 5 = 11$

Now tell them you don't know the input (so label it N) but you do know the output is 13. What two related sentences can we write?

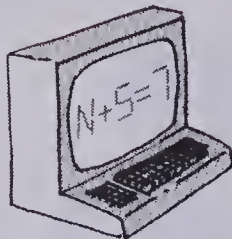
Elicit: $N + 5 = 13$
 $13 - 5 = N$ Then solve to get $N = 8$.

Repeat for other rules until students are comfortable with writing the two related sentences.

USING THE BOOK

Do Exercises 1 to 6 on page 278 orally and discuss. Then assign the balance of the exercises on the page. Following the corrections of these exercises, repeat the Initial Activity for subtraction rules: subtract 3; subtract 9.

Related Sentences



Shaun was using a micro-computer. She saw this equation on the screen. She thought about related sentences.

$N + 5 = 7$
 $N = 7 - 5$
 $N = 2$

The computer solved the equation by using a related subtraction sentence.

Exercises

Solve each pair of equations
What do you notice about the solutions?

1. (a) $N + 2 = 6$
 $N = \blacksquare 4$

(b) $N = 6 - 2$
 $N = \blacksquare 4$
2. (a) $N + 3 = 8$
 $N = \blacksquare 5$

(b) $N = 8 - 3$
 $N = \blacksquare 5$
3. (a) $N + 1 = 13$
 $N = \blacksquare 12$

(b) $N = 13 - 1$
 $N = \blacksquare 12$
4. (a) $N + 4 = 7$
 $N = \blacksquare 3$

(b) $3N = 7 - 4$
5. (a) $N + 5 = 12$
 $N = \blacksquare 7$

(b) $7N = 12 - 5$
6. (a) $N + 8 = 18$
 $N = \blacksquare 10$

(b) $10N = 18 - 8$

Write a related subtraction sentence for each. Then solve.

7. $N + 2 = 6$
 $N = 6 - \blacksquare 2$
 $N = \blacksquare 4$

8. $N + 6 = 10$
 $N = 10 - 6$
 $N = 4$

9. $N + 10 = 15$
 $N = 15 - 10$
 $N = 5$
10. $N + 12 = 25$
 $N = 25 - 12$
 $N = 13$

11. $N + 15 = 35$
 $N = 35 - 15$
 $N = 20$

12. $N + 17 = 42$
 $N = 42 - 17$
 $N = 25$
13. $N + 1.5 = 2.5$
 $N = 2.5 - 1.5$
 $N = 1.0$

14. $N + 2.1 = 3.4$
 $N = 3.4 - 2.1$
 $N = 1.3$

15. $N + 4.2 = 6.7$
 $N = 6.7 - 4.2$
 $N = 2.5$
16. $N + 9.2 = 11.6$
 $N = 11.6 - 9.2$
 $N = 2.4$

17. $N + 5.6 = 13.0$
 $N = 13.0 - 5.6$
 $N = 7.4$

18. $N + 4.9 = 11.3$
 $N = 11.3 - 4.9$
 $N = 6.4$

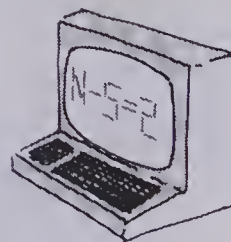
Do Exercises 1 to 6 on page 279 orally and discuss. Then assign the balance of the exercises. Some students may need the function machine to help explain Exercises 1 to 6.

More Related Sentences

Shaun saw this equation on the screen.

The computer solved the equation by printing a related addition sentence.

$$\begin{aligned} N - 5 &= 2 \\ N &= 2 + 5 \\ N &= 7 \end{aligned}$$



Exercises

Solve each pair of equations.

What do you notice about the solutions?

- | | | |
|--|---|---|
| 1. (a) $N - 2 = 6$
$N = \blacksquare 8$ | 2. (a) $N - 1 = 6$
$N = \blacksquare 7$ | 3. (a) $N - 3 = 8$
$N = \blacksquare 11$ |
| (b) $N = 6 + 2$
$N = \blacksquare 8$ | (b) $N = 6 + 1$
$N = \blacksquare 7$ | (b) $N = 8 + 3$
$N = \blacksquare 11$ |
| 4. (a) $N - 5 = 4$
$N = \blacksquare 9$ | 5. (a) $N - 6 = 4$
$N = \blacksquare 10$ | 6. (a) $N - 12 = 20$
$N = \blacksquare 32$ |
| (b) $9N = 4 + 5$ | (b) $10N = 4 + 6$ | (b) $32N = 20 + 12$ |

Write a related addition sentence for each, and then solve.

- | | | |
|---|--|--|
| 7. $N - 7 = 10$
$N = 10 + \blacksquare 7$
$N = \blacksquare 17$ | 8. $N - 6 = 3$
$N = 3 + 6$
$N = 9$ | 9. $N - 3 = 10$
$N = 10 + 3$
$N = 13$ |
| 10. $N - 9 = 1$
$N = 1 + 9$
$N = 10$ | 11. $N - 5 = 4$
$N = 4 + 5$
$N = 9$ | 12. $N - 16 = 38$
$N = 38 + 16$
$N = 54$ |
| 13. $N - 0.5 = 2.5$
$N = 2.5 + 0.5$
$N = 3.0$ | 14. $N - 3.6 = 1.8$
$N = 1.8 + 3.6$
$N = 5.4$ | 15. $N - 2.7 = 1.2$
$N = 1.2 + 2.7$
$N = 3.9$ |
| 16. $N - 4.3 = 4.2$
$N = 4.2 + 4.3$
$N = 8.5$ | 17. $N - 5.6 = 7.6$
$N = 7.6 + 5.6$
$N = 13.2$ | 18. $N - 10.8 = 20.3$
$N = 20.3 + 10.8$
$N = 31.1$ |

Related sentences for subtraction: 279

ACTIVITIES

1. Reinforce addition and subtraction facts (with Level A) by using flash cards.

2. Have students use the function machine as described in the Initial Activity. Each student gives a rule and the output and challenges the other students to write the two related sentences and the solution.

3. Duplicate a sheet with two columns of equations. For each equation in the left-hand column there is a related sentence in the right-hand column in mixed-up order. The students are to match the equations. A third column can be added. In this column are the solutions for the related sentences. These too are mixed-up and the students are to match them with the related sentences.

EXTRA PRACTICE

Write a related sentence for each.

Then solve.

- | | |
|-----------------------|----------------------|
| 1. $M + 3.6 = 19.4$ | 2. $T - 4.6 = 11.2$ |
| 3. $X + 27.3 = 41$ | 4. $Y - 2.8 = 7.6$ |
| 5. $R + 9.8 = 20.01$ | 6. $V - 36 = 14.78$ |
| 7. $D + 0.33 = 10.00$ | 8. $F - 7.35 = 1.99$ |
| 9. $H + 22.36 = 67.1$ | ★10. $4 + N = 11$ |
| ★11. $9 + M = 16$ | ★12. $9 - N = 6$ |
| ★13. $17 - N = 10$ | ★14. $26 - N = 18$ |
| ★15. $13 + N = 28$ | |

Read each problem and match it with the correct number sentence. Then solve the problem using the number sentence.

- Bill had 45 comic books. Sally gave him 18 more. How many does he now have?
- Rock concert tickets cost \$9.60 each. What is the cost of a dozen tickets?
- A comic book has 18 cartoons on each of 45 pages. How many cartoons are there altogether?
- A dozen roses cost \$9.60. How much does one rose cost?
- Nancy made two purchases of \$3.25 and \$1.65. She paid with a ten-dollar bill. How much money did she get in change?
- Ron earned \$10.00 one day and \$3.25 another day. He spent \$1.65. How much did he have then?

____. $10.00 - (3.25 + 1.65) = N$

____. $(10.00 + 3.25) - 1.65 = N$

____. $9.60 \times 12 = N$

____. $45 + 18 = N$

____. $45 \times 18 = N$

____. $9.60 \div 12 = N$

OBJECTIVE

To graph the solution to an equation using decimals in tenths

PACING

- Level A 1-7, 11 on each page
- Level B All
- Level C All

MATERIALS

a sheet of decimal number lines, 0 to 12, graduated in tenths (DM69)

RELATED AIDS

HMS — DM69.

BACKGROUND

The technique used to solve equations involving whole numbers is now used to solve equations involving decimals to tenths.

SUGGESTIONS

Initial Activity With those students that need more reinforcement of the opposite operations (inverse relationship), write these four headings on the chalkboard. Ask the student to fill in each blank as you give the first 3 numbers:

	Start with	Add	Subtract	Answer
(a)	2.2	7	7	<input type="checkbox"/>
(b)	5.7	12.2	12.2	<input type="checkbox"/>

After a number of these ask what the student can conclude. Point out that, just as we can think of subtraction as *undoing* addition, we can think of addition *undoing* subtraction. Hence, we say addition and subtraction are *inverse operations*.

	Start with	Subtract	Add	Answer
(a)	10	9	9	<input type="checkbox"/>
(b)	17.1	7.4	7.4	<input type="checkbox"/>

USING THE BOOK

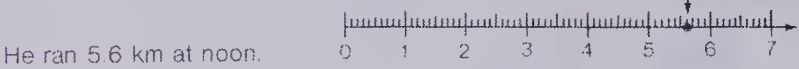
Use the steps in problem solving to solve the problem in the display.

Jogging

Mark was training for the track team.
He ran at noon.
After school he jogged 6.2 km.
His total distance for the day was 11.8 km.
How far did he go at noon?
Show your answer on a number line.



Related sentences N is the distance at noon.
 $N + 6.2 = 11.8$
 $N = 11.8 - 6.2$
 $N = 5.6$
 $5.6 + 6.2 = 11.8$ ← "It works!"

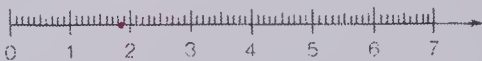


Exercises

Solve. Graph each solution on a number line marked in tenths.

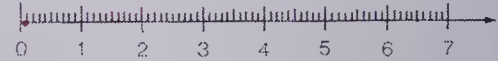
1. $N = 3.2 - 1.3$

$N = \blacksquare 1.9$



2. $N = 1.6 - 1.5$

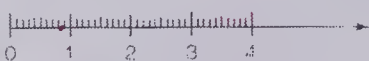
$N = \blacksquare 0.1$



3. $N + 1.4 = 2.3$

$N = 2.3 - 1.4$

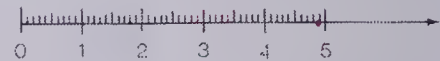
$N = \blacksquare 0.9$



4. $M + 7.3 = 12.2$

$M = 12.2 - 7.3$

$M = \blacksquare 4.9$



5. $M + 0.8 = 3.4$

2.6

6. $K + 16.4 = 20.2$

3.8

7. $K + 1.4 = 1.8$

0.4

8. $K + 4.8 = 6.9$

2.1

9. $K + 9.8 = 13.2$

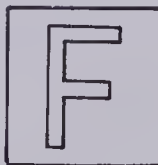
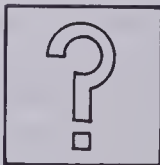
3.4

10. $N + 3.6 = 6.3$

2.7

11. Marianne ran on Saturday. On Sunday she went 6.4 km.

Her total weekend distance was 10.8 km. How far did she run on Saturday? 4.4 km



Draw a diagram.

$+$ $-$
 \times \div

Write number sentence.

Write related sentence.

Solve.

Graphing Decimals

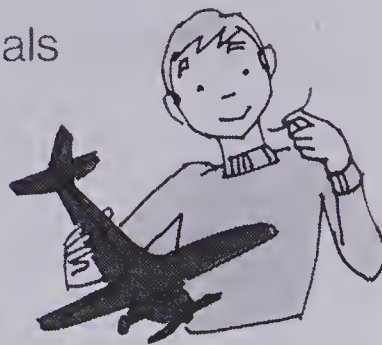
Chris used 3.4 cm of wire on his model.
He had 2.3 cm left.
How much did he have at the beginning?
Show your answer on a number line.

N is the number of centimetres he had at the beginning.

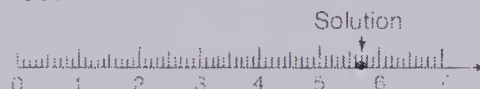
Related sentences
 $N - 3.4 = 2.3$
 $N = 2.3 + 3.4$
 $N = 5.7$

Check: $5.7 - 3.4 = 2.3$ — it works!

He had 5.7 cm at the beginning.



Use a number line marked in tenths.



Exercises

Solve. Graph each solution on a number line marked in tenths.

1. $N = 2.3 + 1.6$

$N = \blacksquare 3.9$



2. $N = 3.6 + 2.7$

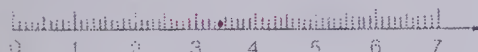
$N = \blacksquare 6.3$



3. $N - 2.1 = 1.3$

$N = 1.3 + \blacksquare 2.1$

$N = \blacksquare 3.4$



4. $N - 4.2 = 1.6$

$N = 1.6 + \blacksquare 4.2$

$N = \blacksquare 5.8$



5. $N - 3.3 = 2.2$

$N = \blacksquare 5.5$

6. $N - 4.4 = 1.1$

$N = \blacksquare 5.5$

7. $N - 4.5 = 0$

$N = \blacksquare 4.5$

8. $N - 5.5 = 1.6$

$N = \blacksquare 7.1$

9. $N - 0.3 = 5.6$

$N = \blacksquare 5.9$

10. $N - 4.9 = 1.1$

$N = \blacksquare 6.0$

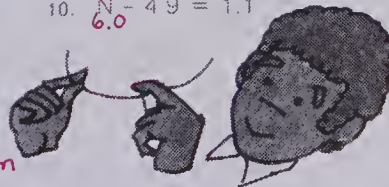
11. Mike had some wire.

He used 6.2 cm.

He now has 11.2 cm left.

How much wire did he have when he started?

17.4 cm



Solving equations and graphing solutions

281

Finally, locate the point on the number line represented by the solution. Hand out the sheet of decimal number lines.

Assign Exercise 1. Check to see that there are no difficulties. Assign balance of page 280 and check them. Assign Exercise 1 on page 281, check it, then assign the balance.

ACTIVITIES

1. Let students work in pairs at the chalkboard. Each pair draws a function machine with N as the input. The first student writes a rule in the machine and an output. The second student writes a pair of related sentences and solves. After 10 turns each the student with the most correct is the winner.

2. Play "Eraser" as described in the Activity Reservoir. Use equations like the ones on pages 280 and 281.

3. See "Input-Output" as described in the Activity Reservoir. Use strips which reflect the skills from these two pages.

EXTRA PRACTICE

1. Chad cracked his piggy bank to buy a ring.

The ring cost \$18.53.

After buying the ring he had \$7.37 left.

How much did he have in his piggy bank? [\$25.90]

2. A merchant received a new bolt of cloth.

During the day she sold 29.4 m.

She had 10.6 m left at the end of the day.

How much cloth was in the bolt originally? [40 m]

3. On a bicycle trip, Marg cycled 26 km on the second day.

By the end of the second day she had cycled 51 km altogether.

How much did she cycle on the first day? [25 km]

4. Mr. Smith caught 3 fish with a total mass of 12.8 kg.

Two were 3.1 kg and 4.6 kg.

What was the mass of the other fish? [5.1 kg]

OBJECTIVE

To use related sentences to solve equations involving multiplication and division

PACING

Level A 1-12 on each page

Level B All

Level C All

RELATED AIDS

HMS — DM71.

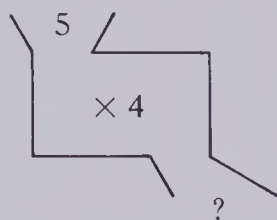
BACKGROUND

Multiplication and division are opposite operations (inverse operations). Solving for n in $n \times 3 = 12$ yields the same answer as solving for n in $n = 12 \div 3$. Similarly the solution of $n \div 4 = 6$ is the same as $n = 6 \times 4$. We restrict the types here to $n \times 3 = 12$ and $n \div 2 = 6$ and leave until later the two types $5 \times n = 10$ and $6 \div n = 2$.

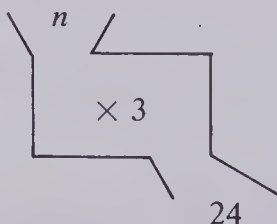
SUGGESTIONS

Initial Activity Use the function-machine method described in Initial Activity on page 278, but use the operations multiplication and division, in turn.

Example



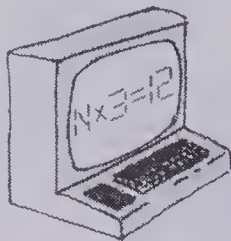
If 5 goes in, what comes out? [20]
Then proceed to this type.



If 24 comes out, what number did I put in? [8]

Repeat both for division rules. Then reinforce the inverse relationship between multiplication and division. Write these on the chalkboard and ask the students to find the results.

Related Sentences for Multiplication



The computer gave Kevin this problem.
He solved it by using related sentences

$$\begin{aligned} N \times 3 &= 12 \\ N &= 12 \div 3 \\ N &= 4 \end{aligned}$$

Check: $4 \times 3 = 12$ ← it works

Exercises

Solve each pair of equations

What do you notice about the solutions?

1. (a) $N \times 2 = 6$
 $N = \blacksquare 3$

(b) $N = 6 \div 2$
 $N = \blacksquare 3$

2. (a) $N \times 3 = 18$
 $N = \blacksquare 6$

(b) $N = 18 \div 3$
 $N = \blacksquare 6$

3. (a) $N \times 4 = 20$
 $N = \blacksquare 5$

(b) $N = 20 \div 4$
 $N = \blacksquare 5$

4. (a) $N \times 5 = 40$
 $N = \blacksquare 8$

(b) $N = 40 \div 5$
 $N = \blacksquare 8$

5. (a) $N \times 8 = 72$
 $N = \blacksquare 9$

(b) $N = 72 \div 8$
 $N = \blacksquare 9$

6. (a) $N \times 9 = 63$
 $N = \blacksquare 7$

(b) $N = 63 \div 9$
 $N = \blacksquare 7$

Write a related division sentence for each, and then solve

7. $N \times 4 = 36$
 $N = 36 \div \blacksquare 4$
 $N = \blacksquare 9$

8. $N \times 3 = 30$
 $N = 30 \div 3$
 $N = 10$

9. $N \times 8 = 56$
 $N = 56 \div 8$
 $N = 7$

10. $N \times 12 = 72$

11. $N \times 15 = 45$

12. $N \times 20 = 400$

13. $N \times 2 = 8.6$

14. $N \times 8 = 9.6$

15. $N \times 10 = 6.4$

16. $N \times 3.2 = 6.4$

17. $N \times 3.1 = 7.75$

18. $N \times 4.3 = 10.32$

282 Related sentences for multiplication

ANSWERS:

10. $N = 72 \div 12$
 $N = 6$

11. $N = 45 \div 15$
 $N = 3$

12. $N = 400 \div 20$
 $N = 20$

13. $N = 8.6 \div 2$
 $N = 4.3$

14. $N = 9.6 \div 8$
 $N = 1.2$

15. $N = 6.4 \div 10$
 $N = 0.64$

16. $N = 6.4 \div 3.2$
 $N = 2$

17. $N = 7.75 \div 3.1$
 $N = 2.5$

18. $N = 10.32 \div 4.3$
 $N = 2.4$

	Start with	Multiply	Divide	Answer
(a)	4	3	3	<input type="checkbox"/>
(b)	5	6	6	<input type="checkbox"/>

Just as we can think of division *undoing* multiplication, we can think of multiplication *undoing* division as in these:

	Start with	Divide	Multiply	Answer
(a)	12	6	6	<input type="checkbox"/>
(b)	24	8	8	<input type="checkbox"/>

Related Sentences for Division

To find N in this equation,

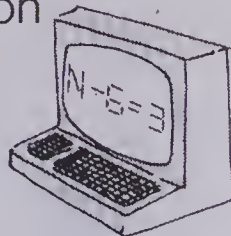
Carol used a related sentence for division:

$$N \div 6 = 3$$

$$N = 3 \times 6$$

$$N = 18$$

Check: $18 \div 6 = 3$ ← "It works!"



Exercises

Solve each pair of equations.

What do you notice about the solutions?

1. (a) $N \div 4 = 5$

$$N = \blacksquare 20$$

2. (a) $N \div 3 = 7$

$$N = \blacksquare 21$$

3. (a) $N \div 4 = 7$

$$N = \blacksquare 28$$

(b) $N = 5 \times 4$

$$N = \blacksquare 20$$

(b) $N = 7 \times 3$

$$N = \blacksquare 21$$

(b) $N = 7 \times 4$

$$N = \blacksquare 28$$

4. (a) $N \div 6 = 9$

$$N = \blacksquare 54$$

5. (a) $N \div 8 = 6$

$$N = \blacksquare 48$$

6. (a) $N \div 9 = 9$

$$N = \blacksquare 81$$

(b) $N = 9 \times 6$

$$N = \blacksquare 54$$

(b) $N = 6 \times 8$

$$N = \blacksquare 48$$

(b) $N = 9 \times 9$

$$N = \blacksquare 81$$

Write a related multiplication sentence, and then solve.

7. $N \div 5 = 9$

$$N = 9 \times \blacksquare 5$$

$$N = \blacksquare 45$$

8. $N \div 10 = 10$

$$N = 10 \times 10$$

$$N = 100$$

9. $N \div 8 = 56$

$$N = 56 \times 8$$

$$N = 448$$

10. $N \div 12 = 6$

11. $N \div 8 = 11$

12. $N \div 9 = 12$

13. $N \div 4 = 8.4$

14. $N \div 23 = 6$

15. $N \div 4.1 = 3.2$

16. $N \div 6.2 = 3.5$

17. $N \div 8 = 13.2$

18. $N \div 0.8 = 0.9$

Related sentences for division 283

USING THE BOOK

Discuss the display at the top of page 282.

Do Exercises 1 to 6 orally, emphasizing the related sentences aspect and that related sentences have the same solution.

Assign the balance of the exercises on page 282. Check. Repeat the procedure for page 283.

ACTIVITIES

1. Reinforce multiplication and division facts (Level A) by using flash cards.

2. Have students use the function machine as described in Initial Activity. Each gives a multiplication or division rule and the output and challenges the other(s) to write the two related sentences and the solution. Students may work in pairs or small groups.

3. Prepare a set of cards made of "3 of a kind".

$n \times 5 = 35$	$n = 35 \div 5$	$n = 7$
-------------------	-----------------	---------

Play "Rummy" as described in the Activity Reservoir.

4. See "Itza Fact!" as described in the Activity Reservoir.

5. See the "Coded Riddles" idea in the Activity Reservoir. Have the students use equations which encompass all four operations.

EXTRA PRACTICE

Write a related sentence for each. Then solve and check.

1. $N \times 3 = 27$ [9]

2. $M \times 9 = 72$ [8]

3. $T \times 6 = 42$ [7]

4. $N \div 4 = 8$ [32]

5. $R \div 8 = 32$ [256]

6. $P \div 7 = 28$ [196]

7. $B \times 12 = 48$ [4]

8. $C \div 20 = 80$ [1600]

9. $D \div 25 = 200$ [5000]

10. $E \times 50 = 500$ [10]

11. $F \times 900 = 9000$ [10]

12. $N \div 80 = 80$ [6400]

13. $F \times 100 = 10\,000$ [100]

14. $D \div 2000 = 0.4$ [800]

15. $M \div 1000 = 0.1$ [10]

16. $N \div \frac{1}{2} = \frac{3}{4}$ [$\frac{3}{8}$]

17. $X \times \frac{1}{5} = \frac{1}{10}$ [$\frac{5}{10}$]

18. $Z \times \frac{2}{5} = \frac{5}{2}$ [$\frac{25}{4}$]

19. $X \div 0.3 = 1.1$ [0.33]

20. $R \times 0.8 = 1.6$ [2]

21. $M \times 1.6 = 1.28$ [0.8]

22. $Q \div 0.2 = 8.6$ [1.72]

23. $C \times 4.1 = 9.43$ [2.3]

24. $T \div 3.2 = 4.4$ [14.08]

ANSWERS:

10. $N = 6 \times 12$
 $N = 72$

11. $N = 11 \times 8$
 $N = 88$

12. $N = 12 \times 9$
 $N = 108$

13. $N = 8.4 \times 4$
 $N = 33.6$

14. $N = 6 \times 2.3$
 $N = 13.8$

15. $N = 3.2 \times 4.1$
 $N = 13.12$

16. $N = 3.5 \times 6.2$
 $N = 21.7$

17. $N = 13.2 \times 8$
 $N = 105.6$

18. $N = 0.9 \times 0.8$
 $N = 0.72$

OBJECTIVE

To solve word problems using equations

PACING

- Level A All
- Level B All
- Level C All

BACKGROUND

While students may be able to solve the problems without using equations, emphasize and insist that the students use equations though they may write directly the simple equation in the form $N = \square \pm \square$

SUGGESTIONS

Initial Activity Discuss the various aspects of a park — birds, trees, flowers, etc. and what it takes to keep a park for people to enjoy. Endeavour to build the need for everyone to co-operate in keeping a park clean and presentable. If appropriate, discuss the negative aspects of vandalism.

USING THE BOOK

Have a student read the problem in the display at the top of the page. Then have all the students close their books and ask: "What do we need to find?" "What are we told that will help us find that?" "Let's use N to represent the number of kilograms of bird seed she has now. What number sentence can we write?" Then proceed to write the related sentence, solution, and answer statement. After assigning the exercises, it may be necessary to work with a small group having difficulty. You may wish to continue using the method just described.

ACTIVITIES

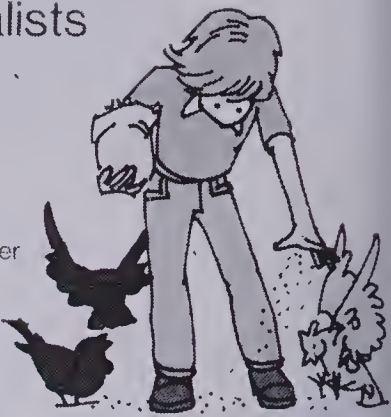
1. Choosing the correct operation is a frequent source of difficulty in solving word problems. You may wish to prepare an activity as described in Activity 1, page 22, to help in this regard.
2. Prepare a puzzle chart as shown for completion by the students.

Facts	Solution	Operation
25, 3	75	multiplication
25, 3	28	
\$4.50, 2	\$2.25	
4.63, 0.5		addition
	\$9.99	subtraction
etc.		

The Young Naturalists

Penny has a bag of bird seed.
She calculates she needs 4 kg more.
In all she needs 16 kg
How much does she have now?

Use N to represent the number of kilograms she has now
She has now + She needs more = She needs altogether
 $N + 4 = 16$
Equation: $N + 4 = 16$
Solve: $N = 12$
Check $12 + 4 = 16$ ← It works!
Statement: Penny has 12 kg of bird seed now.



Exercises

Write an equation to solve each problem.

1. Jeff planted birch trees last week.
He planted 23 more this week.
In total he has now planted 56.
How many trees did he plant last week?
Use N to represent the number of trees planted last week.
Planted last week + planted this week = planted altogether
 $N + 23 = 56$
Equation: $N + 23 = 56$
Solve: $N = 33$
Check: $33 + 23 = 56$
Statement: Jeff planted 33 trees last week.
2. Nadine helped clean up the park.
She worked 6 h each day.
She worked 30 h in all.
How many days did she work? 5d
 $N = 30 \div 6$
 $N = 5$
3. Connie was painting picnic tables.
There were 42 tables in total.
She painted 6 each day.
How many days did she paint? 7d
 $N = 42 \div 6$
 $N = 7$
4. Craig went fishing for pickerel.
He gave away 4.3 kg of fish to friends.
He kept one fish with a mass of 2.8 kg for himself.
What mass of fish did he catch altogether? 7.1 kg
 $N = 4.3 + 2.8$; $N = 7.1$
5. Marco counted 42 more Canada Geese than Alphonse.
Alphonse counted 156 geese.
How many did Marco count? 198
 $N = 156 + 42$; $N = 198$

3. Have each student collect a picture related to parks and make up one question that can be solved by using a number sentence involving addition, subtraction, multiplication, or division. The pictures can be glued to tag and the problem written beneath the picture. A solution should be written on the back. These are posted on the bulletin board. Students select questions and do them.

EXTRA PRACTICE

1. There was 28.5 ha added to a park. It now is 159 ha.
How large was it originally? [130.5 ha]

2. The number of swans has been tripled.
There are 27 swans now.
How many were there originally? [9]
3. A playing field has been divided into 3 fields of equal sizes.
Each new field is 14.5 ha.
How large was the original field? [43.5 ha]
4. The pond has been increased to 15 000 m².
Originally it was only $\frac{1}{3}$ this size.
What was the original size? [5000 m²]

Ordered Pairs

Martin's class voted on a class sweater.



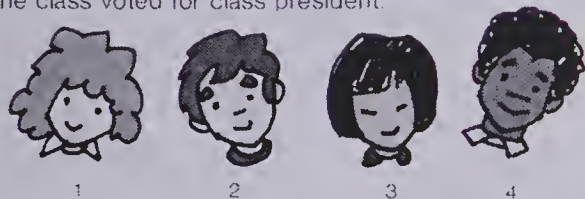
We can show the information by using ordered pairs.

(1,4) (2,7) (3,15) (4,1) (5,0)

Exercises

- In the ordered pair (2,7), what does the 2 mean? What does the 7 mean? **Sweater #2** **7 votes**
- What does the ordered pair (3,15) mean? **Sweater #3 received 15 votes.**
- Does (1,4) mean the same as (4,1)? Explain. **No** **(1,4) means sweater #1 received 4 votes.** **(4,1) means sweater #4 received 1 vote.**

The class voted for class president.



Candidate	Number of Votes
1	4
2	5
3	2
4	16

- Write the information showing the results of the voting by using ordered pairs.
(1,4), (2,5), (3,2), (4,16)
- Harry wrote the information this way:
(4,1) (5,2) (2,3) (16,4)
Explain what Harry meant by his ordered pairs.

Ordered pair concept 285

OBJECTIVE

To use ordered pairs in a basic way

PACING

Level A All
Level B All
Level C All

VOCABULARY

ordered pairs

BACKGROUND

An "ordered pair" is a pair of numbers or attributes that are used to describe a situation. The fact they are ordered indicates that the element in the first place relates to a specific situation and the element in the second place relates to a different but particular situation. Hence the reader must remember to what each element relates. As an example, in the display in (1,4) the 1 identifies the sweater and the 4 tells the number of votes that particular sweater received.

USING THE BOOK

While this page can be assigned for seatwork, a group discussion may better facilitate the development of the objective. Assigning Exercises 4 and 5 will provide you with the opportunity to see if the students have achieved the objective.

ACTIVITIES

1. Have students list other ordered pairs and how they can be used. For example: 3rd Ave. and 5th St. might be (3,5); the temperature at any given time might be (10:30, 26°C), etc.

2. Have students make up a code for writing coded messages. Example: (1,A), (26,B), (2,C), (25,D), (3,E), (24,F), ..., (13,Y), (14,Z). Decode this message: 10, 6, 1, 11, 3, 26, 8, 1, 18, 25, 5, 20, 4; 5, 10; 24, 17, 20. [skateboarding is fun]

3. Tell students they are to design a parking lot for cars. They will want to be able to assign a parking stall to a customer so the customer can easily find it. Further, they will want to be able to locate a given car given its placement. [There will be numerous solutions. One might be an ordered pair approach based on (row, stall) system; that is row 3, stall 5.]

ANSWERS:

5. (4,1) - 4 votes for candidate #1. (5,2) - 5 votes for candidate #2. (2,3) - 2 votes for candidate #3.
(16,4) - 16 votes for candidate #4.

OBJECTIVE

To plot a point in the first quadrant given the ordered pair

PACING

- Level A All
- Level B All
- Level C All

MATERIALS

graph paper — preferably 1 cm square and 10 × 10 at least

RELATED AIDS

HMS — DM69.

BACKGROUND

René Descartes (1596-1650), a French mathematician, devised the system of associating points in a plane with ordered number pairs. This enables mathematicians to draw the graph of a line. This brings together algebra and geometry: points are associated with number pairs and lines are associated with equations. At this level we only deal with points and number pairs.

SUGGESTIONS

- Initial Activity** Review the meaning of the words *vertical* and *horizontal*. Emphasize that in graphing:
- (a) the vertical line is named first (using the horizontal number line)
 - (b) the horizontal line is named second (using the vertical number line)
 - (c) the meaning of “ordered pair”.

USING THE BOOK

Provide the students with graph paper. Show them how to mark and label the horizontal and vertical number lines. Then show them how to locate the point (6,8). Label. Ask them to locate each of these points: (2,3), (2,6), (5,6), (5,3). If they join these points they will have a square. Then ask what the ordered pair is for each point labelled in the display.

Assign the exercises. Exercises 1 and 2 are self-checking.

Graphing Ordered Pairs

Mathematicians have agreed to always name the numbers in a definite order

They name the vertical line first: (6,)

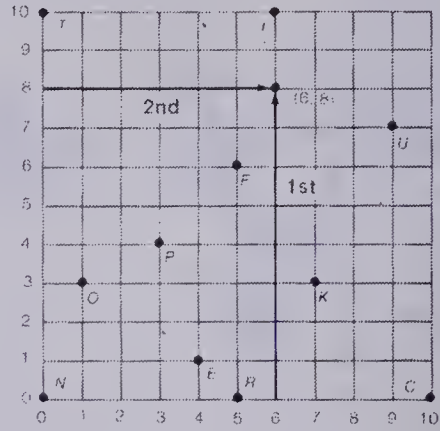
They name the horizontal line second: (,8)

To indicate the numbers are in a definite order, brackets are placed around the numbers:

(6,8)

This is called an **ordered pair**.

The name for point K is (7,3).



Exercises

1. Refer to the display.

Copy and place the letters in the blanks to answer the question

What do cats strive for?

(3,4) (9,7) (5,0) (5,0) (5,6) (4,1) (10,0) (0,10) (6,10) (1,3) (0,0)

P U R R F E C T I O N

2. Copy and complete the chart.

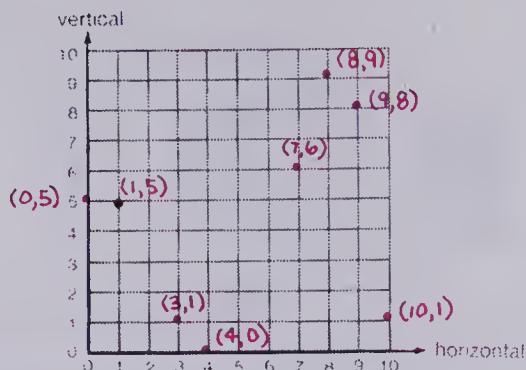
1st Number	5	2	8	6	10	3
2nd Number	1			2		
Ordered pair	(5,1)	(2,6)			(10,8)	
Letter			T			C

Do the letters form a word? **METRIC**

6; 10; 8; 4
(8,10); (6,2); (3,4)
M; E; R; I

3. Use graph paper.
Draw and label two
lines: one vertical
one horizontal

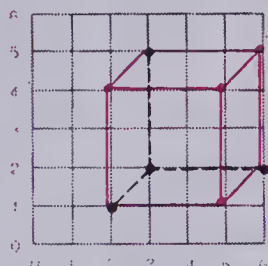
Graph these ordered pairs.
(1,5), (3,1), (8,9), (4,0),
(10,1), (0,5), (7,6), (9,8)



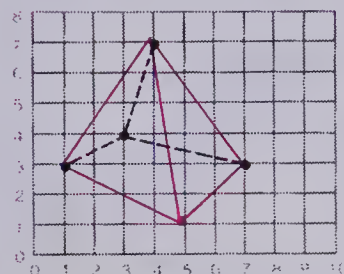
4. Pictures can be drawn by graphing
ordered pairs and joining the points.
Mark these points on a grid.

(2,1), (2,4), (3,5), (3,2),
(6,5), (6,2), (5,1), (5,4)

Join the points to make a cube.



5. Draw a pyramid by using these points
(1,3), (4,7), (7,3), (5,1), (3,4)

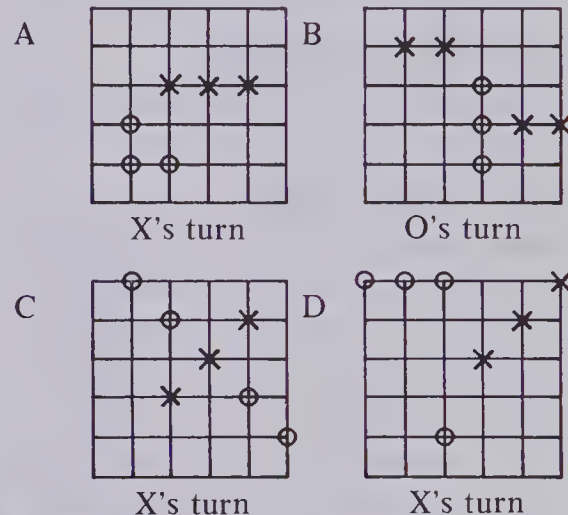


6. Draw a triangular prism by using these points.
(1,2), (2,6), (5,6), (6,1), (4,2), (3,1)

ACTIVITIES

1. Prepare a challenge card or activity
such as: Steve and Karen were playing
a game of "Tic-Tac-Toe". They each
had a 5×5 grid. They took turns
naming ordered pairs and marking
the points. The first to get four in a
row wins.

Write the ordered pair that will win
the game in each situation for the
player indicated.



Play the game with a partner.

2. Play "Radar Scopes" as
described in the Activity Reservoir.

3. Challenge students to code a
picture made graphing ordered pairs.
These codes are given to the balance
of the class who draw the pictures by
connecting one point to the next with
line segments. A barn for instance
might be: (1,1), (1,3), (2,5), (3,6), (4,5),
(5,3), (5,1), (3,1), (3,2), (2,2), (2,1), (1,1).

4. To integrate locating points on
a grid with practising computational
skills, see "500 Grand" as described
in the Activity Reservoir. Note,
however, that the grid in this game
locates spaces on the grid, *not*
intersection points.

OBJECTIVE

To generate ordered pairs using the function machine

PACING

- Level A 1-5, 7, 8, 10-15
- Level B 1-5, 7, 8, 10-17
- Level C 6-17

MATERIALS

Bristol board for the function machine

RELATED AIDS

CALC. ACTIVITY MASTERS — 45, 56, 78.

SUGGESTIONS

Initial Activity Make a large display-size function machine and name it. (Let students use their imagination.) Make it so that it can hold cards for Input, Rule, and Output. Use this in conjunction with input-output tables to help introduce the procedures that will be used on these two pages. Have the students supply the appropriate information for the various inputs, outputs, and rules.

USING THE BOOK

On the chalkboard, draw an input-output table. Give the group a rule such as “add 26”. As you give them inputs, the students take turns writing the “outputs” in the appropriate spaces. Then move to double operations for the rule such as “multiply by 5 and add 8”. Finally, write the rule on the chalkboard in this form: “ $5 \times \blacksquare + 8$ ”. Point out that “ \blacksquare ” stands for “input” and that “ \blacktriangle ” stands for “output”.

Generating Ordered Pairs

Input

Rule:
Add 3

Output

Input	1	2	3	5
Output	4	5	6	8

Input

Multiply
by 3
and
Add 1

Output

Input	1	2	3	10
Output	4	7	10	31

Exercises

Complete each table.

1.

Input

Add 7

Output

Input	0	1	2	5	10
Output	7				

8 9 12 17

2.

Input

Multiply
by 6

Output

Input	0	3	9	10	12
Output					

0 18 54 60 72

3.

Input

Subtract
4

Output

Input	5	10	15	20	25
Output					

1 6 11 16 21

4.

Input

Divide
by 3

Output

Input	3	6	9	12	30
Output					

1 2 3 4 10

5.

Input

Divide
by 10

Output

Input	10	20	30	1
Output				

1 2 3 0.1

6.

Input

Multiply
by 9 4

Output

Input	0	1	2	5	10
Output					

0 9.4 18.8 47 94

7.

Input

Multiply
by 2
and
Add 1

Output

Input	1	2	3	5	10
Output					

3 5 7 11 21

8.

Input

Multiply
by 3
and
Add 2

Output

Input	1	3	5	10	15
Output					

5 11 17 32 47

9.

Input

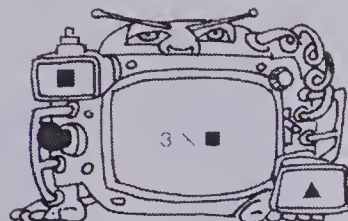
Multiply
by 5.2
and
Subtract
2.3

Output

Input	1	2	3	5	6
Output					

2.9 8.1 13.3 23.7 28.9

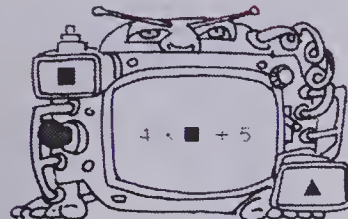
10.



■	1	2	3	4	10
▲	3				

6 9 12 30

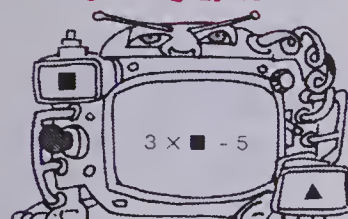
12.



■	0	1	2	5	10
▲	9				

5 13 25 45

14.



■	2	4	6	8	10
▲					

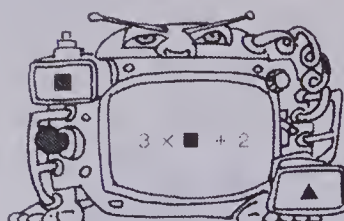
1 7 13 19 25

16. Rule: Multiply by 2 and add 2.

Input	1	2	3	5	8
Output					

4 6 8 12 18

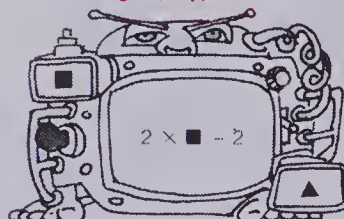
11.



■	1	2	3	4	10
▲	5				

8 11 14 32

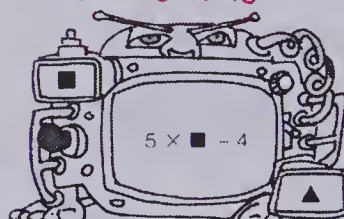
13.



■	2	3	5	6	10
▲					

2 4 8 10 18

15.



■	1	3	5	7	9
▲					

1 11 21 31 41

17. Rule: $3 \times \blacksquare + 5 = \blacktriangle$

■	0	2	4	6	8
▲					

5 11 17 23 29

Generating ordered pairs 289

ACTIVITIES

1. Play the game "High Score".
Make 3 dice labelled: 2, 3, 4, 5, 6, 7.
Make 2 dice labelled: +, -, \times , \div , blank, blank.

Rules: Each player takes turns
throwing the 5 dice.

Player makes a high score with
the 5 dice, e.g.,

$3 \times 5 + 2$. Score: 17.

When a blank comes up, the student
can identify what the operation is.
Player with the highest score after 10
plays is the winner.

(Note: If \div operation cannot be used,
then points are taken off the score,
e.g., $3 \times 5 \div 2$. Since this is not a
whole number and 3×5 is the
highest score possible, and $\div 2$
cannot be used, reduce score by
amount on die not used. Total score is
13 on this throw.)

2. Students, working in pairs,
give each other a one- or two-step rule
and a set of input numbers. The
receiver is to complete the output
numbers. The originator of the puzzle
may use a calculator to check the
outputs.

3. Students prepare an input and
output chart using a secret rule. They
write these charts on cards and
exchange cards to see if others can
identify the rule. (Two-step rules are
extremely hard to identify and
therefore you may wish to restrict the
game to one-step rules.)

4. See "Input-Output" as
described in the Activity Reservoir.

EXTRA PRACTICE

Find the outputs.

Rule	Input (i.e., \square)					
$\square \times 5 + 3$	0	1	2	5	10	15
$\square \div 6 + 9$	0	6	12	18	30	33
$(\square + 3) \times 5$	1	2	3	4	5	10
$(\square + 7) \div 7$	0	7	14	21	49	77
$(\square \div 6) \times 6$	0	1	2	3	6	12
$(\square \times 4) \div 4$	0	2	3	5	8	10

OBJECTIVE

To graph the ordered pairs generated by a function machine

PACING

Level A All
Level B All
Level C All

MATERIALS

graph paper (DM69)

RELATED AIDS

HMS — DM69.

SUGGESTIONS

Initial Activity Review the procedures that you developed on pages 288 and 289 to generate ordered pairs, especially the \blacksquare , \blacktriangle designation for input, output. Use a rule such as $2 \times \blacksquare$ to generate the first five ordered pairs. Once the ordered pairs have been determined, graph the ordered pairs in the same manner that was used for pages 286 and 287.

USING THE BOOK

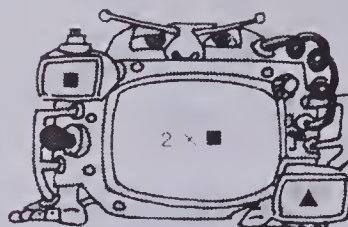
Check the ordered pairs and graph that was demonstrated in the Initial Activity against those shown at the top of the page. Note the designation of the horizontal axis with a \blacksquare and the vertical axis with a \blacktriangle .

The graphs for Exercises 1 and 2 are given to help students get used to labelling the number lines in an acceptable manner. Caution the students to not draw the graph in the text. Distribute graph paper to the students.

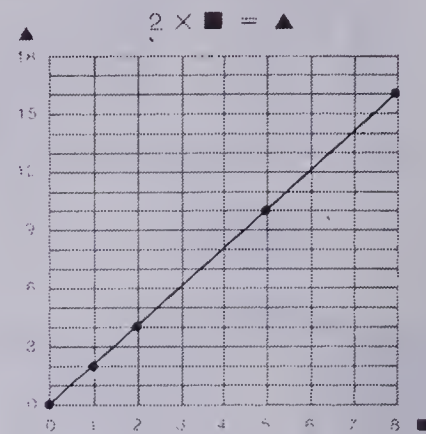
These two pages bring together two skills developed separately earlier: (a) generating ordered pairs, (b) graphing ordered pairs. Hence, students may master these pages rather quickly.

Graphing Points

Complete the table.
Graph the ordered pairs



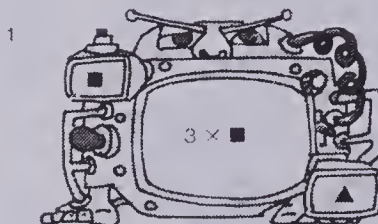
\blacksquare	0	1	2	5	8
\blacktriangle	0	2	4	10	16
Ordered pairs	(0,0)	(1,2)	(2,4)	(5,10)	(8,16)



Draw a line through the points.

Exercises

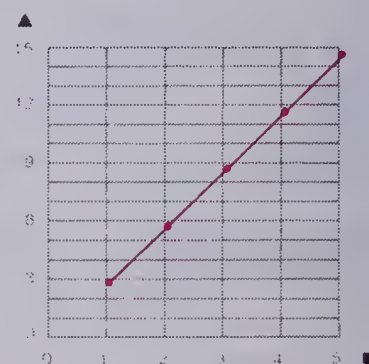
Complete the table:
Graph the ordered pairs.
Join the points.



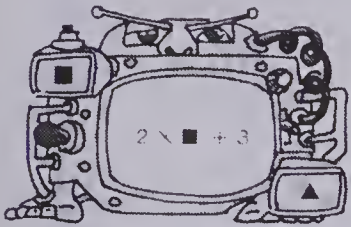
\blacksquare	1	2	3	4	5
\blacktriangle					
Ordered pairs					

3; 6; 9; 12; 15

(1,3) (2,6) (3,9) (4,12) (5,15)

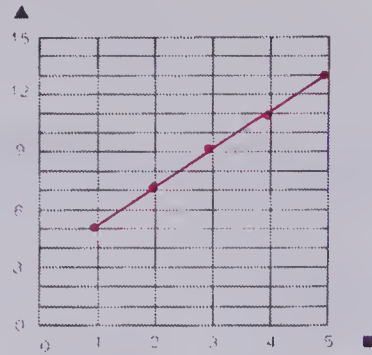


2.

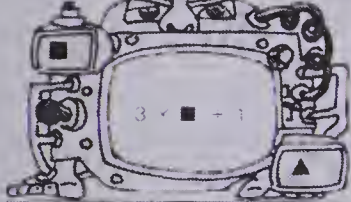


\blacksquare	1	2	3	4	5
\blacktriangle					
Ordered pairs					

5, 7, 9, 11, 13

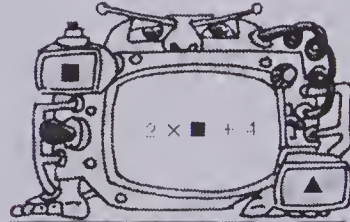
(1, 5), (2, 7), (3, 9),
(4, 11), (5, 13)

3.



\blacksquare	1	2	3	4	5
\blacktriangle					
Ordered pairs					

4, 7, 10, 13, 16

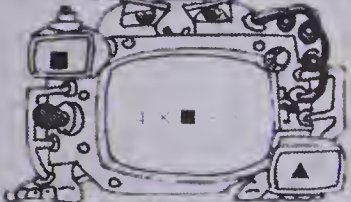
(1, 4), (2, 7), (3, 10),
(4, 13), (5, 16)

\blacksquare	1	2	3	4	5
\blacktriangle					
Ordered pairs					

6, 8, 10, 12, 14

(1, 6), (2, 8),
(3, 10), (4, 12),
(5, 14)

5.

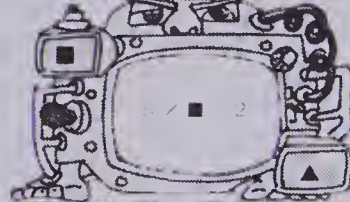


\blacksquare	1	2	3	4	5
\blacktriangle					
Ordered pairs					

1, 5, 9, 13, 17

(1, 1), (2, 5), (3, 9),
(4, 13), (5, 17)

6.



\blacksquare	1	2	3	4	5
\blacktriangle					
Ordered pairs					

3, 8, 13, 18, 23

(1, 3), (2, 8), (3, 13),
(4, 18), (5, 23)

ACTIVITIES

1. Play "Eraser" with a group. The teacher acts as the leader. A part of the body is erased by each student for each error in the output part of the chart. The leader gives the rule and the inputs.

2. Ask the students to play "Radar Scopes" as described in the Activity Reservoir.

3. Each student prepares a graph using a secret rule (and input-output charts). These graphs are glued to tag and identified. The graphs are exchanged and the other students try to determine the rule. (You may need to restrict the rule to one step though some students may appreciate the challenge of a few two-step rules.) Answers can be placed on the reverse side for easy checking.

OBJECTIVE

To use integers on a thermometer

PACING

Level A All
Level B All
Level C 3-7

MATERIALS

a large thermometer or a display thermometer (-20 to 30) with a moveable "red" line

BACKGROUND

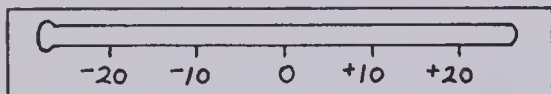
Mathematicians considered these questions:

(1) $3 - 5 = ?$

(2) $\leftarrow \begin{array}{ccccccc} & ? & ? & ? & ? & 0 & 1 & 2 & 3 \dots \end{array} \rightarrow$

(3) $5 + ? = 3$

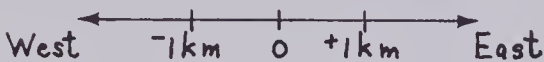
To solve these, mathematicians invented negative numbers. Explain that man often sets up a number scale with 0 as a point on the scale but then he/she wishes to identify points on both sides of the zero. The thermometer is an example.



Zero degrees is the temperature at which water begins to freeze (or ice begins to melt — do not go into this now). A temperature lower than zero definitely means water will freeze, while temperature higher than zero means water will not freeze.

When speaking of temperatures, we refer to "-3" as "minus three". In mathematics "-3" is read "negative three".

Also we often indicate a starting point by zero. Then we may indicate 1 km east by +1, and 1 km west by -1.



This concept of "opposites" is a very important concept to develop.

SUGGESTIONS

Initial Activity Use a demonstration thermometer. Discuss temperatures above zero such as:

30°C (very hot weather)

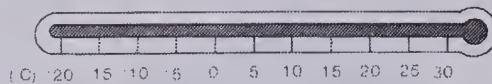
20°C (comfortable)

5°C (cool)

0°C (water freezes).

Temperatures

You have used thermometers like this:



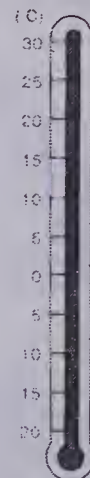
Temperatures greater than 0 are called **positive temperatures**.

Temperatures less than 0 are called **minus temperatures**.

20°C is warmer than -5°C.

-5°C is colder than 10°C.

or like this:



Exercises

1. Which is warmer?

(a) 15°C or 10°C

(b) 10°C or -5°C

(c) 10°C or -10°C

2. Which is cooler?

(a) 20°C or 5°C

(b) 5°C or 10°C

(c) 15°C or 15°C

3. Use "warmer than" or "cooler than" to make these statements true.

(a) 30°C • -10°C

warmer than

(b) 10°C • -20°C

warmer than

(c) -5°C • -15°C

warmer than

(d) -10°C • 5°C

cooler than

(e) 10°C • 20°C

cooler than

(f) -15°C • -10°C

cooler than

4. How many degrees from:

(a) 10°C to 30°C? 20°C

(b) 5°C to 35°C? 30°C

(c) -5°C to 0°C? 5°C

(d) 0°C to 10°C? 10°C

(e) -5°C to 10°C? 15°C

(f) -10°C to 15°C? 25°C

5. The temperature was 10°C. It went up 5°C. What is it now? 15°C

6. The temperature was -10°C. It went up 15°C. What is it now? 5°C

7. The temperature was 5°C. It went down 10°C. What is it now? -5°C

292 Thermometer minus temperatures

Then pose the question: What about temperatures that are colder than zero? How shall we show this? Then, if necessary, provide the answer: We use numbers such as -5, -10, -15, etc. Then print these on the demonstration thermometer (use grease pencil).

USING THE BOOK

Discuss the display at the top of the page. Do the exercises orally with the class as a whole or in small groups. If done in groups, the other children can do some of the activities.

ACTIVITIES

1. Discuss and post temperatures of different places in the world. (If this unit is used in May or June, include

temperatures from the Southern Hemisphere to obtain temperatures below zero.) Discuss the negative temperatures and compare to the low temperatures compared in your region in winter weather.

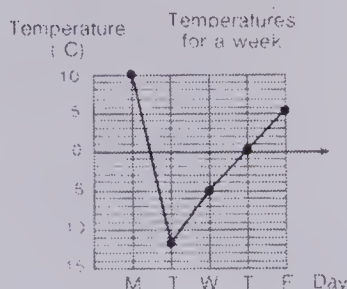
2. Mark 15 temperature cards with numbers between -30°C and +30°C. Students are to arrange the numbers from coldest to warmest.

3. Challenge. The temperature rises 10°C between 05:00 and 11:00. Between 11:00 and 17:00 it rises another 14°C. By 23:00 the temperature has fallen 18°C from what it was at 17:00. How many degrees is it warmer or colder at 23:00 than it was at 17:00? than at 11:00? [18°C colder; 4°C colder]

Graphing Temperatures

Class 6A recorded the temperatures for a week in January.

Day	Mon	Tues	Wed	Thurs	Fri
Temperature (°C)	10	12	5	0	5



Exercises

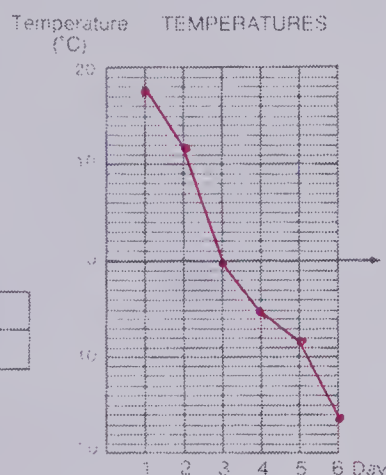
- Which day was the coldest? the warmest? **Monday** **Tuesday**
- How many degrees difference was there in the temperature between:
 - Monday and Tuesday? **22°C**
 - Tuesday and Wednesday? **7°C**
 - the warmest and coldest days? **22°C**

- Draw and label a graph to show these temperatures.

Day	1	2	3	4	5	6
Temperature (°C)	18	12	0	-5	-8	-16

- Draw and label a graph as in Exercise 3. Show this information.

Day	1	2	3	4	5	6	7	8	9	10
Temperature (°C)	15	8	5	6	0	8	-2	3	-10	-18



Graphing means temperatures 293

OBJECTIVE

To graph temperatures using the integers

PACING

Level A All
Level B All
Level C All

MATERIALS

graph paper for students (DM69)

RELATED AIDS

HMS — DM69.

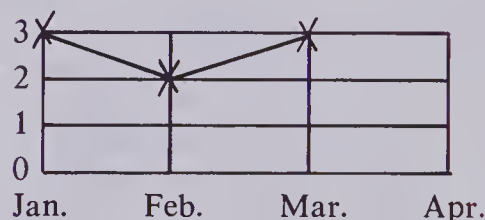
BACKGROUND

We can join the temperature readings on successive days since there are temperature readings at all times between the two points marked. This differs from some graphs such as the one illustrated here.

Example

There are not $2\frac{1}{2}$ students with birthdays between February and March.

Number of Students With Birthdays in Each Month



Line graphs should only be used where there is continuous data between any two points plotted.

SUGGESTIONS

Initial Activity Using 2 cm grid graph paper attached to the chalkboard, ask students to take turns making a graph for the data shown. Illustrate each step in drawing the graph.

- Put on horizontal line and scale. Label.
- Put on vertical line and scale. Label.
- Plot temperatures.
- Join points.
- Write title.

Day	Mon.	Tues.	Wed.	Thurs.	Fri.
Temperature (°C)	+5°	-5°	-20°	-10°	+20°

USING THE BOOK

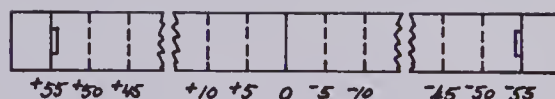
Read through Class 6A's information shown at the top of the page. Relate the data shown in the chart to that pictured in the graph. Emphasize that the information is the same; it is the way of showing the information that is different.

Discuss Exercises 1 and 2 orally using Class 6A's data. Provide graph paper so that the students can complete Exercises 3 and 4.

ACTIVITIES

1. Use the set of cards made in Activity 2 described on page 292. Deal the students 6 cards at random. The student is to identify the month of the year each temperature might be recorded in some city (choose one like Edmonton, Winnipeg, or Sudbury). Have the students draw a graph to show the information.

2. Modify the game "Football" described in the Activity Reservoir. One person chooses the negative end and one plays the positive end.



- Have the student research to find the coldest and warmest temperatures recorded in (a) your province, (b) in Canada, (c) in North America, (d) the world.

EXTRA PRACTICE

- Refer to Exercise 3. What is the difference in temperature between:
 - Day 1 and Day 2? [6°C]
 - Day 1 and Day 3? [18°C]
 - Day 1 and Day 4? [23°C]
 - Day 1 and Day 5? [26°C]
 - Day 1 and Day 6? [34°C]
- Refer to Exercise 4. What is the difference in temperature between:
 - Day 1 and Day 3? [10°C]
 - Day 2 and Day 5? [8°C]
 - Day 5 and Day 8? [3°C]
 - Day 5 and Day 7? [2°C]
 - Day 7 and Day 9? [8°C]
 - Day 9 and Day 10? [8°C]
 - Day 3 and Day 4? [10°C]
 - Day 1 and Day 10? [33°C]

OBJECTIVE

To write and use numbers on the integer number line

PACING

Level A All
Level B All
Level C All

VOCABULARY

integer

MATERIALS

an integer number line, index cards

BACKGROUND

Numbers like -4 and $+4$ are opposites. -4 is to the left of $+4$ on the number line.

-4 is less than $+4$.

Note it is standard practice to always show negative numbers to the left of zero on the horizontal number line. Similarly, negative numbers are below zero on the vertical number line.

The set of integers is made up of positive and negative numbers and zero. Note that zero is neither positive nor negative.

USING THE BOOK

Use an integer number line with only zero shown (draw a line on the chalkboard labelling only 0). Assign each student a number (use index cards). Let each student place his/her number on the number line.

Ask students who had numbers less than $+3$ to raise their hands. Repeat for other numbers. Emphasize that a number to the left or below a given number is the lesser of the two.

Do Exercises 1 to 4 orally either with the class as a whole or with small groups.

ACTIVITIES

1. Play "Bingo" as described in the Activity Reservoir. Have the players write the integers from -12 to $+11$ randomly on their blank grids. As leader, call out expressions such as "6 more than negative 8", "10 less than positive 5", etc.

2. Prepare 2 sets of cards labelled from -10 to $+10$. Shuffle all cards together well. Give one half of the deck to each of 2 players and have them play "flips". Both players (from standing position) simultaneously drop

The Integer Number Line

We can show positive and negative numbers on a number line.

Numbers to the left of zero are **negative** numbers.

Zero is neither **positive** nor **negative**.

Numbers to the right of zero are **positive** numbers.

The *positive* and *negative* numbers along with zero are called **integers**.

Exercises

Copy and complete.

- Use *left* or *right* to make true statements.

-5 is of $+4$
right

$+4$ is of -5
right

-5 is of -10
right

0 is of $+8$
left
- Numbers to the *right* are *greater* than numbers to the *left*.
 Use $>$ to mean *greater than*.
 $-5 > -6$

Numbers to the *left* are *less* than numbers to the *right*.
 Use $<$ to mean *less than*.
 $-7 < -2$

(a) $+10 \bullet +3 >$

(b) $-5 \bullet -2 <$

(c) $-8 \bullet 10 >$

(d) $+8 \bullet -12 >$
- We can show integers on a vertical number line.
Copy and complete.
Use *above* or *below* to make true statements.

-5 0
below

$+5$ 0
above

-5 $+5$
below

$+10$ $+5$
above

-5 10
above

$+5$ -5
above
- Numbers above are *greater* than numbers below.
Numbers below are *less* than numbers above.
Use $>$ or $<$ to mean *greater than* or *less than*.

(a) $-5 \bullet 0 <$

(b) $+5 \bullet 0 >$

(c) $-5 \bullet +5 <$

(d) $+10 \bullet +5 >$

(e) $-5 \bullet -10 >$

(f) $+5 \bullet -5 >$

294 Negative numbers: introduction

1 card each so that it flips several times before reaching the floor (holding cards between thumb and middle finger just before dropping helps). If both cards land face up (i.e., each showing an integer), the player who flipped the greater integer wins all cards on the floor. If one or both cards land face down, players continue flipping more cards till 2 simultan-

eously land face up. The winner is the player with the most cards after a predetermined amount of time.

3. Prepare a challenge such as:
M stands for zero,
N stands for $+1$,
L stands for -1 .

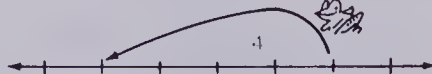
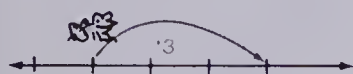
What does (i) Z stand for? (ii) B?
(iii) W? (iv) the first letter of your last name?

Addition on the Integer Line

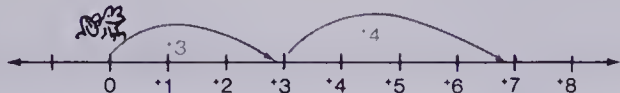
Gurp, the spotted frog, jumped on the integer line.

He jumped this way for $+3$.

He jumped this way for -4 .

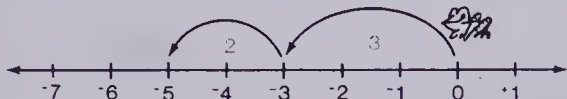


One day he jumped like this. He was 7 units right of where he started.



$$+3 + +4 = +7$$

Another day he jumped like this. He was 5 units left of where he started.

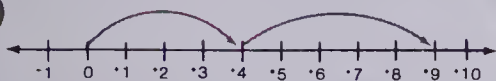


$$-3 + -2 = -5$$

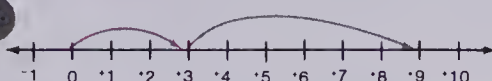
Exercises

Write an integer number sentence for each.

$$+4 + +5 = +9$$

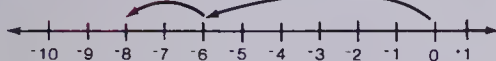


$$+3 + +6 = +9$$



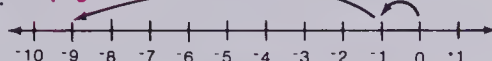
3.

$$-6 + -2 = -8$$



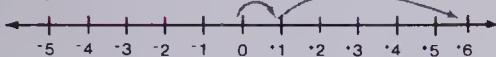
4.

$$-1 + -8 = -9$$



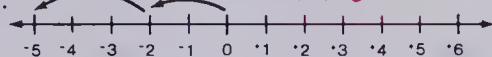
5.

$$+1 + +5 = +6$$



6.

$$-2 + -3 = -5$$



Add. Show each on an integer line.

$$+3 + +7 = +10$$

$$+1 + +6 = +7$$

$$+6 + +3 = +9$$

$$0 + +3 = +3$$

$$-2 + -6 = -8$$

$$-3 + -8 = -11$$

$$-5 + -4 = -9$$

$$0 + -5 = -5$$

$$+14 + +29 = +43$$

$$-19 + -34 = -53$$

$$-42 + -57 = -99$$

$$+23 + +68 = +91$$

$$-82 + -69 = -151$$

$$+84 + +97 = +181$$

$$+106 + +233 = +339$$

$$-431 + -237 = -668$$

Addition of like integers 295

OBJECTIVES

To add 2 positive numbers

To add 2 negative numbers

PACING

Level A 1-18

Level B All

Level C 1, 3, 5, 7-22

MATERIALS

integer number lines

RELATED AIDS

HMS — DM69.

USING THE BOOK

Discuss Gurp and his jumps as indicated in the display at the top of the page. Do Exercises 1 to 6 orally. Have the students draw the integer number lines showing -12 to $+12$ or provide them with a sheet with number lines stencilled. Students need only show the addition on the number line for Exercises 7 to 14. Students are not to use the integer number line to do Exercises 15 to 22. They should be able to find the answer without it.

ACTIVITIES

1. Use the deck of cards made for Activity 2 on page 294. Add more cards and make two decks: one deck of 25 negative numbers and the second deck of 25 positive numbers. Put a 0 card in each deck (but remember 0 is neither positive nor negative). Use one deck at a time to play this game. The students start by drawing 2 cards from one deck and adding the 2 numbers. On the second and subsequent turns, the players draw only one card and add that number to the previous sum. The student with the greatest sum after 10 plays is the winner. (Penalty of 20 points for each error in addition.)

2. Play the game in Activity 1, as a relay game. Form 2 or more teams. The first person of each team draws 2 cards, finds the sum, and tags the next player who draws a card, finds the new sum, and tags the next player, and so on. The first team to complete the relay race is the winner. (Penalty — play moves back 2 players for each error.) Appoint one judge to each team.

3. See "Input-Output" as described in the Activity Reservoir.

OBJECTIVE

To add positive and negative integers

PACING

Level A 1-18

Level B All

Level C 1, 3, 5, 7-22

MATERIALS

integer number lines

RELATED AIDS

HMS — DM69.

USING THE BOOK

Physically acting out the additions will help many students. Draw an integer number line on the chalkboard or floor. Let the students act out the additions shown in the display. Then have another student write the number sentence on the chalkboard.

Do Exercises 1 to 6 orally.

Exercises 7 to 14 may be done physically by those needing the extra help. Students need only use the integer number line as long as they need it.

ACTIVITIES

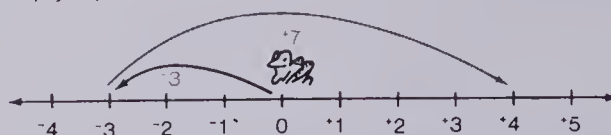
1. Use the 2 decks combined from Activity 1, page 295. Play the game described there, except now sums of this type will occur: $-191 + +28$ and $+89 + -32$.

2. See "Square It" as described in the Activity Reservoir. Replace the numbers on the dot paper with integers.

3. Prepare a challenge such as: If $M = \text{zero}$, $N = +1$, $P = +3$, what is the value of your name? (i.e., $SAM = +6 + -12 + 0 = -6$)

Adding Positive and Negative Integers

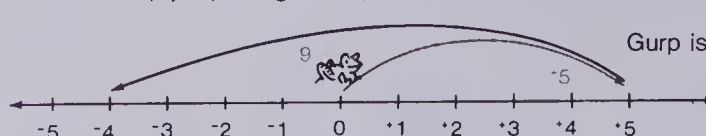
Gurp jumped left first. Then he jumped right.



$$-3 + +7 = +4$$

Gurp is 4 units right of where he started.

This time Gurp jumped right first, then left.

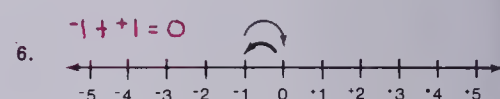
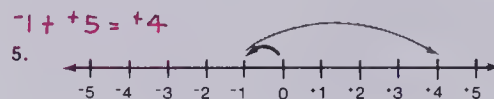
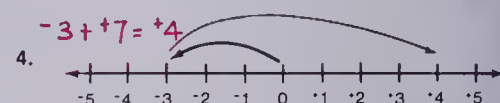
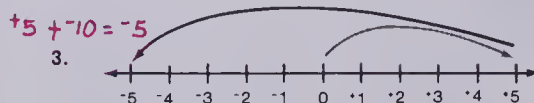
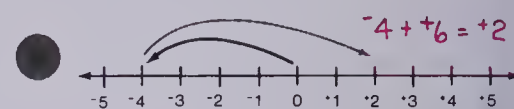
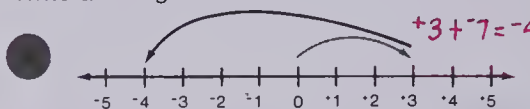


$$+5 + -9 = -4$$

Gurp is 4 units left of where he started.

Exercises

Write an integer number sentence for each.



Add. Show each on an integer line.

1. $+3 + -4 = -1$

8. $+5 + -7 = -2$

9. $+6 + -2 = +4$

10. $+10 + -12 = -2$

2. $-5 + +6 = +1$

12. $-6 + +7 = +1$

13. $-3 + +8 = +5$

14. $-12 + +14 = +2$

15. $+9 + -3 = +6$

16. $-11 + +4 = -7$

17. $-16 + +14 = -2$

18. $+22 + -18 = +4$

19. $-47 + +56 = +9$

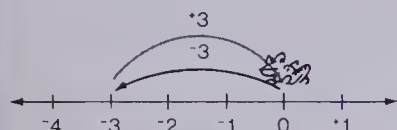
20. $+63 + -72 = -9$

21. $-63 + -47 = -110$

22. $+96 + +104 = +200$

Opposites

Gurp jumped left three units.
Then he jumped right three units.
He was right back where he started.



We say that -3 is the *opposite* of $+3$ and $+3$ is the *opposite* of -3 .

Exercises

Write the opposite of each.

1. $+3$ -3
2. $+7$ -7
3. -2 $+2$
4. -6 $+6$
5. $+9$ -9
6. -11 $+11$
7. $+29$ -29
8. -31 $+31$
9. -104 $+104$
10. $+105$ -105
11. -596 $+596$
12. $+465$ -465

Solve.

13. $-6 + +6 = n$ 0
14. $+12 + -12 = n$ 0
15. $-23 + +23 = n$ 0
16. $+8 + n = 0$ -8
17. $-5 + n = 0$ $+5$
18. $-18 + n = 0$ $+18$
19. $n + -2 = 0$ $+2$
20. $n + +3 = 0$ -3
21. $n + -9 = 0$ $+9$
22. $n + -36 = 0$ $+36$
23. $n + +57 = 0$ -57
24. $-43 + n = 0$ $+43$

Write an integer sentence to solve each problem.

25. In a game, Mark moved ahead 14 spaces. Then he moved back 14 spaces.
What was his total gain? $+14 + -14 = 0$; 0
26. In the same game, Nancy moved back 19 spaces. How many spaces did she have to move forward to get back where she started? $-19 + +19 = 0$; 19

OBJECTIVES

To write the opposite of an integer
To solve an equation using the opposite of an integer

PACING

Level A 1-21
Level B All
Level C 9-26

VOCABULARY

opposites

SUGGESTIONS

Initial Activity Develop the concept of opposites by calling off something like "left" and expect the response "right"; "before" and "after"; "high" and "low"; "gain of 5 metres" and "loss of 5 metres"; "below", "above"; "spend", "earn"; "down", "up".

After a number of these say "+5" and elicit the response "-5". Continue with others such as -7, +4, -23, +4.

USING THE BOOK

You may wish to do the exercises orally with the class as a whole or with groups while the others are engaged in the activities. Emphasize that the sum of two opposites is zero.

Integers and zero 297

ACTIVITIES

1. Prepare an activity card such as:

Make two cubes, one marked 0, +1, +2, +3, +4, +5 and the other marked 0, -1, -2, -3, -4, -5. Throw the two dice and add the numbers. In how many ways can you throw the dice so that the sum is zero? [6] Throw the dice 30 times. Which sums occur most often?

2. Have the children complete the magic squares.

(a)

-2		0
	+1	
		+4

(b)

-3		-7
-1	-6	-5

(c)

+1		
	0	
		-1

How many ways can you find to complete (c)?

3. Have the children complete each.

- (a) In the addition of two positive integers, the sum is [positive].
- (b) In the addition of two negative integers, the sum is [negative].
- (c) In the addition of two opposites, the sum is [zero].
- (d) In the addition of a positive and a negative integer, the sum is [positive or negative].

4. See "Football" as described in the Activity Reservoir.

OBJECTIVE

To subtract with integers

PACING

Level A All

Level B All

Level C 1-15 (odd), 17-21 (odd)

RELATED AIDS

HMS — DM72.

SUGGESTIONS

Initial Activity Review subtracting on the whole number line. Then, use a set of *Loss and Gain* cards consisting of 10 loss cards marked $-10, -9, \dots, -1$ and 10 gain cards marked $+10, +9, \dots, +1$. Give a student (so that the whole group can see) 2 cards, such as $+8$ and -5 . Say: "Each positive card is a gain and each negative card is a loss. What is your total loss or gain?" [gain 3] "I'm going to take away from you a loss card (take the -5). What is your total now? Did your total increase or decrease? Which card might I have given you to bring your total to what it is now?" [$+5$] "Therefore, subtracting -5 is like adding what?" [its opposite, i.e., $+5$].

Repeat the procedure for "taking away a gain of $+8$ " and elicit the conclusion that "subtracting $+8$ is like adding its opposite -8 ."

USING THE BOOK

You may wish to do Exercises 1 to 16 orally discussing thoroughly "subtracting an integer is like adding its opposite."

ACTIVITIES

1. A pair of students uses the Loss and Gain cards described in Initial Activity. Five cards are dealt to each student and totals of the hands are found. The first student removes a card at random from the other's hand and the second gets a new total. The second now draws a card from the hand of the first. This is repeated twice more. The student with the highest remaining total is the winner. (Check: the total should be the sum of the cards remaining.)

2. Use the deck of cards from Activity 1, page 295. Colour at random half of the cards red and form 2 piles, one red and one white. Each student draws 2 cards, one from each pile. The red number is

Subtracting Integers

Compare each pair of equations.

$+4 - +3 = +1$
 $+4 + -3 = +1$

$+4 - +3 = +1$
 $+4 + -3 = +1$

$-5 - +2 = -7$
 $-5 + -2 = -7$

$-5 - +2 = -7$
 $-5 + -2 = -7$

Subtracting an integer is like adding its opposite.

Exercises
Copy and complete each.

1. $+3 - +2 = +1$ $+3 + -2 = +1$	2. $+4 - +5 = -1$ $+4 + -5 = -1$	3. $+2 - +5 = -3$ $+2 + -5 = -3$	4. $+1 - +4 = -3$ $+1 + -4 = -3$
5. $+6 - +2 = +4$ $+6 + -2 = +4$	6. $+5 - +1 = +4$ $+5 + -1 = +4$	7. $+6 - +7 = -1$ $+6 + -7 = -1$	8. $+1 - +3 = -2$ $+1 + -3 = -2$
9. $-3 - +2 = -5$ $-3 + -2 = -5$	10. $-2 - +4 = -6$ $-2 + -4 = -6$	11. $-8 - +5 = -13$ $-8 + -5 = -13$	12. $-10 - +6 = -16$ $-10 + -6 = -16$
13. $0 - +3 = -3$ $0 + -3 = -3$	14. $0 - +6 = -6$ $0 + -6 = -6$	15. $+3 - 0 = +3$ $+3 + 0 = +3$	16. $-7 - 0 = -7$ $-7 + 0 = -7$

★ Use the rule for subtracting integers to do these

17. $+6 - +4 = +10$	18. $+7 - +4 = +11$	19. $+6 - +9 = +15$	20. $+2 - +7 = +9$
21. $-4 - +5 = -9$	22. $-7 - +3 = -10$	23. $-8 - +8 = -16$	24. $-2 - +5 = -7$

subtracted from the white number. After 10 times the player with the most correct is the winner.

3. Have the students complete:

(a) In subtracting a negative number from a positive number, the difference is positive.

(b) In subtracting a positive number from zero, the difference is negative.

(c) In subtracting a negative number from zero, the difference is positive.

(d) In subtracting a positive number from a negative number, the difference is negative.

Graphing Integers

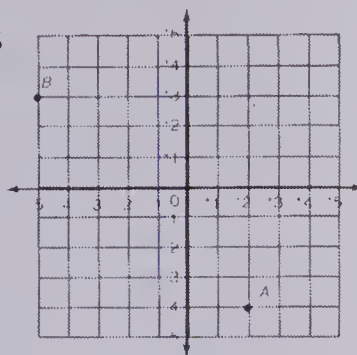
To write the ordered pair for point A.

Think: It is to the *right* of 0 2 units. Write: $(+2, \quad)$

It is *down* from 0 4 units. Write: $(+2, -4)$

The ordered pair is $(+2, -4)$.

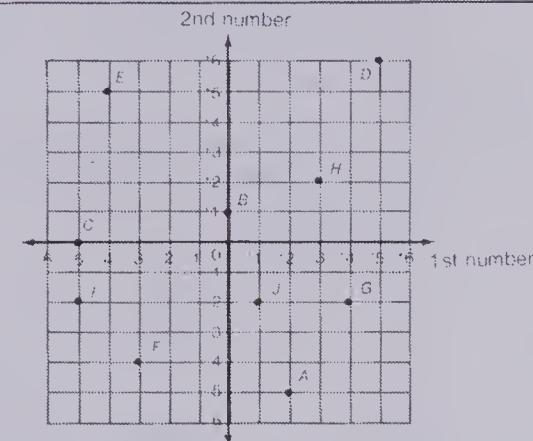
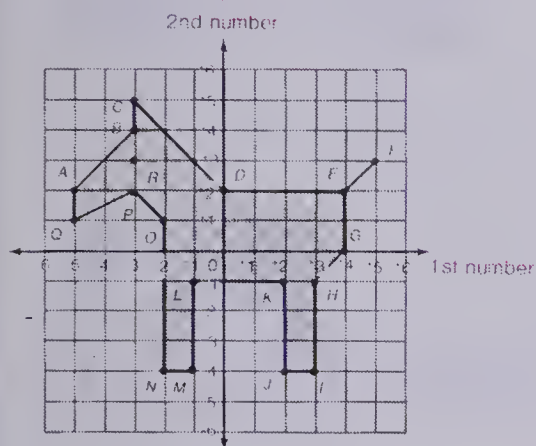
The ordered pair for point B is $(-5, +3)$



Exercises

1. Write the ordered pair for each point.

- | | | | |
|-------|------------|-------|------------|
| (a) A | $(+2, 5)$ | (b) B | $(0, +1)$ |
| (c) C | $(-5, 0)$ | (d) D | $(+5, +6)$ |
| (e) E | $(-4, +5)$ | (f) F | $(-3, -4)$ |
| (g) G | $(+4, -2)$ | (h) H | $(+3, +2)$ |
| (i) I | $(-5, -2)$ | (j) J | $(+1, -2)$ |



2. The graph here shows a dog!

- (a) Find the ordered pair for each point A to Q that has to be joined to the next to draw the dog. The point R is the eye.
- (b) Plot the ordered pairs you find on your own graph paper to check.

Writing ordered pairs for points 299

ANSWERS:

2. (a) A $(-5, +2)$; B $(-3, +4)$; C $(-3, +5)$; D $(0, +2)$; E $(+4, +2)$; F $(+5, +3)$; G $(+4, 0)$; H $(+3, -1)$; I $(+3, -4)$; J $(+2, -4)$; K $(+2, -1)$; L $(-1, -1)$; M $(-1, -4)$; N $(-2, -4)$; O $(-2, +1)$; P $(-3, +2)$; Q $(-5, +1)$; R $(-3, +3)$

OBJECTIVE

To identify and graph points identified by ordered pairs in the four quadrants

PACING

Level A All
Level B All
Level C All

MATERIALS

graph paper (DM69)

RELATED AIDS

HMS — DM69.

BACKGROUND

Pages 299 and 300 deal with essentially the same topic. While the first page deals mostly with identifying points on a four-quadrant grid, the second page is concerned with identification and plotting of points. You may wish to complete these two pages together.

Also, up to this point in the series, ordered pairs such as $(2, 3)$ have been graphed by the "over 2 and up 3" method. Here, you may wish to introduce a new procedure. Illustrate that all points on a line $x = 2$ have an x value of 2 and similarly all points on the line $y = 3$ have a y value of 3. It's where the two lines intersect that the point $(2, 3)$ occurs. Explain also that a point such as $(-3, 5)$ lies somewhere on the line $x = -3$ (draw this line on a chalkboard graph) and *also* somewhere on the line $y = 5$ (draw this line). Elicit from the students that the only point that lies on both lines at once is where the two lines cross (intersect). This point is $(-3, 5)$.

points. Emphasize the similarity of this procedure to that used on page 290.

USING THE BOOK

After discussing the display at the top of the page, direct the students to cover the graph and plot each point on their own graphs. They can check each point after they mark it or wait until they have done all of the exercises on the page.

ACTIVITIES

See those suggested on page 300.

SUGGESTIONS

Initial Activity Review the graphing of integers such as -5 , $+6$, $+8$, -1 on both horizontal and vertical number lines. Review also that numbers to the left of and/or below zero are negative and that those to the right of and/or above zero are positive.

Review if necessary, the graphing of ordered pairs such as $(3, 5)$ as presented on pages 290 and 291.

Draw a horizontal number line on 1 cm graph paper which has been

taped to the chalkboard so that all can see. Again, have volunteers identify several points both to the left and right of zero. Then, superimpose a vertical number line which intersects the horizontal one at zero. Ask the children to now identify a point such as -8 or $+5$. Point out that this is very confusing because there are two places which could be identified for each integer — one on the horizontal and one on the vertical number lines. Discuss the use of ordered pairs such as $(0, -8)$, $(+5, 0)$, and $(-5, +3)$ to identify

OBJECTIVE

To identify and graph points identified by ordered pairs in the four quadrants

PACING

Level A All
Level B All
Level C All

MATERIALS

graph paper (DM69)

RELATED AIDS

HMS — DM69 and DM73.

BACKGROUND

If you have not already done so, see the Background notes on page 299.

SUGGESTIONS

Initial Activity Review if necessary, the identification of points in the four quadrants using ordered pairs as presented on page 299.

USING THE BOOK

Exercises 1 and 2 are self checking in that the "pattern" will be amiss if a point is plotted incorrectly.

ACTIVITIES

1. See "Quadrant" as described in the Activity Reservoir.
2. See "Radar Scopes" as described in the Activity Reservoir.
3. Challenge the student to draw a pattern or picture on graph paper, then write ordered pairs for each point. The ordered pairs are then given to other classmates as challenges.

EXTRA PRACTICE

What is the message?

1st letter: $(-10,4)$, $(-10,8)$, $(-8,6)$, $(-6,8)$, $(-6,4)$.

2nd letter: $(-4,4)$, $(-2,8)$, $(0,4)$. Lift pencil. $(-1,6)$, $(-3,6)$.

3rd letter: $(-1,8)$, $(3,8)$. Lift pencil. $(1,8)$, $(1,4)$.

4th letter: $(4,8)$, $(4,4)$. Lift pencil. $(4,6)$, $(7,6)$. Lift pencil. $(7,8)$, $(7,4)$.

5th letter: $(-4,3)$, $(-4,-1)$.

6th letter: $(0,3)$, $(-2,3)$, $(-2,1)$, $(0,1)$, $(0,-1)$, $(-2,-1)$.

7th letter: $(-5,-3)$, $(-7,-3)$, $(-7,-5)$, $(-5,-5)$, $(-7,-5)$, $(-7,-7)$.

8th letter: $(-4,-3)$, $(-4,-7)$, $(-1,-7)$, $(-1,-3)$.

9th letter: $(1,-7)$, $(1,-3)$, $(3,-7)$, $(3,-3)$.
[MATH IS FUN]

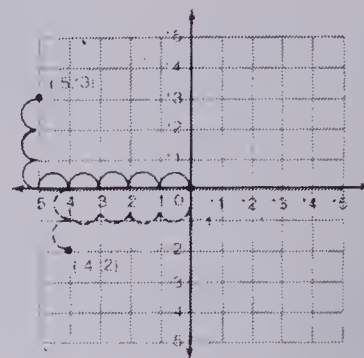
Graphing More Integers

To locate the point $(-5,3)$:

Start at 0, move *left* 5 units (-5), then move *up* 3 units $(-5,3)$.

To locate the point $(-4,-2)$:

Start at 0, move *left* 4 units (-4), then move *down* 2 units $(-4,-2)$.



Exercises

1. Decode the message.

A STITCH IN TIME SAVES NINE

$(-6,1)$, $(-2,-2)$, $(0,0)$, $(-6,-2)$, $(0,0)$

$(-3,4)$, $(-2,-2)$, $(-6,-2)$, $(0,6)$

$(0,0)$, $(-6,-2)$, $(-5,-5)$, $(-6,-5)$

$(-2,-2)$, $(-6,1)$, $(-4,-5)$, $(-6,-5)$, $(-2,-2)$

$(0,-6)$, $(-6,-2)$, $(0,-6)$, $(-6,-5)$

2. Draw the Red-Tail Road Runner.

Start: $(-2,-4)$, $(-1,-1)$, $(-5,-4)$, $(-6,-4)$, $(-8,-3)$.

$(-7,-4)$, $(-6,-7)$, $(-5,-7)$, $(-3,-5)$, $(-2,-2)$.

$(0,-3)$, $(-2,-3)$, $(-3,-2)$, $(-7,-6)$, $(-6,-6)$, $(-7,-7)$.

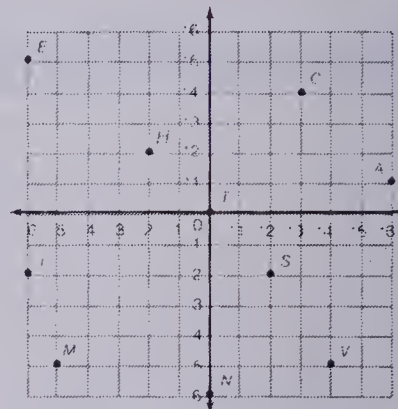
$(-8,-6)$, $(-7,-6)$, $(-4,-1)$, $(-4,0)$, $(-1,-1)$, $(-1,-6)$.

$(-1,-7)$, $(0,-7)$, $(-1,-6)$, $(0,-1)$, $(-2,-3)$.

$(-4,-5)$, $(-4,-6)$, $(-5,-6)$, $(-2,-4)$. Lift pencil.

Put a large dot with a ring around it for an eye at $(-6,6)$.

Colour your Red-Tail Road Runner



Mathematicians

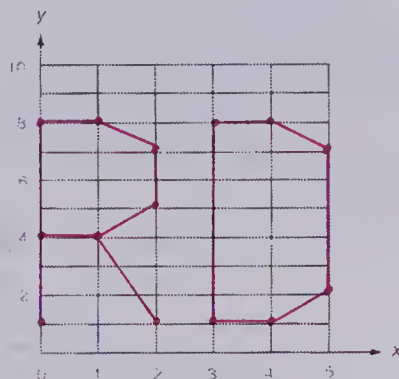
Today's mathematicians often use ideas of great mathematicians who lived many years ago.

1. René Descartes (1596-1650) developed the system of graphing, using ordered pairs. It is called the **coordinate system**.

Copy the information from the charts and graph it, using the coordinate system.
Join the points.

x	0	0	1	2	2	1	0	1	2
y	1	8	8	7	5	4	4	4	1

x	3	3	4	5	5	4	3
y	1	8	8	7	2	1	1



2. Sir Isaac Newton (1642-1727) discovered that objects fall at a certain rate. To find how far (in metres) an object falls in a certain amount of time (in seconds), we can use this rule:



Use this rule to complete this table:

T (s)	0	1	2	3	4
D (m)					

0; 4.9; 19.6; 44.1; 78.4

★ How far would an apple drop in 0.5 s? **1.225 m**

Practice 301

OBJECTIVES

To solve problems
To learn about two famous mathematicians

PACING

Level A 1, 2
Level B All
Level C All

VOCABULARY

René Descartes, Isaac Newton,
coordinate system

MATERIALS

graph paper (DM69)

RELATED AIDS

HMS — DM69.

BACKGROUND

Introduce the page with any of the number of anecdotes about these famous mathematicians:

(a) Much of Descartes' works were not published openly for they contradicted the teachings of the day, especially that of the church. Yet, it was while he was tutoring the Queen of Sweden that he suffered severely of the cold and hard work and died. Royalty accepted his views but the scientific and religious world was more hesitant.
(b) Newton is attributed with the saying "If I have seen a little farther than others, it is because I have stood on the shoulders of giants." (Among the giants were Descartes, Kepler, and Galileo.) Discuss what this quotation means.

Ask: How many years older was Descartes than Newton? How old was Newton when Descartes died? How many years did Newton live after Descartes died?

USING THE BOOK

Some students may need help with this page in that there are two different tasks involved. Work with individuals or small groups of students with common problems.

ACTIVITIES

1. Have the children play the game of "Tic-Tac-Toe" as described in Activity 1, pages 286 and 287. Use four quadrants.

2. Ask the students to use the library to learn one more thing about one (or more) of the mathematicians or about one of the modern-day mathematicians.

3. Challenge the more able students to graph the data in the table of Exercise 2. Discuss the meaning of the graph.

EXTRA PRACTICE

Karl Friedrich Gauss (1777-1855), as an elementary-school child, found the

sum of the numbers 1 to 100 very quickly in his head. He explained it to his teacher this way:

"1 + 100 = 101, 2 + 99 = 101, 3 + 98 = 101, ..., 49 + 52 = 101, 50 + 51 = 101. There are 50 such pairs so $50 \times 101 = 5050$."

Find the sum of these numbers.

- (a) 1 to 10 [55]
- (b) 1 to 20 [210]
- (c) 1 to 30 [465]
- (d) 1 to 40 [820]
- (e) 1 to 50 [1275]
- (f) 1 to 200 [20 100]

- ★(g) 11 to 20 [155]
- ★(h) 11 to 30 [410]
- ★(i) 501 to 600 [55 050]

OBJECTIVE

To interpret and draw pictographs

PACING

- Level A All
- Level B All
- Level C 2, 3

RELATED AIDS

CALC. ACTIVITY MASTERS — 42.

USING THE BOOK

Discuss the advantages of graphs (over number charts):

- (1) the reader gets the message quickly,
- (2) the reader need not “read” detail,
- (3) relative position of an item amongst the other items is more likely remembered than are numbers (comparison).

Discuss the disadvantages.

- (1) not exact,
- (2) does not give detail,
- (3) might be misunderstood.

Discuss the display and the parts of a pictograph:

- (a) title,
- (b) key,
- (c) labels.

Do Exercise 1 orally. Assign the exercises.

ACTIVITIES

1. Ask each student to bring a pictograph to school that they may find in newspapers and magazines. Have each student explain his/her graph. Then make a bulletin-board display of pictographs selected from those brought to class.
2. Ask the students to make a pictograph of something that interests them. Students may work in pairs or small groups. Display a selection of graphs. Discuss the graphs with the class as a whole.
3. Ask the students to make pictographs on some aspects of the energy crisis. The pictographs may relate to ways energy is wasted or conserved or harnessed as well as the extent of the nonrenewable energy sources.

EXTRA PRACTICE

Construct a pictograph to show the data.



























The number of hikers using the trails each month are shown:

Egypt Lake Trail	300	Elk Lake Trail	425
Parker Ridge Trail	650	Sulphur Mt. Trail	775
		Gold Creek Trail	950

Pictographs

We often use pictures in graphs to give information.

Number of Students in Schools in Eagleville










Schools	Enrolment	
Churchill	        	900
Parkhill	      	650
Mt. Royal	     	575
Sunnyside	   	325

Key:  = 100 students



Exercises

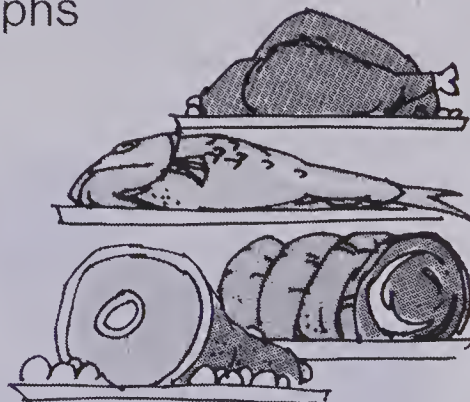
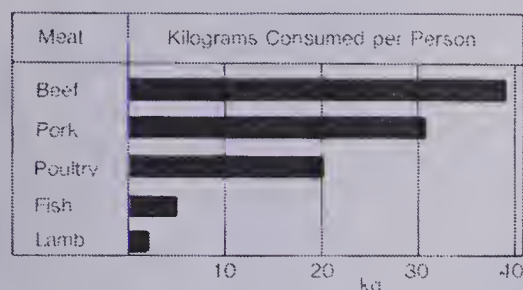
Refer to the display.

1. (a) How many students does each symbol stand for?
(i)  100 (ii)  50 (iii)  25 (iv)  12.5
(b) Which school has the most students? the least? *Churchill* *Sunnyside*
(c) How would you show 175 students? *See pictograph.* (d) What is the title of the pictograph? *Number of Students in Schools in Eagleville*
(e) How many students in each school? *See pictograph.* (f) How many students in all four schools? *2450*
2. The school band had a campaign to raise funds. They sold boxes of chocolate almonds. In September they sold 4000, in October 1500, in November 3750, and in December they sold 5250. Draw a pictograph to show the information. Give the graph a title. Use  to show 1000 boxes. What would each show:  750,  500,  250?
3. A car-manufacturing plant produced 3000 cars in July, 250 in August, 1250 in September, and 4750 in October. Show the data in a pictograph. Be certain to show all the parts necessary to make a good pictograph. Use  = 500 cars.

Bar Graphs

Bar graphs are used to compare data

Canada — Amount of Meat Consumed



Exercises

Use the display.

- (a) Which meat is eaten most often? least often? **Beef** **Lamb**

(b) About how many times more beef is eaten than poultry? than fish? **2** **8**

(c) About how many times more pork is eaten than fish? than lamb? **6** **15**

(d) About how many kilograms of pork are eaten per person annually? **31 kg**

(e) Do you think a similar graph drawn for Australia would be the same? Why? **No** **Lamb would be popular in Australia.**
- The number of foreign-made cars sold in Canada in a year is listed: Toyota — 33 000; Datsun — 28 000; Volkswagen — 22 000; Fiat — 7500; Honda — 18 000; Others — 6000.

Draw a bar graph to show the data. Label fully.
- Use the library. Collect data to make a bar graph of the lengths of these rivers: Amazon, Nile, Mississippi, Mackenzie, Volga, and St. Lawrence.
- Use the library. Collect data to make a bar graph of the population of these cities: New York; London, England; Tokyo; Montreal; Paris; and Hong Kong.

Bar graphs 303

OBJECTIVE

To interpret and to draw bar graphs (from unorganized data)

PACING

Level A 1-3
Level B All
Level C 1, 3, 4

MATERIALS

graph paper (DM69)

RELATED AIDS

HMS — DM69 and DM74.

USING THE BOOK

Elicit from the students what messages they get from the graph in the display, e.g.,

- more beef than any other meat eaten,
- less mutton eaten than fish, etc.,
- each Canadian eats about 20 kg of poultry.

Do Exercise 1 orally. Assign Exercise 2. Discuss the steps in drawing a bar graph — an important step is determining the scale to be used. Assign Exercises 3 and 4. Provide assistance if necessary. (If your class has difficulty, you may wish to do Exercise 2 as a group project.)

ACTIVITIES

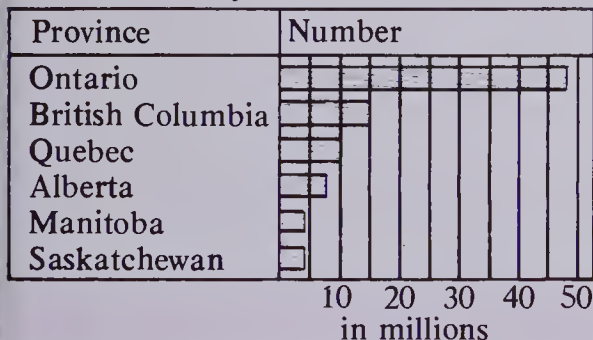
Ask the students to show the same information collected in Activities 1, 2, or 3 on page 302 in bar-graph form.

half of the males?

- How many female taxpayers are there in the age group 45-54?
 - How many male taxpayers are there between 25-44?
 - Why is a double bar used?
3. Draw a double bar graph to show the number of boys and girls in each of 5 classes in your school.

EXTRA PRACTICE

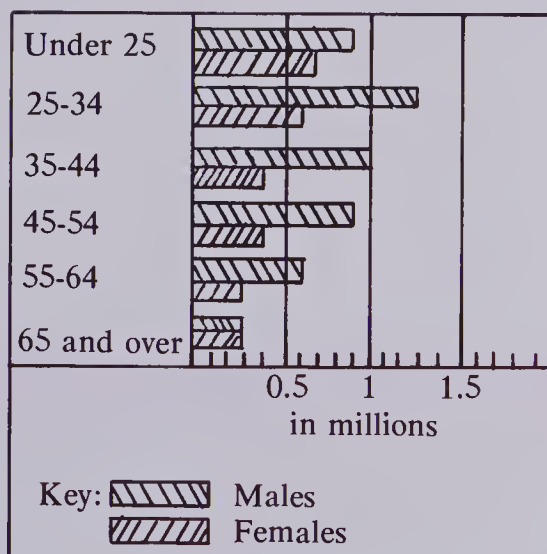
1. Books Circulated in Public Libraries By Provinces



- What is the graph about?
- Which province lends the most books? the least?
- How many books are loaned by Manitoba? B.C.?
- Why would you expect that more books would be loaned in Ontario than in any other province?

2. Sometimes a double bar graph is used.

Number of Taxpayers By Age and Sex



- What two sets of facts are compared?
- How many male taxpayers are there under 25 years?
- In which age group is the number of females about one

OBJECTIVE

To interpret and to draw a broken line graph

PACING

Level A All
Level B All
Level C All

MATERIALS

graph paper

VOCABULARY

broken line graph (DM69)

RELATED AIDS

HMS — DM69.

BACKGROUND

Different graphs are used for different purposes. Bar graphs are generally used for comparisons. Broken line graphs are usually used to show trends.

USING THE BOOK

Discuss the display at the top of the page by doing Exercises 1 to 7 orally. Assign Exercises 8 to 13.

ACTIVITIES

1. Have the students record and show by a broken line graph the temperatures either every hour from beginning of school day to lunch time or every day at noon for a week.

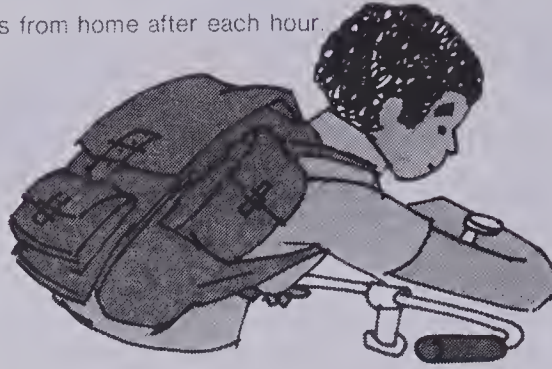
2. Have students bring in examples of broken line graphs collected from newspapers and magazines. Have the students explain the trend indicated by each. Display a selection of the graphs.

3. Ask the students to collect data and make a broken line graph of something that interests them. Remind them that broken line graphs usually are used to show trends of how things are happening.

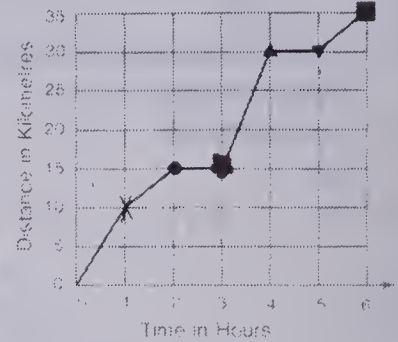
Time and Distance

A broken line graph is often used to show *change* over several periods of time.

Tewari went on a bike trip. He recorded the distance he was from home after each hour.



Distance Tewari Was from Home



Exercises

Refer to the graph above

1. After 1 h he was 10 km from home. He placed an X at the point (1,10). What does (1,10) represent? *1h and 10 km*
2. After 2 h he was 15 km from home. He placed a ● at a point to represent this. What is the ordered pair for the point ●? *(2,15)*
3. How far was he from home after 3 h? *15 km*
4. How far was he from home after 4 h? *30 km*
5. What does the point marked ▲ represent? *(4,30) After 4 h he was 30 km from home.*
6. When did Tewari rest on the trip? *3rd hour*
7. In which hour did Tewari travel the greatest distance? *4th hour*

BRAINTICKLER

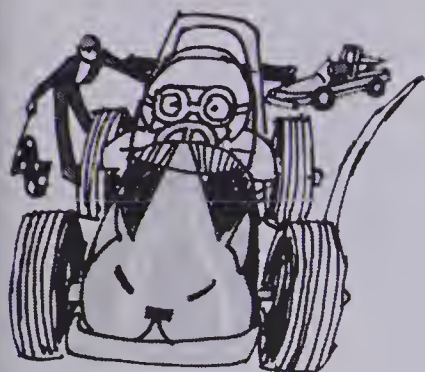
A **palindrome** reads the same forward as backward. The years 1661, 1771, and 1881 are palindromes. What years are the next *three* palindromes?

304 Line graphs

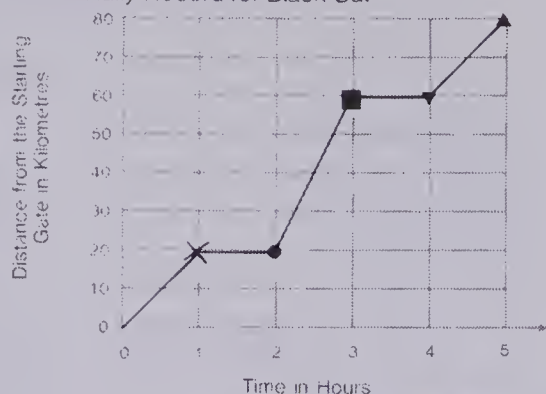
ANSWERS:

Braintickler: 1991, 2002, 2112

During a car rally Black Cat's progress was recorded on this broken line graph.



Car Rally Record for Black Cat



8. How far had Black Cat travelled in (a) 1 h? *20 km* (b) 2 h? *20 km*
9. How far did Black Cat go in the 5th hour? *20 km*
10. Black Cat had a breakdown after 1 h.
How does the graph show this?
By a horizontal line from (1,20) to (2,20)
11. Black Cat had a second breakdown.
When did the second breakdown occur?
4th hour
12. During which two hours did Black Cat travel the farthest? *Hour 2 and hour 3*
13. (a) Draw a broken line graph similar to the one above for The Blue Streak in the same car rally.

Time (h)	0	1	2	3	4	5	6	7	8
Distance from Starting Gate (km)	0	10	60	80	80	140	160	160	200

- (b) When did the driver stop to rest? *4th hour and 7th hour*

Line graphs 305

EXTRA PRACTICE

1. Construct a broken line graph to show the trend in this data.

Population of Canada

1861	3 500 000
1871	4 000 000
1881	4 500 000
1891	4 900 000
1901	6 000 000
1911	7 400 000
1921	8 900 000
1931	10 500 000
1941	11 900 000
1951	14 000 000
1961	18 000 000
1971	22 000 000
1981	24 500 000

- ★2. In Alberta there has been a marked population movement from the country to the city.

Percent of Total Population Living in the Country and in the City

	1906	1916	1926	1936	1946	1956	1966	1976	1986
Country	69	62	62	62	56	44	31	28	
City	31	38	38	38	44	56	69	72	

- (a) Show this data on one graph using two broken line graphs.
- (b) Can you account for
 - (i) the level period from 1916 to 1936?
 - (ii) the sudden climb in the city percentages in the forties, fifties, and sixties?
- (c) Why do you think there is a tapering off of movement to cities in the late sixties and early seventies?
- (d) About what year was there an even population mix between the country and city areas?
- (e) If the trend continues, what percent of the people will live in cities in 1986?

OBJECTIVE
To interpret circle graphs

PACING
Level A 1-11
Level B All
Level C 6-12

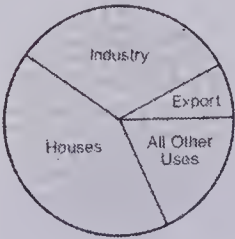
RELATED AIDS
HMS — DM69.
CALC. ACTIVITY MASTERS — 54.

SUGGESTIONS
Initial Activity Tune Up. Find N.
50% of 30 = N 40% of 60 = N
30% of 80 = N 80% of 20 = N
90% of 70 = N 10% of 10 = N
35% of 20 = N 60% of 85 = N

USING THE BOOK
Emphasize that a circle graph allows the reader to quickly make comparisons regarding the information being presented. As an example, in the first circle graph, more lumber is used in house building than in any other use shown.
Do Exercises 1 to 9 orally eliciting responses from the students. Assign the balance.

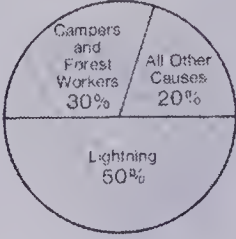
Circle Graphs

How Lumber Is Used



The largest use of lumber is in house construction

Causes of 380 Forest Fires in Blue Lake Park



Lightning is the biggest cause of forest fires in this park.

- Exercises**
We can use circle graphs to *compare* and *find* facts quickly.
1. According to the graph, what happens to the smallest portion of the lumber? *It is exported.*
 2. About what portion of all the lumber is used in industry? *$\frac{1}{3}$*
 3. What does "all other uses" mean? List some other uses
 4. The amount of lumber used altogether in "all other uses" and "export" is about the same as what other use? *Industry*
 5. Approximately what portion of the lumber is used in houses? *$\frac{1}{2}$*
 6. What portion of all forest fires is caused by lightning? *50%*
 7. What is meant by "all other causes"? List some other causes.
 8. About what portion of all forest fires is caused by campers and forest workers? *30%*
 9. *How many* forest fires were caused by lightning? *190*
by all other causes? *76*

ACTIVITIES

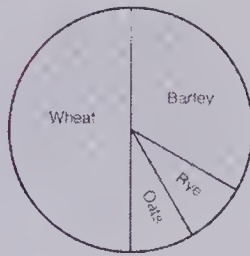
1. Have students bring in examples of circle graphs collected from newspapers and magazines. Have students explain the comparisons made by the circle graphs.

2. Ask students which type of graph they prefer to read — of those used in this chapter. Ask them to tell why.

3. In one township, the use of land is distributed as shown. Have the students draw the circle graph to show this. (*Hint: 40% of 360 gives the number of degrees for the sector angle for urban use.*)

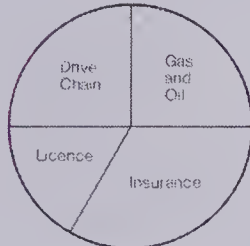
Forests — 20%
Roads — 10%
Mining — 5%
Farming — 25%
Urban — 40%

10. (a) What is the circle graph about? *Farmer Elias' income from crops.* Farmer Elias' Income from Crops
- (b) From which crop does Farmer Elias have the largest income? *Wheat*
the smallest income? *Rye and oats*
- (c) About one third of the income is from which crop? *Barley*
- (d) About what portion of the income is from wheat? $\frac{1}{2}$
- (e) What two crops yield about the same income? *Rye and oats*



11. (a) What portion of all cost was gas and oil? $\frac{1}{4}$
- (b) About how much did he spend on a drive chain? on oil and gas? *\$9*
- (c) What fraction of all cost was the licence? insurance? *\$12*
\$6
- (d) What fraction of all cost were the licence and insurance together? $\frac{1}{2}$

Jay's Cost of Operating His Motorbike



Total cost is \$36

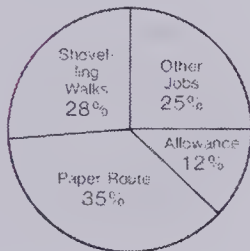
12. Copy and complete the table.

Source	Amount
Allowance	<i>\$ 48</i>
Paper route	<i>\$ 140</i>
Shovelling sidewalks	<i>\$ 112</i>
Other jobs	<i>\$ 100</i>
Total	<i>\$400</i>

How can you check your work?

Total amount should be \$ 400.

Debbie's Income



Total income is \$400.

OBJECTIVE

To locate places on a globe using the ordered pair (longitude, latitude)

PACING

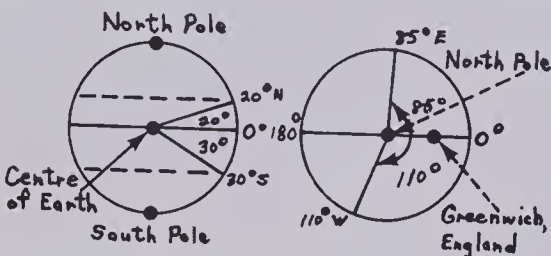
Level A Optional
Level B Optional
Level C All

MATERIALS

globes and/or world maps utilizing Mercator projection

BACKGROUND

Lines of longitude are numbered east and west from the Prime Meridian on which Greenwich, England is located. Lines of latitude are numbered north and south of the Equators. Degrees of latitude and longitude are measured from the centre of the earth. Hence, the latitudes are numbered to 90°N and 90°S . Whereas the lines of longitude are numbered to 180°E and 180°W .

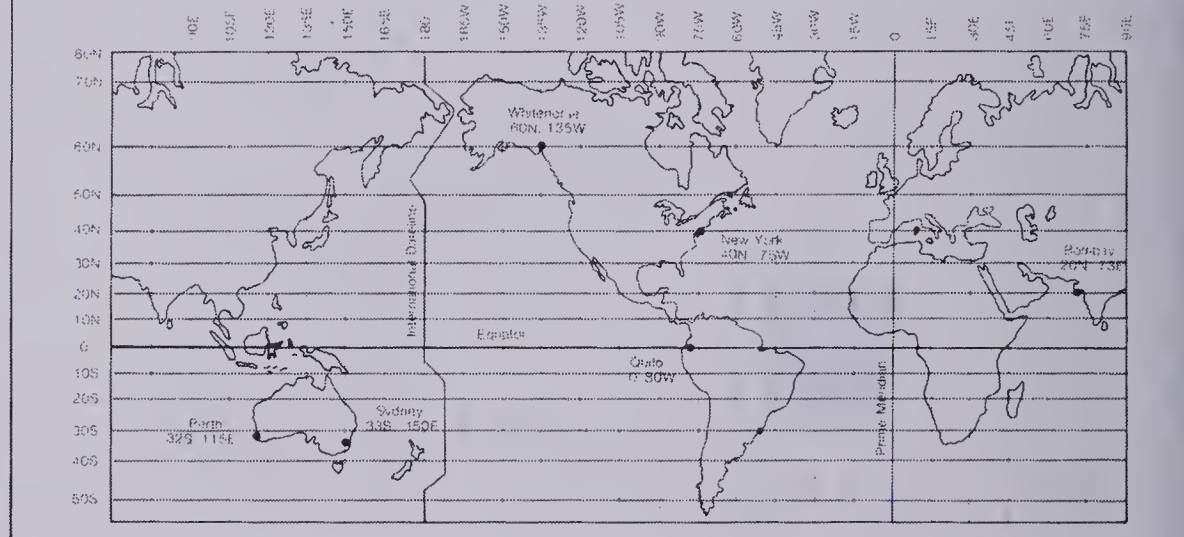


Latitude and Longitude



A globe is a model of the earth. To help locate points on the earth, geographers have agreed to draw lines of **longitude** (blue) and **latitude** (red) on the model to form a grid.

This map of the world has been transformed onto a flat surface. The lines of longitude and latitude have been drawn in colour:
longitude — blue
latitude — red.



308 Longitude and latitude

USING THE BOOK

If only one globe is available, it might be best to rotate groups through this exercise with the other groups doing the Chapter Test and Cumulative Review, or some extension topics. Though the globe and map on page 308 show both longitude and latitude, many globes show only occasional lines of longitude in an attempt to keep their information legible. Keep this in mind when selecting globes for use with this exercise. It is best to have the students working in groups, each with a globe. Failing this, a large map of the world drawn with Mercator projection — similar to that in the display on page 308 — will be satisfactory. Have the students read the information on page 308. Either on the globe or map, or, in the text, have the students move their fingers along the line of longitude: 30°E , 60°W , etc. Then have them move them along the line of latitude: 10°N , 20°S , 50°N , etc. Ask why the lines of latitude are often called “parallels of latitude”.

Exercises

- The equator is a line of latitude. What is its number? **0**
- As we leave the equator and move toward the North Pole, the lines of latitude are numbered 10N, 20N, What does the N indicate? **North**
- As we leave the equator and move toward the South Pole, the lines of latitude are numbered 10S, 20S, What does the S indicate? **South**
- Name a place on (a) 20N, (b) 30S, (c) 60N, (d) 32S, (e) Equator.
- Lines of longitude go from the North Pole to the South Pole.
Longitude lines are numbered starting with 0.
Name two countries through which the zero longitude line passes. **Possible answers: England, France, Spain, etc.**
- As we go east from the 0 line, the longitude lines are numbered 15E, 30E,
What does the E indicate? **East**
- As we go west from the 0 line, the longitude lines are numbered 15W, 30W,
What does the W indicate? **West**
- How high are the lines of longitude numbered? **180** How high are the lines of latitude numbered? **80**
- Name a place on each line.
(a) 75W (b) 115E (c) 30E (d) 135W (e) 150E
- Locate these places.
(a) Moscow — 55N, 40E (b) Ottawa — 45N, 75W
(c) Fiji Islands — 20S, 179E (d) Rio de Janeiro — 23S, 45W
- What city is identified by each location?
(a) 60N, 135W **Whitehorse** (b) 40N, 75W **New York** (c) 20N, 73E **Bombay**
(d) 30N, 30E **Cairo** (e) 30S, 30E **Durban** (f) 0, 80W **Quito**

Activity

- Use a globe or map on which longitude and latitude lines are drawn. Work with a partner.
Name a city or place with its longitude and latitude (20N, 75E).
Challenge your partner to find it in 1 min. Take turns.
The person locating the most places is the winner. Make your own rules.

Then ask them to move their fingers along longitude 75W and latitude 40N until their fingers meet. What city is located?

Do Exercises 1 to 9 orally in class or small groups. Assign Exercises 10 and 11.

ACTIVITIES

- Have the students try the Activity suggested at the bottom of page 309.
- Plan an "Around the World Trip" identifying the places you would like to visit by using ordered pairs of (longitude, latitude). Then give your itinerary to a classmate who is to list the places you would like to visit.
- Research the history of longitude and latitude.

ANSWERS:

- (a) Bombay (b) Durban (c) Whitehorse (d) Perth (e) Quito
- (a) New York (b) Perth (c) Cairo (d) Whitehorse (e) Sydney

OBJECTIVE

To evaluate achievement of the chapter objectives

PACING

- Level A All
- Level B All
- Level C All

MATERIALS

graph paper (DM69)

RELATED AIDS

HMS — DM69 and DM75.

USING THE BOOK

Each student should do this test independently under supervision. Assistance should be given only when the instructions are not understood. After the work has been corrected, you should provide appropriate remedial work. You may wish to reteach if a large number of students had difficulty with a particular topic or concept.

The following chart will help in this regard. The specific objectives are listed in the Chapter Overview (see page 276).

An alternate Chapter Test can be found in the Holt Mathematics System Duplicating Masters available for use with this grade level.

Test Item	Objective	Text Page Number
1-4	A	278, 279, 282, 283
10, 11	B	277, 281
12, 13	C	288, 289
14	D	299
15, 16	E	302-307
5-9	F	296, 298

Chapter Test

Write related sentences to solve each.

1. $N + 5 = 9$
 $N = 9 - 5, N = 4$
2. $N - 6 = 13$
 $N = 13 + 6, N = 19$
3. $N \times 4 = 48$
 $N = 48 \div 4, N = 12$
4. $N \div 3 = 12$
 $N = 12 \times 3, N = 36$

Calculate.

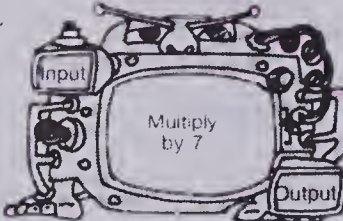
5. $+3 + -5 = -2$
6. $-5 + -7 = -12$
7. $-4 + +9 = +5$
8. $+3 - +2 = +1$
9. $+5 - -4 = +9$

Graph the solutions

10. $M + 6 = 9.5$
11. $N - 0.3 = 20$

Complete each table.

12.



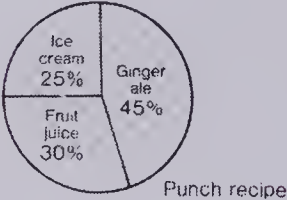
Input	0	1	2	5	10
Output					

0 7 14 35 70

14. Draw and label a graph to show these ordered pairs.

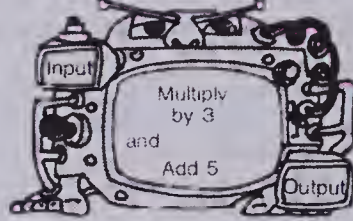
(5,2), (4,-3), (-2,5).
(0,-4), (-2,-5), (-4,0)

16.



Brian wants to make 8 L of fizzy-float. How much ice cream does he need? 2 L

13.



Input	0	1	2	5	8
Output					

5 8 11 20 29

15. Draw a broken line graph to show these data recorded at a car rally.

Log Record of Grasshopper						
Time (h)	0	1	2	3	4	5
Distance from Starting Gate (km)	0	80	80	180	300	450

Cumulative Review

Add.

$$\begin{array}{r} 1. \quad 23 \\ 645 \\ 56 \\ + 3479 \\ \hline 4203 \end{array}$$

$$\begin{array}{r} 2. \quad 11.45 \\ 3.57 \\ 4.03 \\ + 38.59 \\ \hline 57.64 \end{array}$$

Subtract.

$$\begin{array}{r} 3. \quad \$4651.28 \\ - 1976.49 \\ \hline \$2674.79 \end{array}$$

$$\begin{array}{r} 4. \quad 8.103 \\ - 0.698 \\ \hline 7.405 \end{array}$$

Multiply.

$$\begin{array}{r} 5. \quad 259 \\ \times 36 \\ \hline 9324 \end{array}$$

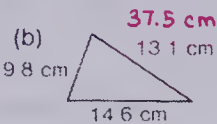
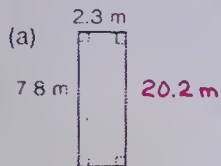
$$\begin{array}{r} 6. \quad 852.1 \\ \times 1.6 \\ \hline 1363.36 \end{array}$$

Divide.

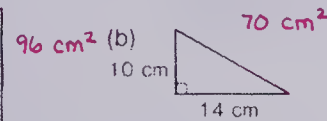
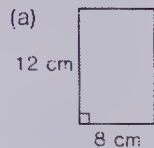
$$7. \quad 2.5 \overline{)75} \quad \begin{array}{l} 30 \end{array}$$

$$8. \quad 2.3 \overline{)12.88} \quad \begin{array}{l} 5.6 \end{array}$$

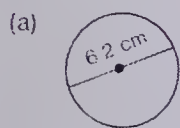
9. Calculate the perimeter.



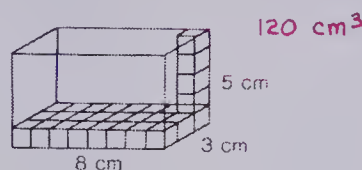
10. Calculate the area.



11. Calculate the circumference ($\pi = 3.1$).



12. Calculate the volume.



Calculate.

$$13. \quad \frac{1}{4} + \frac{1}{8} = \frac{3}{8}$$

$$14. \quad \frac{7}{10} - \frac{4}{5} = \frac{3}{10}$$

$$15. \quad \frac{4}{5} \times \frac{3}{5} = \frac{12}{25}$$

$$16. \quad \frac{2}{5} \div \frac{2}{3} = \frac{3}{5}$$

Calculate.

$$17. \quad 26\% \text{ of } 40 = 10.4$$

$$18. \quad \text{Find } X: \frac{3}{7} = \frac{X}{21} \quad 9$$

19. A bank charges 12% interest each year.

What is the interest on a loan of \$550? \$66

OBJECTIVE

To review and test selected concepts and skills previously covered

PACING

Level A All
Level B All
Level C All

USING THE BOOK

This page may be used for diagnostic and remedial as well as review purposes. Students should check their work, correct any errors, and review the pages that contain any problems of the type they missed. Some students can do this on their own while others may need help. If a large number of students have a particular problem incorrect, you may want to reteach that topic to the groups, then assign a duplicated worksheet to reinforce that topic or, refer to an appropriate skill card in the BFA Computational Skills Kit II or BFA Problem Solving Kit II.

Test Item	Text Page Number
1	4
2	6
3	5
4	9
5	65
6	76
7	130
8	132
9	105
10	108, 109
11	107
12	115
13, 14	195
15	208
16	211
17	230
18	225
19	249

CHAPTER 11 OVERVIEW

This chapter extends the development of the three transformations: translations, rotations, and reflections, as well as enlargements and tessellations. Both line and rotational symmetry are developed as outgrowths of reflections and rotations respectively. Enlargements are used as an example of a transformation that does not preserve size. Geometric constructions are introduced from both the traditional compasses and straightedge approach and the transformational reflection approach. It is recommended that the teacher choose one approach for most students and encourage other students to experience both approaches.

OBJECTIVES

- A For a given object, draw the image and identify the corresponding parts under (a) a translation, (b) a reflection, (c) a rotation, and (d) an enlargement
- B To construct (a) a segment congruent to a given segment, (b) an angle congruent to a given angle, (c) the perpendicular bisector of a given segment, (d) the angle bisector of a given angle, (e) parallel lines; to locate the centre of a given circle; and to use the constructions to draw certain polygons
- C To identify shapes with rotational and line symmetry and to identify the line of symmetry in a figure

BACKGROUND

A translation (slide) is a motion along a line in a plane. The image is congruent to the object and has the same attitude (that is, it is not turned or rotated). Corresponding angles and sides are congruent. Only the position of the object has changed.

A reflection (flip) is a motion in space much like the flipping of a page or the image produced in a mirror. The image is congruent to the object and the mirror image of the object (that is, the order of the vertices is reversed from that of the object). The corresponding angles and sides are congruent. The distance of each set of corresponding points from the line of symmetry (i.e., flip line) are equal.

A rotation (turn) in a plane is turned (pivoted) about the centre of rotation. The image is congruent to the object. The attitude of the image may be different than that of the object. Corresponding angles and sides are congruent.

The three motions preserve the size and shape of any figure. Enlargements do not preserve size but produce similar figures in which the comparison of the measures of corresponding sides result in equal ratios.

The constructions are based on geometric principles to be enhanced. The compasses and straight-edge constructions are based on the fact that all points on arcs of equal radii are equidistant from the centre of the arc. The reflection constructions are based on the two facts: (a) corresponding points on the object and image are equidistant from the flip line, (b) the segment joining corresponding points of the object and image is perpendicular to the flip line.

MATERIALS

tracing paper
scissors
paper for folding
plastic semitransparent mirrors
graph paper
student compasses
rulers
heavy construction pages (tag)

CAREER AWARENESS

Pattern Designer [323]

The designer of patterns is employed by many and varied manufacturers. Most cloth is either woven with a pattern in the weaving or is printed after weaving. Wallpapers and floor coverings usually have patterns.

In most cases, the pattern is one that is repeated in a regular way. The basic unit for a pattern is a shape that tessellates — that is, one that will fill a plane space or cover a surface. Some of these shapes and how they can be modified to tessellate are illustrated on page 323. The teacher may follow a discussion of this career with Activity 2 on page 323.

Most designers have studied art and commercial art or industrial designing. Often designers are full-time employees of manufacturing or designing companies but many designers are free-lancers who work independently and then offer their designs to commercial firms.

Tune Up

Calculate.

1. (a) 6×0.1 **0.6** 2. (a) 0.1×3 **0.3** 3. (a) 26×0.1 **2.6** 4. (a) 0.1×14.2 **1.42**
 (b) 6×0.01 **0.06** (b) 0.01×3 **0.03** (b) 26×0.01 **0.26** (b) 0.01×14.2 **0.142**
 (c) 6×0.001 **0.006** (c) 0.001×3 **0.003** (c) 26×0.001 **0.026** (c) 0.001×14.2 **0.0142**
 (d) 6×0.0001 **0.0006** (d) 0.0001×3 **0.0003** (d) 26×0.0001 **0.0026** (d) 0.0001×14.2 **0.00142**
5. (a) 0.1×0.1 **0.01** 6. (a) 200×300 **60 000** 7. (a) 250×300 **75 000** 8. (a) 160×200 **32 000**
 (b) 0.1×0.01 **0.001** (b) 200×320 **64 000** (b) 250×330 **82 500** (b) 160×210 **33 600**
 (c) 0.1×0.001 **0.0001** (c) 200×340 **68 000** (c) 250×360 **90 000** (c) 160×220 **35 200**
9. (a) $100 \div 10$ **10** 10. (a) $100 \div 100$ **1** 11. (a) $100 \div 1000$ **0.1** 12. (a) $32 \div 32$ **1**
 (b) $10 \div 10$ **1** (b) $10 \div 100$ **0.1** (b) $10 \div 1000$ **0.01** (b) $32 \div 32$ **10**
 (c) $1 \div 10$ **0.1** (c) $1 \div 100$ **0.01** (c) $1 \div 1000$ **0.001** (c) $32 \div 0.32$ **100**
13. (a) $42 \div 10$ **4.2** 14. (a) $42 \div 0.1$ **420** 15. (a) $10 \div 0.1$ **100** 16. (a) $100 \div 0.1$ **1000**
 (b) $42 \div 100$ **0.42** (b) $42 \div 0.01$ **4200** (b) $10 \div 0.01$ **1000** (b) $100 \div 0.01$ **10 000**
 (c) $42 \div 1000$ **0.042** (c) $42 \div 0.001$ **42 000** (c) $10 \div 0.001$ **10 000** (c) $100 \div 0.001$ **100 000**
17. (a) $7.8 \div 6$ **1.3** 18. (a) $13.5 \div 9$ **1.5** 19. (a) $12.8 \div 32$ **0.4** 20. (a) $71.3 \div 2.3$ **31**
 (b) $12.4 \div 3.1$ **4** (b) $1.84 \div 2.3$ **0.8** (b) $11.6 \div 2.9$ **4** (b) $7.04 \div 3.2$ **2.2**
 (c) $25.44 \div 5.3$ **4.8** (c) $514.5 \div 4.9$ **105** (c) $330 \div 7.5$ **44** (c) $30.09 \div 1.7$ **17.7**

Practice: multiplication and division 313

OBJECTIVE

To practise multiplication and division involving powers of 10

PACING

Level A Optional
 Level B Optional
 Level C Optional

USING THE BOOK

Assign Exercises 1, 4, 5, 6, 10, 12, 14, and 17 first to diagnose areas needing more practice. Depending on the errors made by individual students, assign more only where difficulties exist. Use this chart to help make up the assignment.

Exercise	Topic	Page
1-5	Multiplying by 0.1, 0.01, 0.001, 0.0001	73
6-8	Multiplying 3-digit by 3-digit numbers	66-67
9-11, 13	Dividing by 10, 100, 1000	95
12, 17-20	Dividing using decimal divisors	130-132
14-16	Dividing by 0.1, 0.01, 0.001	127, 128, 133, 135

ACTIVITIES

See the various games and ideas in the Activity Reservoir for additional drill ideas, if it is needed. Also, see the Activities listed for the pages charted under Using the Book.

EXTRA PRACTICE

1. 4×200 [800]
 8×200 [1600]
 12×200 [2400]
 16×200 [3200]
2. 3×800 [2400]
 6×800 [4800]
 9×800 [7200]
 12×800 [9600]
3. 35×500 [17 500]
 35×1000 [35 000]
 35×1500 [52 500]
 35×2000 [70 000]
4. 50×50 [2500]
 50×500 [25 000]
 50×5000 [250 000]
 $50 \times 50 000$ [2 500 000]
5. $456 \div 456$ [1]
 $456 \div 45.6$ [10]
 $456 \div 4.56$ [100]
 $456 \div 0.456$ [1000]
6. $693 \div 231$ [3]
 $69.3 \div 231$ [0.3]
 $6.93 \div 231$ [0.03]
 $0.693 \div 231$ [0.003]
7. $100 \div 25$ [4]
 $10 \div 25$ [0.4]
 $1 \div 25$ [0.04]
 $0.1 \div 25$ [0.004]
8. $57.5 \div 125$ [0.46]
 $97.5 \div 0.65$ [150]
 $92.25 \div 0.369$ [250]
 $3.174 \div 0.345$ [9.2]

OBJECTIVE

To be able to identify and draw examples of slides, reflections, and rotations

PACING

Level A All

Level B All

Level C All

VOCABULARY

reflections, rotations

MATERIALS

tracing paper, scissors

SUGGESTIONS

Initial Activity Since this is a review, you may ask the students to bring to class pictorial examples of the three transformations, then discuss them in class and make a bulletin-board display using some of the better examples.

If necessary, use tracing paper and/or the semitransparent mirror.

USING THE BOOK

Discuss Exercises 1 to 3 with the students. For each ask, "What remains the same in a slide (reflection, turn)? What is different in a slide (reflection, turn)?"

Then, having re-established the properties of each, assign Exercise 4.

ACTIVITIES

1. Ask the students to cut from their comic strips a pattern for making illustrations using the three transformations. They can be separate or combined. They should colour the patterns. The students may select those most suited for a bulletin-board display. This exercise may be combined with and reflect a theme in some other aspect of the school day, i.e., social studies, physical education, clean-up day, etc.

2. Provide the students with a series of patterns (or let them make their own) to make a wallpaper design using one or more of the three transformations.

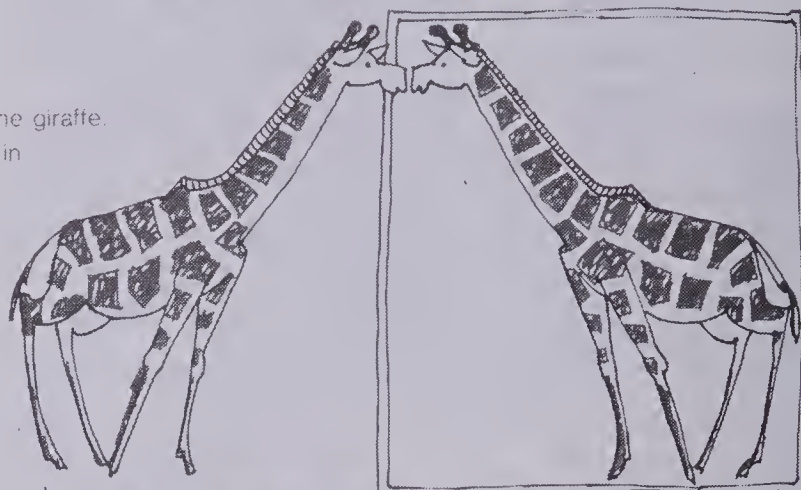
3. If you have not already done so, investigate patterns using both geometric shapes and numbers. See page 173, Activity 1. Be sure to use slides, flips, and turns in the sequences.

Slides, Flips, and Turns

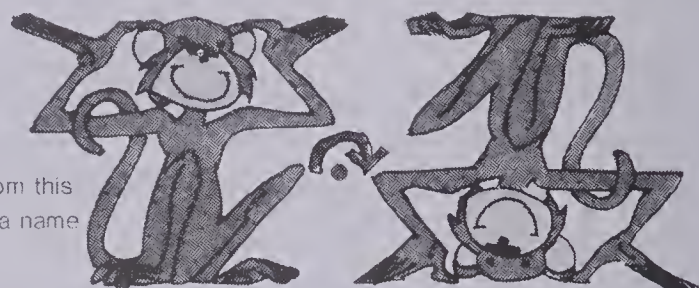
1. Polly, the parrot, slid along her roost.



2. Lonesome Harry, the giraffe, sees his reflection in the mirror



3. Flipping Fred, the monkey, has turned to sit on his head



4. Choose animal cartoons from this or other books. Give each a name and show them in.

- (a) slides
 - (b) reflections (flips)
 - (c) turns (rotations)
- Colour.

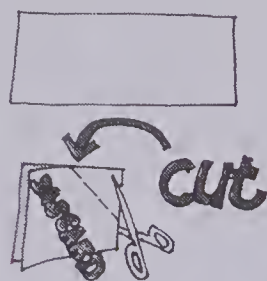
Folding, Cutting, and Measuring

1. Fold a piece of paper once and cut off a corner as shown. Open the shape.

(a) Measure the angles and sides.

(b) Name the shape you have. **Isosceles triangle**

(c) Draw on the line of symmetry.



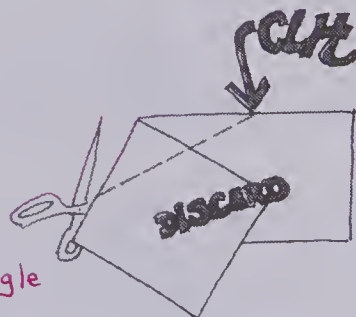
2. Fold a piece of paper once with an oblique fold

(a) Predict the shape you will get when you cut off the corner.

(b) Cut off the corner. Was your prediction correct?

(c) Measure the sides and angles. Name the shape you have. **Scalene triangle**

(d) Draw on the lines of symmetry.



3. Fold a piece of paper twice as shown

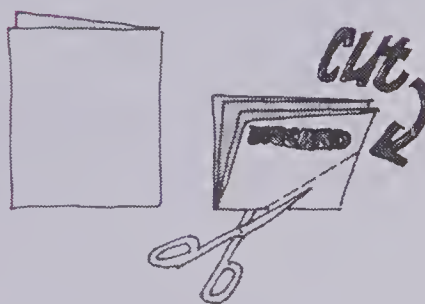
(a) Predict the shape you will get when you cut off the corner.

(b) Cut off the corner. Open it. Was your prediction correct?

(c) Draw on the lines of symmetry.

(d) Name the shape you have. **Quadrilateral**

(e) Fold another piece of paper twice as shown. Can you cut it to produce a square? **Yes**



OBJECTIVE

To identify symmetry through folding and cutting paper

PACING

Level A All

Level B All

Level C All

VOCABULARY

oblique, prediction

MATERIALS

paper for folding and cutting, scissors

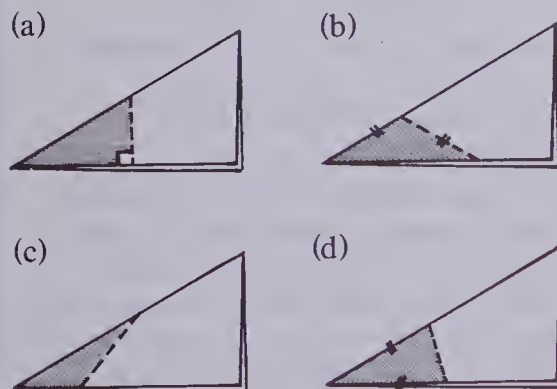
USING THE BOOK

Since this page is designed as an activity in which students are expected to make some "discoveries", it is best handled by ensuring that the students read and interpret the instructions, do the activities, then and only then discussing with small groups their "discoveries".

Activity shapes and symmetry 315

ACTIVITIES

1. Have the students fold 4 pieces of paper each as shown and mark on a dotted line as indicated.



Ask the students to predict what the shape of the shaded portion will be after the cut on the dotted line has

been made. The student can then make the cut and check their prediction.

2. Have each student cut from different coloured paper, a 10 cm × 10 cm square. Each student is to cut the square in 4 pieces (using straight segments) in any way they wish. The 4 pieces are then placed in envelopes and the envelopes labelled A, B, C, etc. The students then select an envelope and try to reassemble the pieces to form a square. The students keep a record of those they complete until all have been successfully reassembled. This exercise may go on for weeks.

3. See the "Tangram" ideas listed in the Activity Reservoir for more geometric puzzles.

OBJECTIVES

To identify corresponding parts of an object and its reflection image
To draw a reflection image on the coordinate plane

PACING

Level A All
Level B All
Level C All

VOCABULARY

corresponding vertices, corresponding sides

MATERIALS

graph paper (DM69).

RELATED AIDS

HMS—DM69.

BACKGROUND

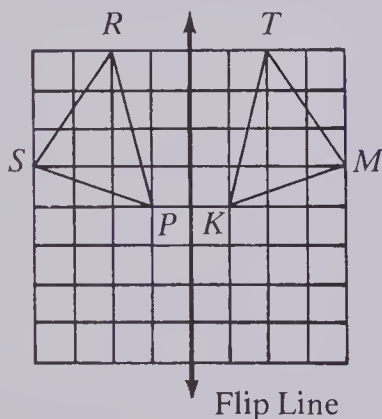
A shape and its flip image are congruent shapes. Corresponding sides and angles of shapes are congruent.

SUGGESTIONS

Initial Activity Review reflections by using semitransparent plastic mirrors or by using tracings or cutouts.

USING THE BOOK

After discussing the display at the top of the page, draw a shape and its image on the chalkboard graph, as shown.



Ask the students to name the corresponding

- (a) vertices
- (b) sides

The students may check using a semi-transparent mirror.

Emphasize:

- (a) corresponding angles are congruent,
- (b) corresponding sides are congruent,

Reflections

Draw the mirror image of $\triangle ABC$.

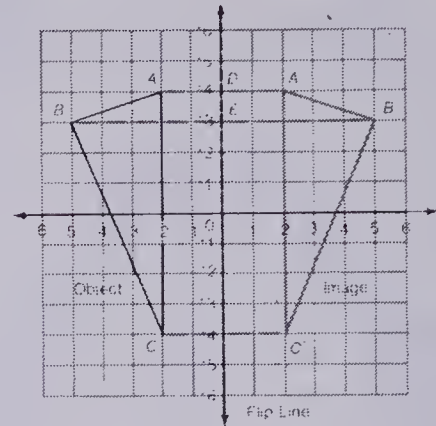
Corresponding vertices:

$A \rightarrow A'$
 $B \rightarrow B'$
 $C \rightarrow C'$

Corresponding sides:

$AB \rightarrow A'B'$
 $BC \rightarrow B'C'$
 $AC \rightarrow A'C'$

$\triangle A'B'C'$ is a reflection of $\triangle ABC$.



Check using a transparent mirror.

Exercises

1. Look at $\triangle ABC$ and its reflection above.

- (a) Is line AA' parallel to line BB' ? **Yes**
- (c) Is line BB' at right angles to the flip line? **Yes**

- (b) Is line AA' at right angles to the flip line? **Yes**

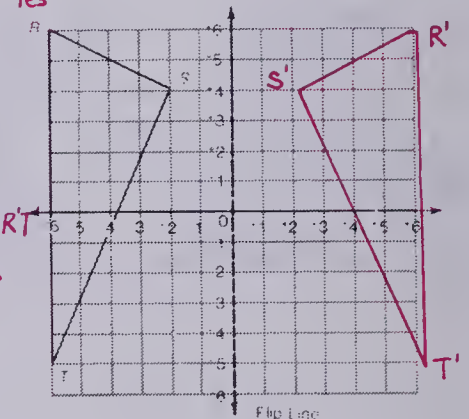
- (d) Is $AD = A'D$? **Yes** (e) Is $BE = B'E$? **Yes**

2. Draw $\triangle RST$ on a grid.
Draw the mirror image.
Label each vertex.

- (a) Name the corresponding:

- (i) vertices $R \rightarrow R', S \rightarrow S', T \rightarrow T'$
- (ii) sides $RS \rightarrow R'S', ST \rightarrow S'T', RT \rightarrow R'T'$

- (b) Are the two triangles congruent? **Yes**



316 Drawing reflections

- (c) the two shapes are congruent.

Do Exercise 1 orally. Assign Exercise 2. You may wish to do pages 316 and 317 in one period.

ACTIVITIES

1. This game requires 2 players and a judge. A flip line (line of reflection) is drawn on a grid paper or dot paper. The first player picks a point on the grid and marks it with an X. The second player locates the flip image of the first point, marks it with a O, then picks another point and marks it O. The first player is to locate the flip image of this second point O and marks it X. The players continue. The player with the most correct after 10 turns each is the winner. (The judge

uses a semitransparent mirror to check.)

2. Play the game in Activity 1, but draw segments joining any two points.

3. See "Radar Scopes" as described in the Activity Reservoir.

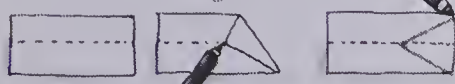
EXTRA PRACTICE

Provide each student with grid paper and have them draw a suitable-size four-quadrant arrangement. Each student is to draw a simple diagram on the grid. Students exchange sheets and draw the reflection image over the flip line of one or both of the axes.

Regular Polygons and Symmetry

Follow the steps to make each polygon.

1. Start with a rectangle.



- (a) Fold in half. Unfold.
(b) Fold one corner to crease line and mark point.
(c) Unfold. Draw lines.

(d) Cut out an equilateral triangle.

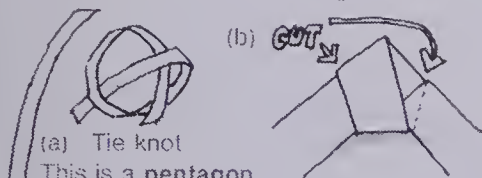
2. Start with a rectangle.



- (a) Fold, and draw line.
(b) Cut off top. Unfold.

This is a square.

3. Start with a narrow rectangle.



- (a) Tie knot. This is a pentagon.

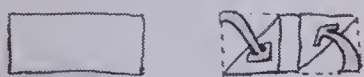
4. Start with an equilateral triangle.



- (a) Fold in corners to centre.

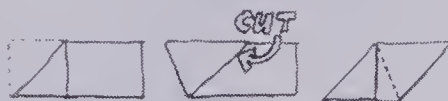
This is a hexagon.

5. Start with a rectangle.



- (a) Fold corners.
(b) Cut along crease line to produce a parallelogram.

6. Start with a rectangle.



- (a) Fold.
(b) Fold again and cut.
Unfold.

This is a rhombus.

7. Use the above polygons. Draw on all lines of symmetry.

★ 8. List the properties of each shape.

- (a) equilateral triangle (b) square (c) pentagon (d) hexagon (e) parallelogram
(f) rhombus

Lines of symmetry 317

OBJECTIVES

To fold and cut paper to make certain polygons

To draw lines of symmetry on certain shapes

PACING

Level A All

Level B All

Level C All

MATERIALS

paper, scissors, plastic mirrors

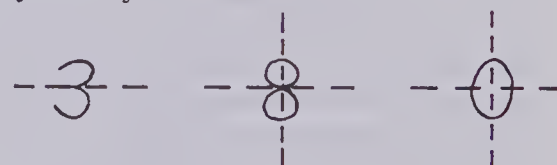
USING THE BOOK

Folding paper to make polygons is enjoyable if appropriate time is taken to allow the students success. After making each polygon the student makes the lines of symmetry on it. The models should be large enough to make handling easy.

The summary exercise, Exercise 8, should be studied and discussed thoroughly with the class noting the various names and properties and looking for patterns. (The regular polygons have the same number of lines of symmetry as they have sides.)

ACTIVITIES

1. Draw each numeral on the chalkboard. Discuss those that have symmetry and draw on the lines of symmetry.



2. Ask students to explore the various letters of the alphabet to identify those that have symmetry. They should draw each letter that has symmetry and draw on the line or lines of symmetry.

3. Have students make a list of symmetric words. The students can check by using the semitransparent mirror.

Example



4. Refer to the "Polycubes" activity on page 49, Activity 3. How many of the seven polycubes have a line of symmetry?

OBJECTIVES

- To identify corresponding parts of a shape and its image after a slide
- To draw a slide image on a coordinate plane given one vertex of the image

PACING

- Level A 1-3, 5
- Level B All
- Level C 1, 3-5

MATERIALS

graph paper (DM69)

RELATED AIDS

HMS — DM69.

BACKGROUND

A shape and its slide image are congruent shapes. Corresponding sides and angles of a shape and its slide image are congruent (same size).

USING THE BOOK

Lead the students through the display at the top of the page in which T' is located by a rule: right 6, down 2. Then locate O' and P' using the same rule. Some students may profit from tracing, cutting out, and sliding the original triangle to match it with the image. This may be necessary in all the exercises to fully convince some students. Exercise 5 illustrates a practical use of a slide: "The area of a parallelogram is the same as that of a rectangle with the same height and base." (Do not force the verbal statement but develop the concept.) The students may profit from cutting out the shaded triangle and sliding it along to its new position.

Corresponding Parts in Slides

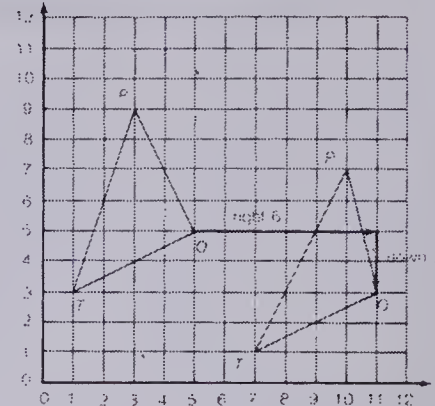
We can draw the image of a shape given the position of one corresponding vertex of the image

T' is the image of T .

Construct the slide image of $\triangle TOP$.

Step 1 To go from T to T' is a slide using the rule "right 6, down 2".

Step 2 Locate O' and P' using the "right 6, down 2" rule.

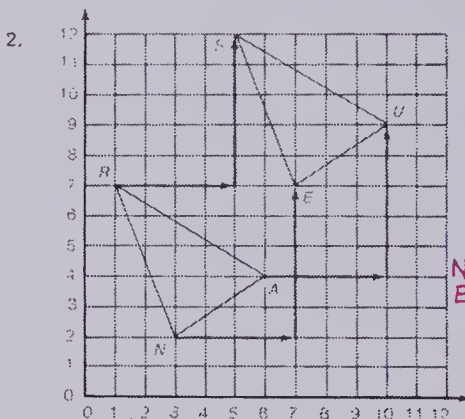


Exercises

Use graph paper labelled as shown. Draw $\triangle SET$.

$S \rightarrow (5,9)$ $E \rightarrow (2,6)$ $T \rightarrow (4,3)$

- The image of S is $S' \rightarrow (10,7)$.
 - How far right? **5** down? **2**
 - To locate T' , we use the rule "right **5**, down **2**".
 - Locate E' using the rule.
 - Draw $\triangle SET'$.



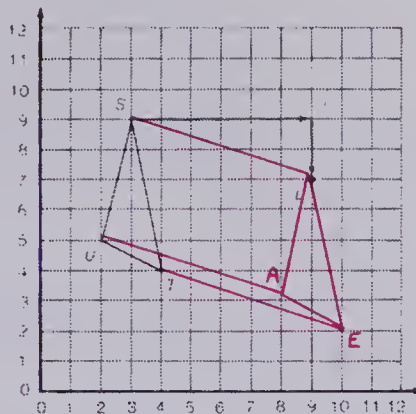
- $R \rightarrow S, A \rightarrow U, N \rightarrow E$
- $RA \rightarrow SU, AN \rightarrow UE, RN \rightarrow SE$
- $\angle R \rightarrow \angle S, \angle A \rightarrow \angle U, \angle N \rightarrow \angle E$
- $R(1,7) \rightarrow S(5,12); A(6,4) \rightarrow U(10,9); N(3,2) \rightarrow E(7,7)$
 $\triangle SUE$ is the slide image of $\triangle RAN$.

Name the corresponding:

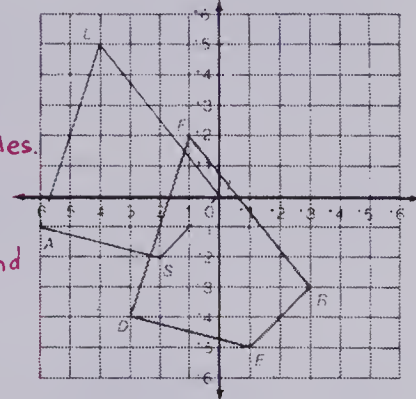
- vertices
- sides
- angles

- Write the ordered pairs of each set of corresponding vertices.

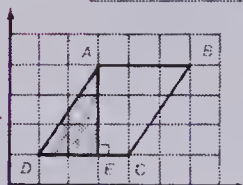
3. $\triangle STU$ and $\triangle LEA$ will show a slide.
- (a) Find the rule and complete $\triangle LEA$.
Right 6, down 2.
- (b) Draw lines connecting corresponding vertices
 Are these lines parallel? *Yes*
- (c) Are $\triangle STU$ and $\triangle LEA$ congruent? *Yes*



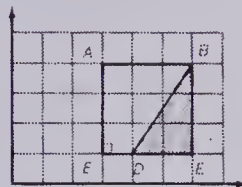
4. $FRED$ and $LISA$ are related by a slide.
 Write a statement about:
- (a) FR and LI *FR and LI are corresponding sides.*
- (b) $\angle FRE$ and $\angle LIS$ *$\angle FRE$ and $\angle LIS$ are corresponding angles.*
- (c) segment FL and segment DA *Segment FL and segment DA are parallel.*
- (d) $FRED$ and $LISA$ *$FRED$ and $LISA$ are congruent.*



5. Draw the parallelogram $ABCD$.
 Draw AE .
 Slide $\triangle AED$ right so that AD is matched to BC .



- (a) Will the area of the parallelogram $ABCD$ be the same as the area of rectangle $ABE'E$? Why?
Yes
- (b) Calculate the area of the rectangle. *9 square units*
- (c) Calculate the area of the parallelogram. *9 square units*

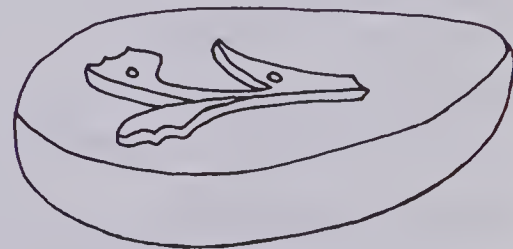


Drawing slides 319

ACTIVITIES

1. Have students play the game in Activity 1 on page 316, modified to involve slides. The first player draws a simple shape and names the slide (e.g., 5R, 6D). The second student draws the image, then draws the original and names another slide for the first player to draw. The player with the most correct after 5 turns each is the winner.

2. Make potato prints. Students may bring potatoes. Slice a potato into halves. Draw a design on one half and cut away the surplus potato using a pen knife or scissors.



You may cut out bits within the design. Use squared paper or sheets of paper that have been folded to form large squares. Apply paint to the design part of the potato and print in the squares of the paper. Use slides to produce a design.

3. Have the students make wall-paper patterns using slides, flips, and slides and flips combined. Colour and display.

4. Find examples of slides in and about the school. You may extend this to include selections from magazines.

OBJECTIVES

To draw a rotation image

To use rotations to show that the area of a triangle is one half the area of the corresponding parallelogram (of the same height and base)

To show that the area of a trapezoid is one half the area of the corresponding parallelogram

To experience optical illusions

PACING

Level A All

Level B All

Level C All

MATERIALS

graph paper (DM69), scissors

RELATED AIDS

HMS — DM69, DM76, and DM77.

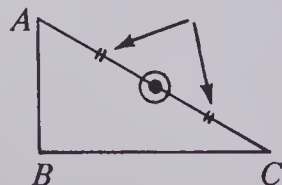
BACKGROUND

A shape and its rotation image are congruent shapes. The corresponding sides and angles are also congruent.

USING THE BOOK

The student must make the appropriate cutouts and turn them on the page to reinforce the point that the triangle is in fact one half of the rectangle. While Exercises 1 and 2 are easier "to see", and may be done by drawing rather than cutting and tracing, Exercise 3 needs the actual experience. The student should be able to respond with "the triangle is one half of the parallelogram" or "the parallelogram is twice the triangle". Exercises 4 and 5 are optional and may be considered challenges. Again cutouts are encouraged. Be sure to draw the students' attention to the symbol used to show equal lengths of some of the line segments in the drawings on this page.

Example



Discuss the optical illusions. Ask the students to check each of their answers.

Rotations

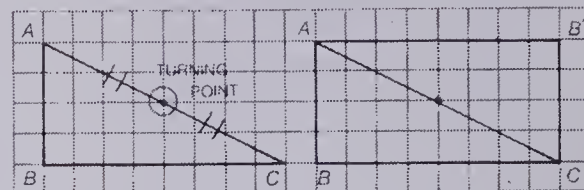
Draw $\triangle ABC$ on graph paper.

Make a cutout of this triangle.

Rotate the triangle

$\frac{1}{2}$ turn about the turning point.

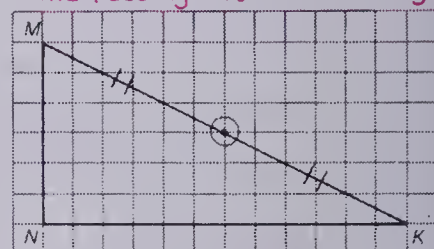
Trace the image



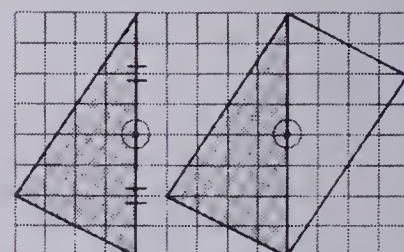
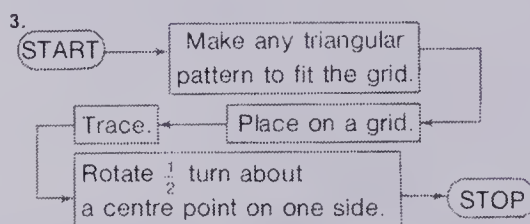
Exercises

Refer to the display.

1. (a) What is the shape made by the two triangles? *A rectangle*
 (b) Compare the areas of the rectangle and the triangle. Which is larger and by how much? *The rectangle is twice as large.*
 (c) Name the corresponding (i) vertices, (ii) sides.



2. Repeat the steps in the display using this triangle.
 Answer the questions in Exercise 1.



3. (a) What is the name of the new shape formed? *Parallelogram*
 (b) How many triangles make the parallelogram? *2*
 (c) Compare the areas of the parallelogram and the triangle.
 Which is larger and by how much? *The parallelogram is twice as large as the triangle.*

320 Drawing rotations

ANSWERS:

1. (c) (i) $A \rightarrow A, B \rightarrow B, C \rightarrow C, B' \rightarrow B$ (ii) $AB \rightarrow AB, BC \rightarrow BC, CB' \rightarrow AB, AB' \rightarrow CB$

2. (a) *A rectangle* (b) *The rectangle is twice as large as the triangle.*

(c) (i) $M \rightarrow M, N \rightarrow N, K \rightarrow K, N' \rightarrow N$ (ii) $MN \rightarrow MN, NK \rightarrow NK, KN' \rightarrow MN, MN' \rightarrow KN$

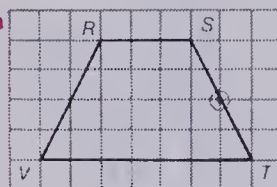
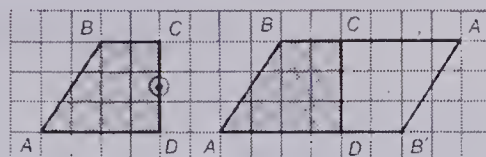
- ★4. Half turn the trapezoid $ABCD$ around the turning point.

(a) What is the name of the shape $ABA'B'$? **Parallelogram**

(b) How many trapezoids make a parallelogram? **2**

(c) Compare the areas of the parallelogram with the original trapezoid. Which is bigger and how much? **The parallelogram is twice as large.**

(d) Name the corresponding (i) vertices, (ii) sides.



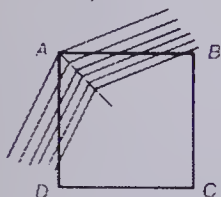
- ★5. Repeat Exercise 4 for trapezoid $RSTV$.

(a) **Parallelogram** (b) **2** (c) **The parallelogram is twice as large.**

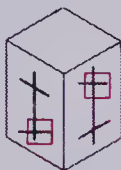
Optical Illusions

Things are *not* always what they appear! Answer each question, and *then* measure to check.

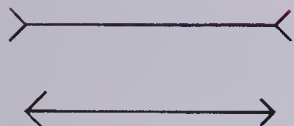
1. Is $\angle A$ a square corner? **Yes**



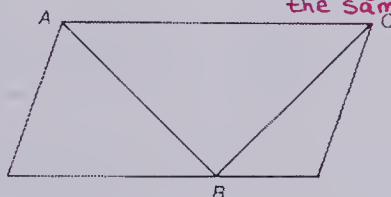
2. Which angles are *actually* right angles?



3. Which is longer? **They are both the same length.**



4. Which is longer, AB or BC ? **They are both the same length.**



ACTIVITIES

1. Have some students make large bulletin-board working models for Exercises 2, 3, 4, and 5. The cutout, in colour, has a circular region around the point of rotation so a thumbtack can be placed here to hold the working area. The students can then turn the shape to match the original, and thereby complete the parallelogram. Encourage students to discuss the models with classmates.



Thumbtack

2. Ask the students to make wallpaper patterns using rotations, and rotations and/or slides and/or flips.

3. Ask students to research and draw examples of other optical illusions to show the others in class.

4. Some students may wish to explain why optical illusions occur. They may then be able to draw some original designs of optical illusions based on these principles.

ANSWERS:

4. (d) (i) $A \rightarrow A$, $B \rightarrow B$, $A' \rightarrow A$, $B' \rightarrow A$, $B' \rightarrow B$
(ii) $AB \rightarrow AB$, $BC \rightarrow BC$, $CA' \rightarrow DA$, $A'B' \rightarrow AB$, $AD \rightarrow AD$, $DB' \rightarrow CB$

5. (d) (i) $R \rightarrow R$, $V' \rightarrow V$, $R' \rightarrow R$, $V \rightarrow V$ (ii) $VR \rightarrow VR$, $RS \rightarrow RS$, $SV' \rightarrow TV$, $V'R' \rightarrow VR$, $VT \rightarrow VT$, $TR' \rightarrow SR$

OBJECTIVES

To make designs using rotational symmetry

To identify patterns which have rotational symmetry

PACING

Level A All

Level B All

Level C All

VOCABULARY

rotational symmetry

MATERIALS

scissors, tracing paper

BACKGROUND

In rotational symmetry:

(a) there is one point (axis) of rotation,

(b) as the shape turns it occupies one or more different positions that look exactly the same,

(c) the number of positions that look the same is called the order of the rotational symmetry.

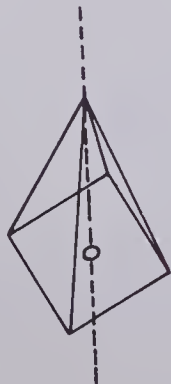
USING THE BOOK

This page, designed as an activity, is self-explanatory, but some students may need help following the instructions in Number 1. Use the term "rotational symmetry" periodically to reinforce the term.

ACTIVITIES

1. Have students prepare original designs with rotational symmetry (this will involve knowing some basic principals of rotational symmetry) or, some students may collect pictures illustrating rotational symmetry.

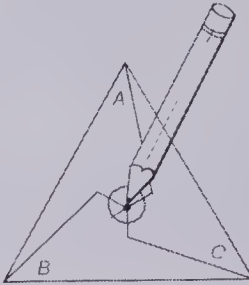
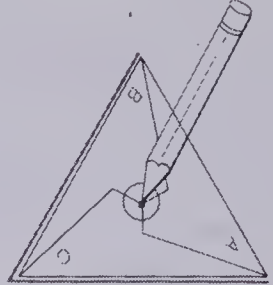
2. Ask students to identify solids that have rotational symmetry. Remember, a solid will have an axis of symmetry.



3. In many countries rubber sink stoppers are of this shape (part of a

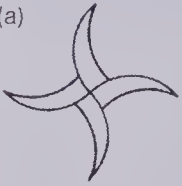
Rotational Symmetry


- Trace, label, and cut out the pattern in Figure 1. Place it on your page and trace it. Label it in your book as shown in Figure 1. Put a pin at \odot . Turn it $\frac{1}{3}$ of a turn until it fits the original drawing as shown in Figure 2. Note that the shape fits. Turn it again $\frac{1}{3}$ of a turn. Does it fit again? **Yes**

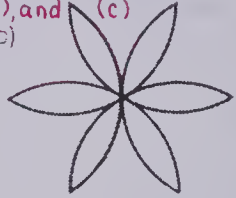




This shape has **rotational symmetry**.

- Trace and cut out each of the following shapes. Which ones have rotational symmetry? **(a), (b), and (c)**

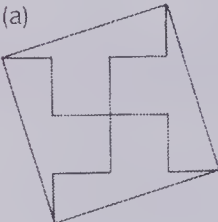
(a) 

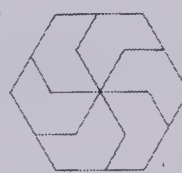
(b) 

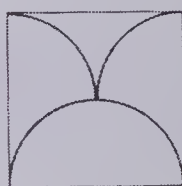
(c) 

(d) 

- Trace. Which ones have rotational symmetry? **(a) and (b)**

(a) 

(b) 

(c) 

- Create 3 designs that have rotational symmetry.

322 Rotational symmetry

cone) while other countries sometimes have stoppers of this shape (spherical).



Is there an advantage, one over the other?

4. Refer to the "Polycubes" activity on page 49. Which of the shapes have rotational symmetry?

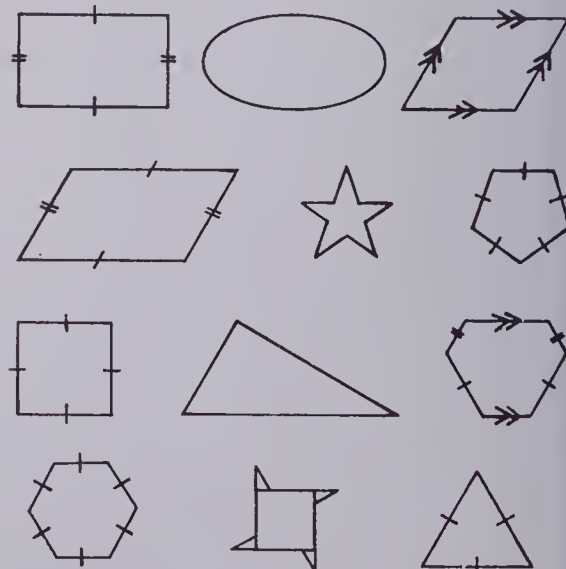
EXTRA PRACTICE

Cut from tag a series of shapes. Students are to trace each and identify which have rotational symmetry. (★ Some students may be challenged to identify the order of symmetry of each shape which has rotational

symmetry.)

Draw a series of shapes and ask the student to identify which has:

- line symmetry,
- rotational symmetry.



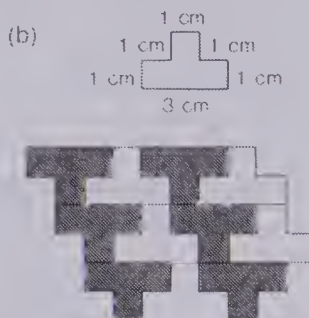
Tessellations

A **tessellation** is a repeated pattern that can be used to cover a surface.

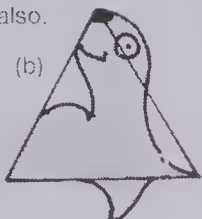
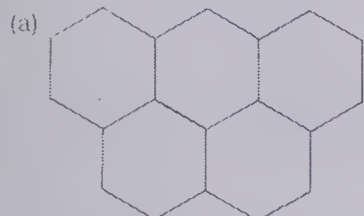
Exercises

1. Make each pattern.

Trace it to make a surface-covering pattern. You may want to make patterns using the two shapes together. Colour.

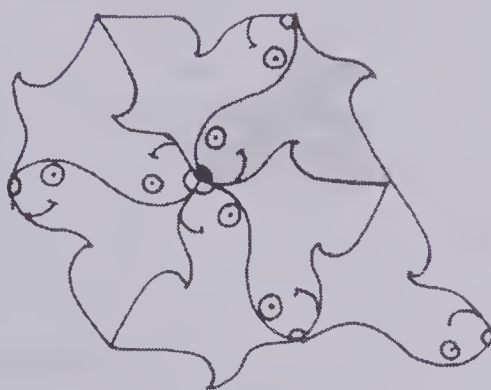
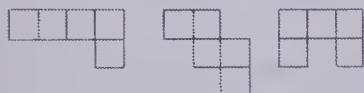


2. Other shapes will cover a surface also.



3. Pentominoes are made from 5 squares attached along at least one complete edge

- (a) Make all of the different pentominoes.
- (b) How many different pentominoes are there?
- (c) How many tessellate?
- (d) Colour your patterns.



This shape is based on an equilateral triangle. It tessellates. Draw a shape that tessellates.

OBJECTIVE

To make simple tessellation patterns using a shape that tessellates

PACING

Level A 1, 2
Level B All
Level C All

MATERIALS

scissors, duplicated patterns of shapes

VOCABULARY

tessellation, tessellates

BACKGROUND

A shape tessellates (produces a tessellation) if it produces a pattern which does not overlap nor leave spaces in the pattern. (Circles will not tessellate; squares do.)

SUGGESTIONS

Initial Activity Duplicate sufficient numbers of each shape to be used by the students. The students can cut the shapes out.

USING THE BOOK

Ask the students to make different patterns without overlapping the shapes and without leaving spaces in the pattern. They should colour their patterns.

Tessellations 323

ACTIVITIES

1. Exercise 2(b) indicates how interesting shapes can be made that tessellate. Challenge the better students to make their own shapes that tessellate.

Refer the students to:

Escher, M.C. *The Graphic Work of M.C. Escher*, Ballantine Books, New York.

Ranucci, E.R., J.L. Teeters *Creating Escher-Type Drawings*, Creative Publications, Palo Alto, California.

2. Bring, or have students bring, samples of wallpaper and floor coverings to class. Then identify the shape or basic pattern that is repeated (and tessellates). You may then ask

the class to design their own wallpaper or floor covering pattern. They are to ensure that the pattern will tessellate when it is repeated.

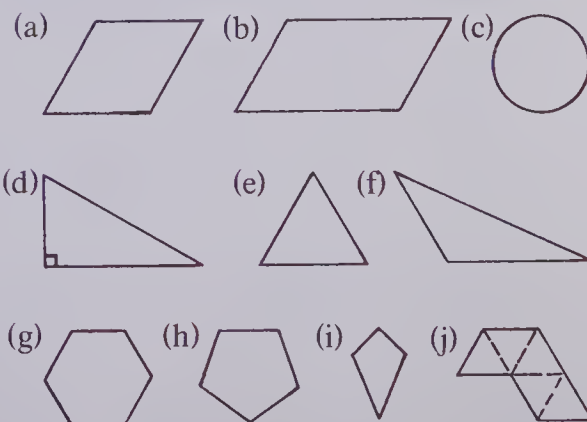
3. Have students find examples of tessellations in magazines or in their environment.

4. See the "Tangram" ideas in the Activity Reservoir.

5. Have some students report on M.C. Escher and his art.

EXTRA PRACTICE

Have the students illustrate with examples or counter-examples to show which of these shapes will and will not tessellate.



OBJECTIVE

To draw the perpendicular bisector of a segment

PACING

Level A One method only
Level B One method
Level C Both methods

VOCABULARY

perpendicular bisector

MATERIALS

plastic semitransparent mirror (A commercial product is the MIRA.)
compasses, straightedge

BACKGROUND

Two methods of constructing a perpendicular bisector of a segment are shown. The teacher should decide, in light of the needs of the school and of the students, whether to do one or the other or both.

Care must be taken to keep the mirror perpendicular to the page. If a commercially-prepared product is not used, a piece of 2 mm thick red semi-transparent plastic can be glued to a wooden block so that the plastic is held perpendicular to the page. The edge of the plastic against the page should be bevelled (see diagram).

USING THE BOOK

In teaching the use of the plastic mirror, it helps students if you tell them that the mirror is usually placed where the solution line will be. Hence for the perpendicular bisector (explain this term) the mirror would initially be placed across the line near its midpoint, then adjusted so one point is mapped onto the other. Discuss the two conditions: perpendicular and bisector.

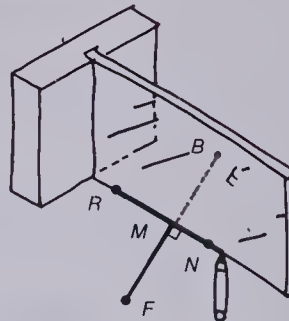
Perpendicular Bisector

We can use a transparent mirror to construct the perpendicular bisector of a segment.

Place the mirror across FB so that the image of point F is on top of point B .

Draw line RN .

Line RN is the **perpendicular bisector** of line FB .



Exercises

1. Draw a line segment DR 10 cm long.

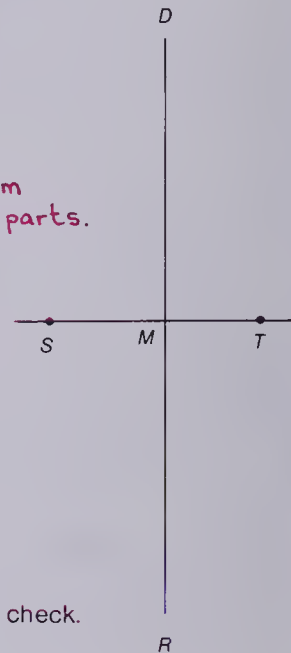
Use your plastic mirror to draw the perpendicular bisector as in the diagram.

- (a) How long is line segment DM ? line segment MR ? **5 cm**
What does *bisect* mean? **To divide into 2 equal parts.**

- (b) Measure $\angle DMS$. **90°**

What does *perpendicular* mean? **A line at right angles to another line.**

- (c) Write these instructions in your own words:
"Draw the perpendicular bisector of segment DR ."



2. Draw these segments:

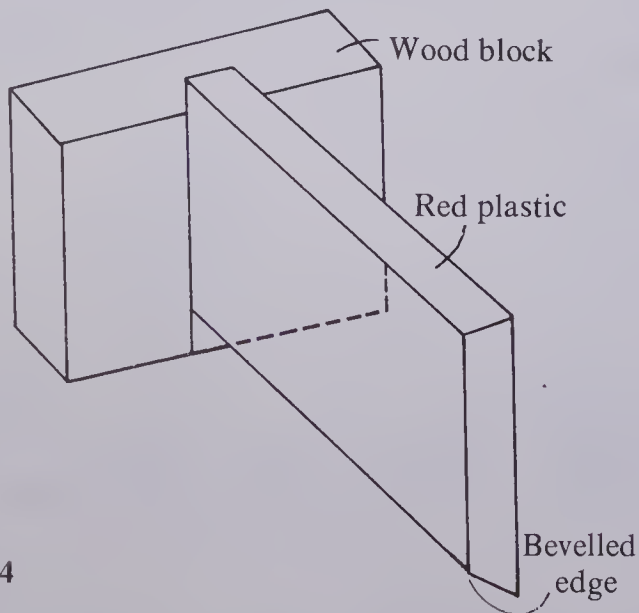
- (a) 8 cm (b) 12 cm (c) 15 cm

Construct the perpendicular bisector of each. Measure to check.

4. Draw a line segment RD .

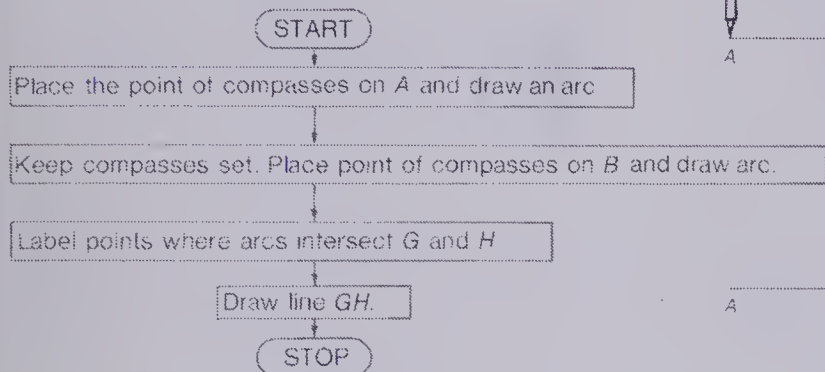
Construct three different line segments that are perpendicular to RD .

324 Perpendicular bisector, transparent mirrors

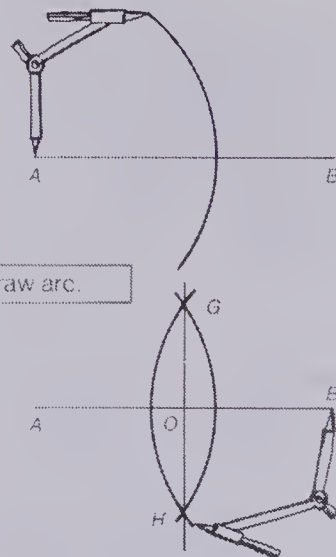


Bisecting the Perpendicular Another Way

We can use compasses and a straightedge to construct the perpendicular bisector of a line segment.



GH is the perpendicular bisector of AB.
Use a plastic mirror to check.



Exercises

- Draw a line segment AB in your book.
Use the above method to construct the perpendicular bisector of the segment.
 - Measure $\angle AOG$. Is it 90° ? **Yes**
 - Measure AO and OB. Are they equal? **Yes**
 - Is it important how far the compasses are opened? **The compass opening must be greater than one half of the line segment being bisected.**
- Draw a large triangle.
 - Construct the perpendicular bisector of each side
 - Extend the bisectors so that they intersect
 - Write a statement about the intersection point. **The intersection point is inside an acute-angled triangle and outside an obtuse-angled triangle.**
 - What does *bisect* mean? Write a statement **Bisect means to divide into 2 equal parts.**
- Draw a line segment SD.
Construct three different line segments that are perpendicular to SD.

Perpendicular bisector compasses 325

Many students find the compasses very difficult to handle and manipulate. Therefore, not only do the students need to learn how to perform the construction, they need to learn how to manipulate the compasses. Encourage the students to do the best they can and to practise.

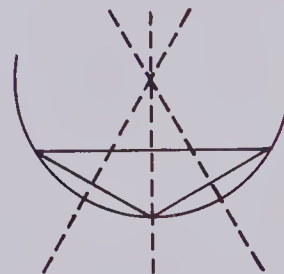
If students experience both methods, you may let them decide which they will use when they need to draw perpendicular bisectors.

Some students may question why they have to do these constructions the hard way (with the compasses) when it is so easy with the mirror. Hence, the student may be granted permission to do it whichever way he/she prefers once each method has been experienced.

Students should be cautioned as to the correct use and method of using compasses. Emphasize that the compasses must not be adjusted after the first arc (the radius must remain the same).

ACTIVITIES

- Have the students draw a triangle and its extended bisectors as in Exercise 2 on page 325. Then, using the point of intersection of the bisectors as a centre point, have them draw (using compasses) a circle so that the circumference passes through each vertex of the original triangle.



- Challenge the students to define each:
bisect; perpendicular; perpendicular bisector.

OBJECTIVE

To bisect an angle

PACING

Level A One method

Level B One method

Level C Both methods

MATERIALS

semitransparent plastic mirror,
compasses, straightedge

BACKGROUND

Two methods of constructing the bisector of an angle are shown. The teacher should decide whether to do one or both methods.

USING THE BOOK

For Exercise 1, suggest that the students place the plastic mirror where they think the bisector will be. Then adjust the mirror so that one ray, ray RP maps onto the other ray, ray RF . They then draw a ray along the mirror edge.

When the students use the compasses, emphasize that the second and third arcs must have the same radius, i.e., the same setting of the compasses.

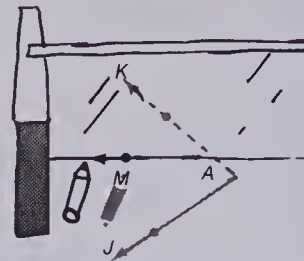
Angle Bisector

We can use the plastic mirror to bisect an angle.

Place the mirror between the two rays so that the image of ray AJ fits on ray AK .

Draw ray AM .

Ray AM bisects $\angle KAJ$.

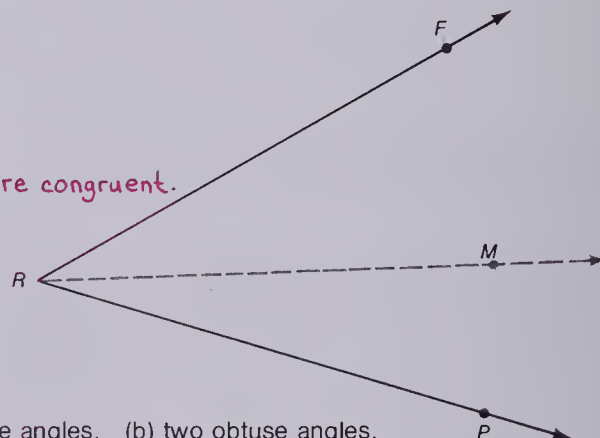


Exercises

1. Draw a large angle FRP .
Use your mirror to bisect it.

(a) Measure each of the new angles. How do the sizes compare? *They are congruent.*

- (b) Write this instruction in your own words:
"Bisect an angle."



2. Draw and bisect (a) two acute angles, (b) two obtuse angles.

3. Draw any large triangle.

(a) Bisect each angle.

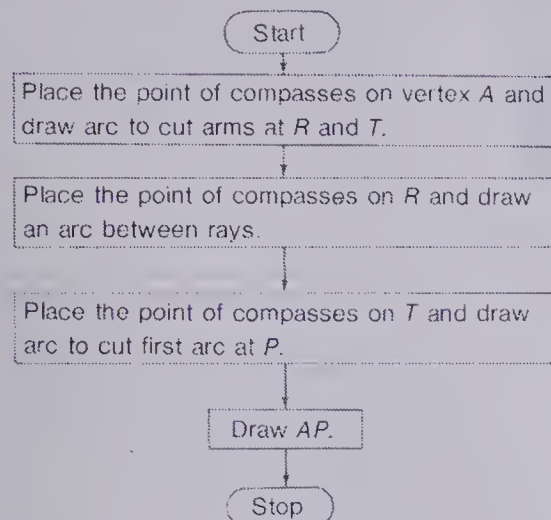
(b) Extend the bisectors so that they intersect.

(c) Write a statement about the intersection point.

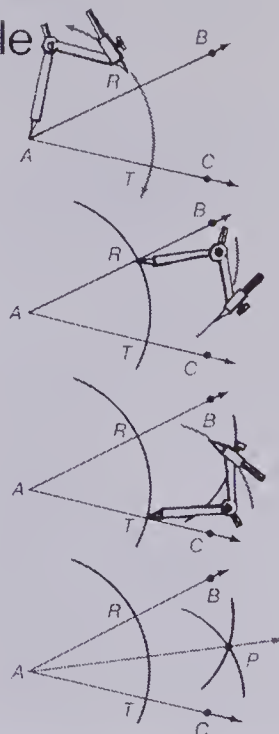
The intersection point is inside the triangle.

Bisecting an Angle

We can use compasses to bisect an angle.



Line AP bisects $\angle BAC$.
Use a plastic mirror to check.

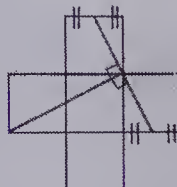


Exercises

- Draw any angle DAN .
Bisect it.
Check, using a protractor.
- Draw a triangle.
 - Bisect each angle.
 - Extend the bisectors so that they intersect.
 - Write a statement about the intersection point.

BRAINTICKLER

Make this pattern by tracing a square 5 times. Cut out. Draw in the red lines. Cut on the red lines. Use the 4 pieces to make one square.



Bisecting an angle using compasses 327

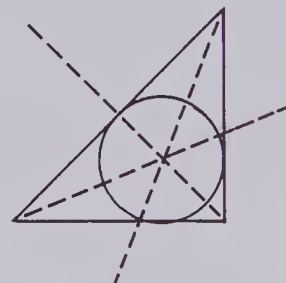
ACTIVITIES

1. Have the students check their work by measuring the initial angle and each angle formed. Remind the students that their measuring exercises produce only approximate numbers, hence, small variations may appear. Discuss the differences as they occur.

2. Have students draw a square and bisect the angles. What is the point of intersection called? [centre of the square or centre of rotation] Does this also happen with parallelograms?

3. Challenge the students to find a way to trisect an angle. Remind them that using a protractor is not allowed in geometric constructions. Suggest that they research in the library unsolved mathematics problems — this being one. (There are ways to trisect an angle but none using standard Euclidian methods.)

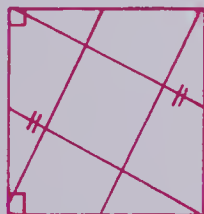
4. In Exercise 3 on page 326 and Exercise 2 on page 327, have the students draw an inscribed circle using the point of intersection of the three bisectors as centre.



ANSWERS:

2. (c) The intersection point is inside the triangle.

Braintickler:



OBJECTIVE

To locate the centre of a circle

PACING

Level A All
Level B All
Level C All

MATERIALS

semitransparent plastic mirror

BACKGROUND

This construction is not done using compasses and straightedge at this time.

USING THE BOOK

Ask the students to draw a circle using cylindrical tin cans. Then assist the students to follow the directions in the display at the top of the page. The image of one half of the circle must match the half behind the mirror. The point of intersection of any two diameters is the centre.

An alternate method of illustrating the principle contained on this page is to have each student cut out the circles traced using the cans. The circle is folded to form two matching halves. This is repeated again. Where the two fold lines intersect is the centre of the circle.



The Braintickler is another optical illusion.

ACTIVITIES

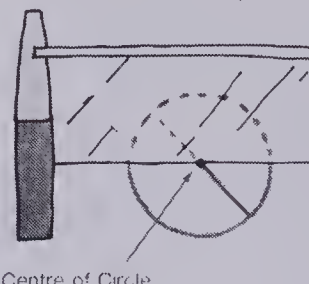
1. Ask the students to write, in flow-chart form, the instructions for locating and checking the centre of a circle.

2. Ask the students to write a flow chart for the alternate method of locating the centre of a circle cut from paper.

3. See "Road Rally" as described in the Activity Reservoir.

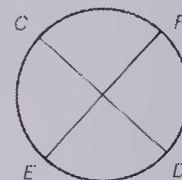
Locating the Centre of a Circle

We can use the plastic mirror to locate the centre of a circle. Place the mirror on the circle so that one half matches the other half. Draw a line along the mirror. Repeat. Draw another line. The two lines intersect at the *centre of the circle*.



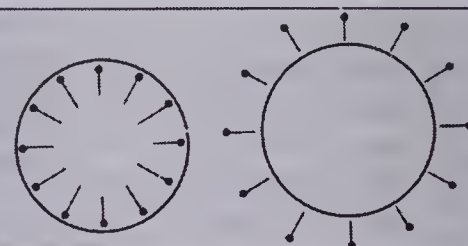
Exercises

- Use a round tin can to draw a circle.
 - Locate the centre of the circle. Use compasses to check that you have located the centre.
 - What is each segment CD and EF called? **Diameter**
- Repeat Exercise 1 with two other circles.
- Trace this arc.
 - Find the centre of the circle of which it is part.
 - Use compasses to complete the circle.



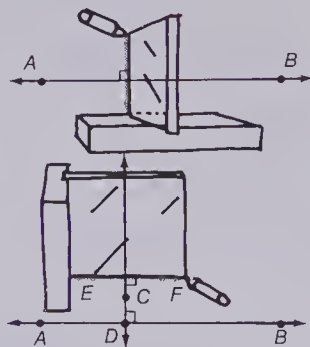
BRAINTICKLER

Which circle is larger? **They are both the same size.**
Guess first. Then check.



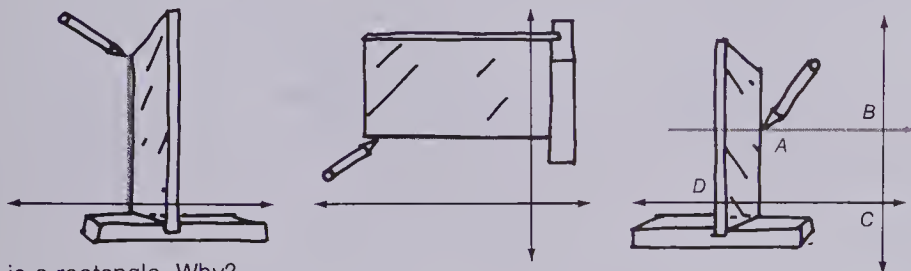
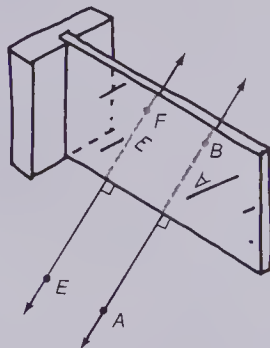
Constructing Parallel Lines

We can draw a line parallel to another line by using a plastic mirror.
Draw a line DC perpendicular to AB .
Draw a line EF perpendicular to DC .
 EF is parallel to AB .



Exercises

1. Draw a line AB .
Construct a line EF parallel to AB .
To check: Place your mirror across the two lines so AB fits on itself. Is EF on itself?
If it is, the two lines are parallel.
2. Draw a pair of parallel lines:
(a) 3 cm apart (b) 10 cm apart
3. Draw a rectangle using the method shown.



$ABCD$ is a rectangle. Why?

4. Draw a square by using the method in Exercise 3.

Parallel lines 329

OBJECTIVE

To draw a line parallel to a given line

PACING

Level A 1
Level B 1, 3
Level C All

MATERIALS

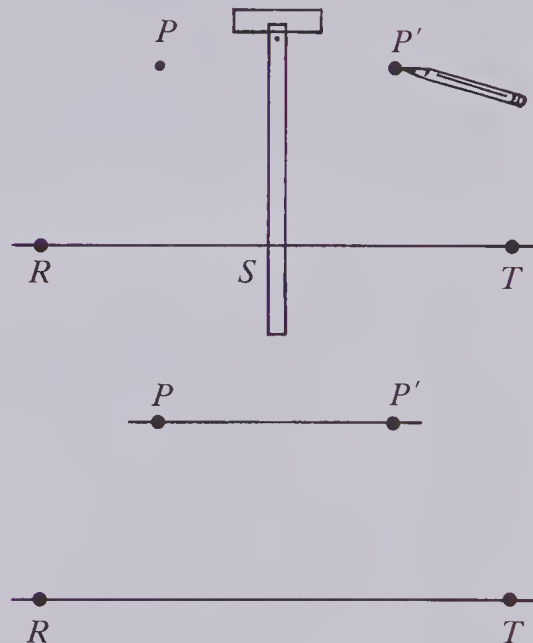
semitransparent plastic mirror

BACKGROUND

This construction is not done using compasses and straightedge at this time.

There are two methods of drawing a line parallel to a given line: the method illustrated and the following.

Place a dot P where you want the other line to go. Place the mirror as indicated so that ray SR maps onto ray ST . Look in the mirror and mark the image P' of P behind the mirror. Draw PP' . PP' is parallel to RT .



While this method has some advantages over the one in the text, the one in the text is more basic and easier understood by the student.

USING THE BOOK

Illustrate at the chalkboard or on the overhead projector how to construct a line parallel to a given line. Assign the exercises and move about helping those students needing it.

ACTIVITIES

1. Ask the students to draw an angle consisting of two segments with a common endpoint (as illustrated). Then have them construct two parallel lines to make a parallelogram.



2. Challenge the students to find another way to draw a line parallel to a given line. (See Background.)

3. Use the "Tangram" pieces as described in the Activity Reservoir. Have the students find how many patterns they can make with at least one pair of parallel sides. Have them trace each pattern.

OBJECTIVES

To draw a segment congruent to a given segment

To copy an angle

PACING

Level A All

Level B All

Level C All

MATERIALS

compasses and straightedge

RELATED AIDS

HMS — DM78.

BACKGROUND

These constructions are not done using the semitransparent mirror.

USING THE BOOK

First Display. To copy segment MN we draw a ray PR . Using the compasses, we measure off the length of MN and mark it on the ray. Illustrate the methods using the overhead projector or chalkboard.

Second Display. Draw the students' attention to the fact that the $\angle DET$ is to be copied and the other drawings in the display illustrate the steps in making this copy.

ACTIVITIES

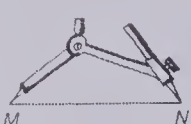
1. Ask the students to draw any parallelogram. Then ask them to copy the parallelogram using the techniques they have learned to date. There are a number of ways in which this can be done. Discuss each different method used.

2. Four soldiers came to a river they could not swim. The only way across was by a small boat owned by two small boys. The boat would hold one soldier or two boys, but not a soldier and a boy nor two soldiers. How can the soldiers get across the river using only the boat and oars? [Solution: Both boys row across the river and one rows back to let a soldier row across. The second boy rows back and the process is repeated until all are across.]


3. A legendary traveller with a wolf, a goat, and a huge cabbage, came to a river. He found a small boat that could hold only himself and one of his possessions. The wolf cannot be left alone with the goat and the goat cannot be left alone with the

Copying a Line Segment

We can use compasses to make a line segment congruent to MN .



Set compasses to correct length.



Place the point on a new point P and draw an arc. Draw a line from P to arc. Label the intersection R .

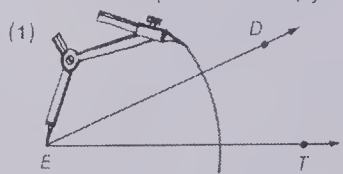
PR is congruent to MN .

Exercises

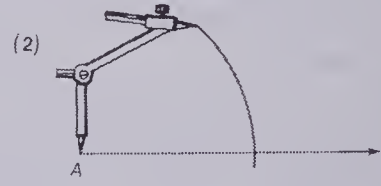
Draw a segment KJ on your page.
Construct another segment TS congruent to KJ .

Copying an Angle

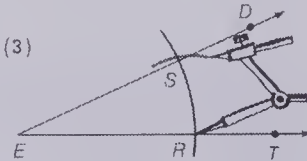
Follow the steps as we copy $\angle DET$.



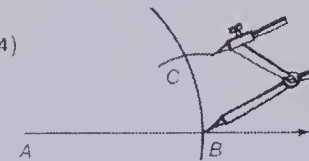
(1)



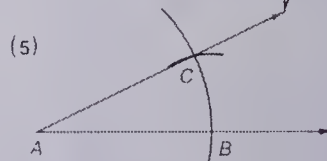
(2)



(3)



(4)



(5)

$\angle CAB$ is a copy of $\angle DET$.

Exercise

Draw any $\angle DET$. Use the method shown to copy it.

cabbage. How could the traveller, using the boat only, get himself and his possessions across the river? [Solution: The traveller takes the goat across first and returns to get either the wolf or cabbage. He takes this item across, but takes the goat back to leave on the bank while he takes the third possession across, then returns to get the goat.]

Make a Puzzle

1. Draw a square $ABCD$ 10 cm on each side.
Mark points E, F, G, H each 2 cm from a corner as shown.

Draw segments HF and GE .

Cut along the lines to get 4 pieces.

Draw and cut out a 6 cm square.

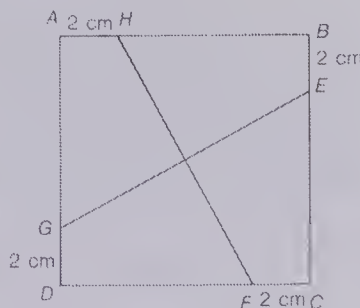
The challenge:

Combine the 5 pieces to form a large square

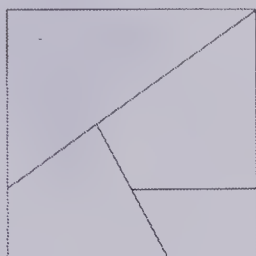


6 cm

6 cm



2. Draw and cut out a 10 cm square.
Cut it into any 4 pieces.
Challenge a classmate to reassemble the pieces into a square.

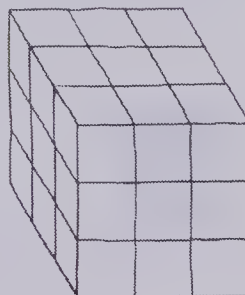


3. Glue 27 cubes together. (Use as little glue as possible.)

Paint the outside of the new large cube.

- (a) How many small cubes have only 1 face painted? **6**
- (b) How many small cubes have exactly 2 faces painted? **12**
- (c) How many small cubes have exactly 3 faces painted? **8**
- (d) How many small cubes have exactly 4 faces painted? **0**
- (e) How many small cubes have no faces painted? **1**

If necessary, take your cube apart to check.



OBJECTIVE

To construct and solve puzzles based on shapes

PACING

Level A All
Level B All
Level C All

MATERIALS

heavy paper or tag, rulers, scissors

BACKGROUND

Spatial perception can be strengthened through suitable experiences. The puzzles on this page, as well as others throughout this series, are designed to provide these experiences. Also, the puzzles are designed to be more easily solved if logical steps are followed — logical problem-solving steps. One such procedure is illustrated in the subparts of Exercise 3.

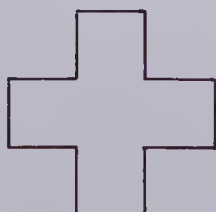
USING THE BOOK

Encourage the students to work independently (assist where reading problems exist). If necessary, allow two students to work together for one may inspire the other. If blocks are unavailable, or to check the answers for Exercise 3, see pages 180 and 181.

Activity 331

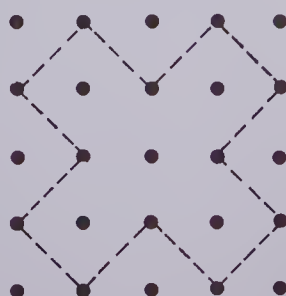
ACTIVITIES

1. Provide the students with 5 by 5 dot paper. Challenge the students to join dots to form a cross like this.



There are to be 5 dots remaining inside and 8 dots outside.

Solution:



2. Allow pairs of students to make a design using the tangram pieces. Each student traces the outlines of the design, then the two students exchange designs and try to re-form the pieces to make the design. As another version, the pair may decide to use any 6 (or 5) of the pieces to form a design. The piece not used is not identified.

3. See "Square It" as described in the Activity Reservoir.

OBJECTIVE

To construct common solid shapes

PACING

Level A All

Level B All

Level C All

MATERIALS

construction paper, rulers, scissors

VOCABULARY

polyhedron, polyhedra, tetrahedron, octahedron, hexahedron, hexagonal

RELATED AIDS

HMS — DM69.

USING THE BOOK

Many students will be able to draw their own nets especially if dot paper and grid paper are available for those based on rectangles. Triangular dot paper is required for those based on the equilateral triangles. You may wish to provide some students with nets for tracing.

For the plural form of polyhedron, both polyhedra and polyhedrons should be accepted.

ACTIVITIES

1. Have the students print names on the polyhedra and suspend them from the ceiling or display them where suitable.

2. Have the students find pictures that illustrate the shapes shown on the page as well as others. Prepare a bulletin-board display.

3. Have the students assemble the unused polyhedra to form a futuristic city. They will want to arrange streets, avenues, etc.

4. Have the students calculate (a) the volume, (b) the surface area, of as many shapes as they can.

Making More 3D Shapes

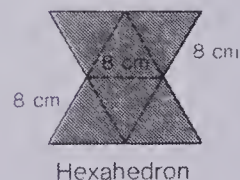
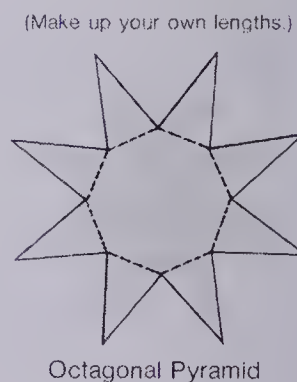
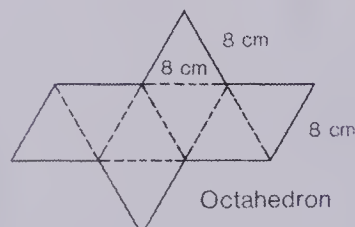
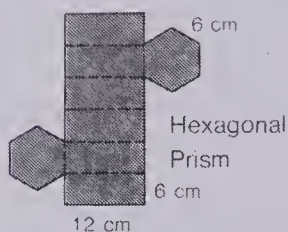
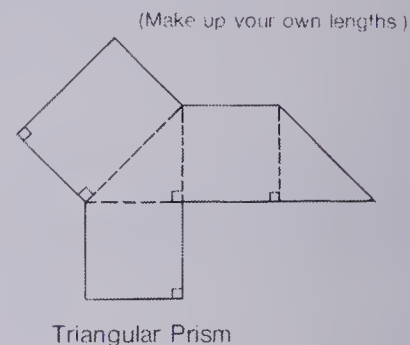
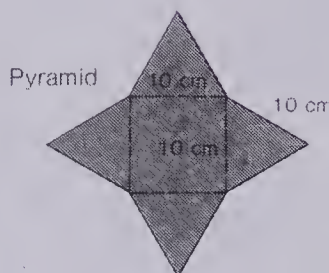
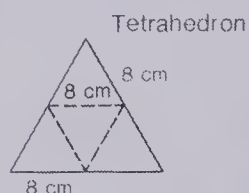
Activity

Construct each of the following nets for making polyhedra.

Use coloured paper or heavy tag.

Cut out the nets and assemble them.

Label each with a name and suspend them in your classroom.



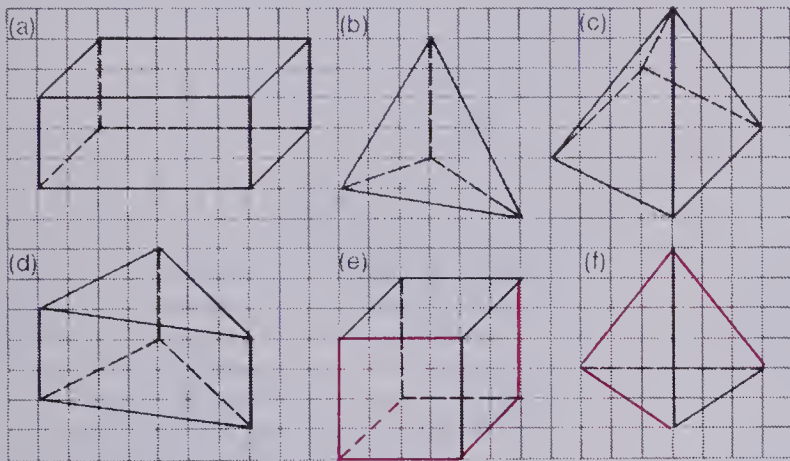
Drawing Polyhedra

1. Copy and complete this chart for the 3D shapes you made from page 332.

Name of Polyhedra	Number of Vertices (V)	Number of Faces (F)	Number of Edges (E)	$V + F - E =$ ■
tetrahedron	4	4	6	$4 + 4 - 6 = 2$
pyramid	5	■ 5	■ 8	$5 + 5 - 8 = 2$
triangular prism	■ 6	5	■ 9	$6 + 5 - 9 = 2$

What do you notice about the solution to $V + F - E$? *It always equals 2.*

2. Copy and name each polyhedron on graph paper. Complete parts (e) and (f).



3. Use graph paper and draw your own polyhedra. Name each one.

Drawing polyhedra 333

ANSWERS:

1. hexagonal prism: $V=12, F=8, E=18, 12+8-18=2$
 hexahedron: $V=8, F=6, E=12, 8+6-12=2$
 octahedron: $V=6, F=8, E=12, 6+8-12=2$
 octagonal prism: $V=14, F=10, E=22, 14+10-22=2$

2. (a) Rectangular prism (b) Tetrahedron (c) Square-based pyramid
 (d) Triangular prism (e) Cube (f) Tetrahedron

OBJECTIVES

To draw certain polyhedra on graph paper

To recognize Euler's relationship

PACING

Level A All

Level B All

Level C All

MATERIALS

polyhedra constructed from page 332, graph paper — 0.5 cm squares is most appropriate (DM69)

RELATED AIDS

HMS — DM69.

BACKGROUND

The relationship $V + F - E = 2$ is attributed to and named after Euler (pronounced Oiler).

USING THE BOOK

Have the students copy in their workbooks the chart in Exercise 1, and complete it for all the models made to date. Encourage them to discover the relationship — if necessary, through discussions with small groups.

Students should be encouraged (instructed) to draw several of each of the polyhedra shown. Only through practice will they remember how and be able to draw the figures at some later date.

ACTIVITIES

1. Provide the students with graph paper. Ask them to draw futuristic city using polyhedra as the basic shapes.

2. See "Road Rally" as described in the Activity Reservoir.

3. See the "Snatch and Match" variation of the "Fraction Cards" idea described in the Activity Reservoir.

OBJECTIVE

To enlarge a given pattern that is on squared paper

PACING

Level A All
Level B All
Level C All

MATERIALS

squared paper with, if possible, squares larger than those on these pages

RELATED AIDS

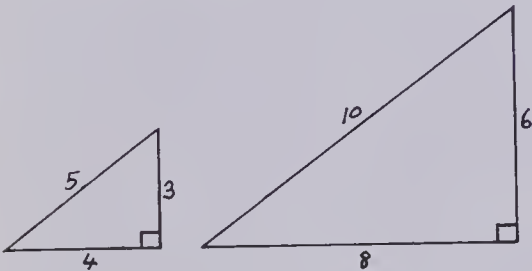
HMS — DM79.

BACKGROUND

A shape and its enlargement are similar shapes.

SUGGESTIONS

Initial Activity Discuss similar shapes referring to the ratio of the length of the sides.



$$\frac{3}{6} = \frac{4}{8} = \frac{5}{10}$$

Then state that enlargements can be made easily using grid paper. As an application of enlargements, you may calculate the height of a light standard, telephone pole, or tree. Stand a metrestick upright, measure the length of its shadow, then measure the length of the shadow cast by the tree.

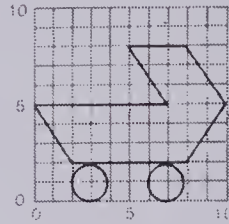
$$\frac{\text{(a) height of metrestick}}{\text{(b) length of its shadow}} = \frac{\text{height of tree (c)}}{\text{length of tree's shadow (d)}}$$

$$\frac{a}{b} = \frac{c}{d}$$

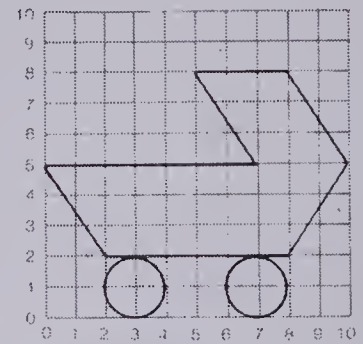
$$c = \frac{a \times d}{b}$$

Enlargements

Cathy wanted to draw this picture larger.



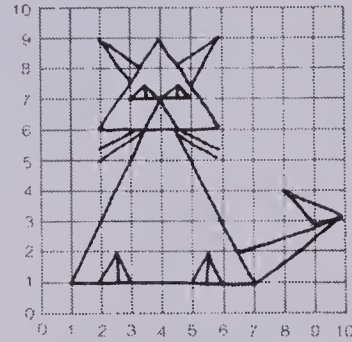
To do it, she used grid paper with larger squares.



Exercises

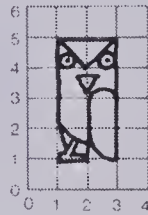
Use 1cm square grid paper.

1. Copy the drawing of Triangle Cat.

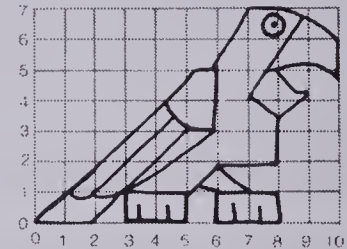


2. Enlarge each bird, and then colour each one.

(a)

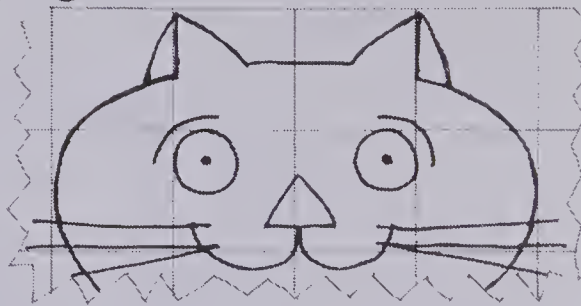
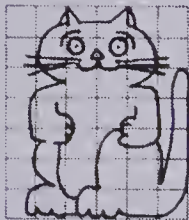


(b)



Copying Cartoons

Janet is copying this cartoon figure.

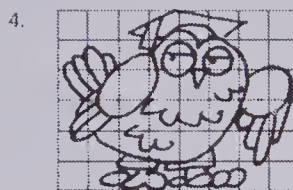
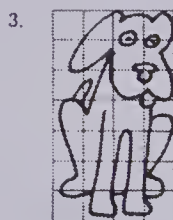
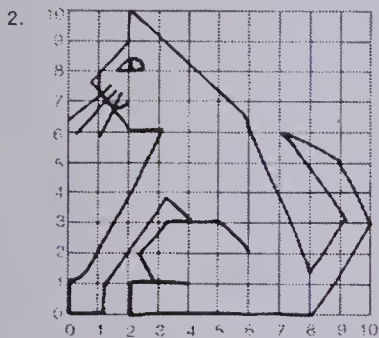


She is using grid paper with larger squares.
She copies each square carefully, and goes on to the next.

Exercises

1. Use a grid with 2cm squares.
Copy the cartoon figure in the display.

Copy each cartoon figure.



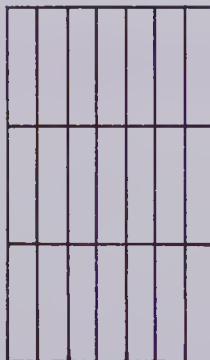
Enlarging curved figures on a grid 335

USING THE BOOK

Treat these two pages as one lesson. Ask the students to cover the right-hand side of the display (i.e., the enlargement). Then direct them to label squared paper with horizontal and vertical number lines. Then choose a point in the pattern — say the point at (0,5). Locate this point on their grids. Then choose another point — say (2,2). Locate this on their grids. Join the two points with a straight line. Continue this until the pattern is complete. (Some students may wish to locate the second point by the “2 over and 3 down” method. While this is not incorrect, the practice comes in using the ordered pair. Note also, that statements like “3 down and 2 over” break the ordered pair rule of the horizontal movement first, then the vertical move. Therefore, if students use this method, set the practice of using the left, right movement first.)

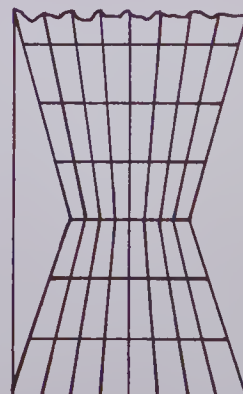
ACTIVITIES

1. Ask the students to colour their pictures.
2. How many triangles are in Triangle Cat?
3. Ask the students to bring their favourite comic strip character. Over these they draw a square grid system. Have them enlarge the character.
4. Provide the students with rectangular grid paper. Ask them to use this grid to enlarge any of the pictures. What is the result?



5. Provide the student with this grid system shown. Ask them to use

this grid to enlarge a picture. What is the result?



OBJECTIVES

To locate points on maps superimposed with grids, using an ordered pair approach
To practise multiplication and division by 10

PACING

- Level A All
- Level B All
- Level C 3; 16-34 (even); Tune Up is optional

RELATED AIDS

CALC. ACTIVITY MASTERS — 82.

USING THE BOOK

In these exercises we name the squares by using ordered pairs: the letter first and the numeral second. Work through the display locating Saint John, having told them it is in square H4. When the students have done this, do Exercise 1. Check where the students place their fingers by getting pairs of students to check each other. Assign the balance of the exercises or do the exercises orally.

The Tune Up may indicate a problem area. If this occurs, complete several examples and have the students “state a rule” for multiplying and dividing by 10. (See pages 60 and 95.)

Using Maps



Nicholle and Gustolf were looking at this map. Gustolf asked, “Where is Saint John?”
Nicholle replied, “It is in square H4.” Can you find Saint John?

Exercises

Refer to the map in the display.

1. Locate square D5. Name a city in this square. **Moosonee**
 2. Locate square F6. Name three cities in this square. **Wabush-Labrador City, Mt. Wright, Gagnon**
- Locate each of the following places by using the name of the square
3. Red Lake **A5**
 4. Goose Bay **H7**
 5. Mattagami **E4**
 6. Kirkland Lake **D4**
 7. Sault Ste. Marie **C3**
 8. Halifax **H3**
 9. Ottawa **E3**
 10. Dryden **A4**
 11. Hamilton **D2**
 12. Ft. George **D6**
 13. Locate the city in J6. In H4 there is a city with a similar name **St. John's**
How do the two names differ? **Saint John**



Name the city in each of these squares.

14. A5 **Prince Rupert** 15. E6 **Ft. Smith** 16. G3 **The Pas** 17. J3 **Fort Albany** 18. D5 **Rainbow Lake**
 19. E3 **Saskatoon** 20. C4 **Jasper** 21. E5 **Uranium City** 22. D2 **Medicine Hat** 23. I2 **Sioux Lookout**

Give the name of the square in which each city is located.

24. Vancouver **B3** 25. Edmonton **D4** 26. Prince George **B4** 27. Calgary **D3**
 28. Winnipeg **G2** 29. Regina **F2** 30. Saskatoon **E3** 31. Thunder Bay **I1**

Tune Up

Multiply.

- | | | | | |
|-----------------------------------|-----------------------------------|--------------------------------------|--|--------------------------------------|
| 1. 23.5×10
235 | 2. 15.6×10
156 | 3. 76.45×10
764.5 | 4. 156.35×10
1563.5 | 5. 76.41×10
764.1 |
|-----------------------------------|-----------------------------------|--------------------------------------|--|--------------------------------------|

Divide

- | | | | |
|----------------------------------|----------------------------------|------------------------------------|----------------------------------|
| 6. $75.4 \div 10$
7.54 | 7. $46.5 \div 10$
4.65 | 8. $138.4 \div 10$
13.84 | 9. $13.1 \div 10$
1.31 |
|----------------------------------|----------------------------------|------------------------------------|----------------------------------|

ACTIVITIES

1. Play "Relay". Make teams. Using a large display map of the world or North America (with Mercator projection), give each team the name of a city with its location in the ordered pair form. Have the players find their city. The first wins a point for their team. The team with the most points after every player has had a turn is the winner.

2. Refer to the Activity Reservoir for suitable activities to reinforce computational skills as needed by the students in your class.

3. Play "Concentration" as described in the Activity Reservoir. Use card sets such as:

Calgary

Alberta

Halifax

Nova Scotia

Winnipeg

Manitoba

OBJECTIVE

To evaluate achievement of the chapter objectives

PACING

- Level A All
- Level B All
- Level C All

RELATED AIDS

HMS — DM80.

USING THE BOOK

Each student should do this test independently under supervision. Assistance should be given only when the instructions are not understood. After the work has been corrected, you should provide appropriate remedial work. You may wish to reteach if a large number of students had difficulty with a particular topic or concept.

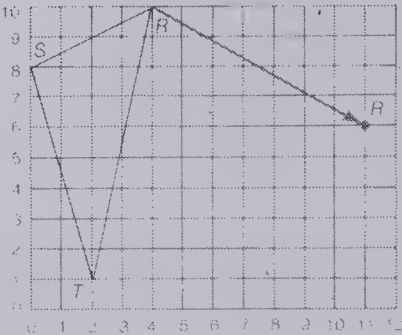
The following chart will help in this regard. The specific objectives are listed in the Chapter Overview (see page 312).

An alternate Chapter Test can be found in the Holt Mathematics System Duplicating Masters available for use with this grade level.

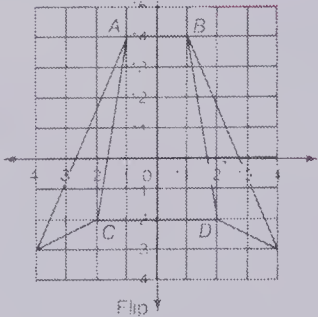
Test Item	Objective	Text Page Number
1-3	A	316, 318, 320
4, 5	B	324-327
6, 7	C	317, 322

Chapter Test

- 1. Copy this shape. Draw the slide image.

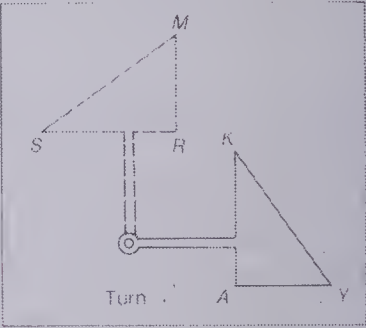


- 2. Write what you know about lines AB and CD. Lines AB and CD are parallel.

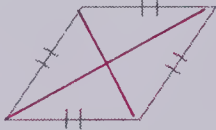


- 3. Name the corresponding (a) sides (b) vertices

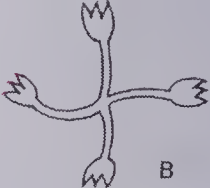
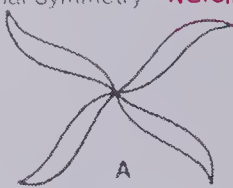
- (a) $KY \rightarrow SM$
 $YA \rightarrow MR$
 $AK \rightarrow RS$
- (b) $K \rightarrow S$
 $Y \rightarrow M$
 $A \rightarrow R$



- 4. Draw an angle. Bisect it
- 6. Draw a large rhombus as shown. Draw all the lines of symmetry.



- 5. Draw a line segment. Construct its perpendicular bisector.
- 7. Identify which shape or shapes have rotational symmetry. Neither



Cumulative Review

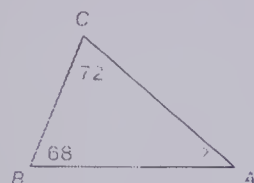
1. 2 407 000 000 **billion, hundred million**
Complete, two **hundred** four **hundred** seven **hundred**

2. The 7 in 2 347 is seven **thousandths**

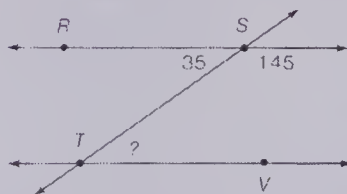
3. $10^4 = 10\,000$

4. Draw an angle. Use a protractor to measure it.

5. Calculate the measure of $\angle A$. **40°**



6. What is the measure of $\angle STV$? **35°**



Multiply

7. 23.4×6.5
152.10

8. 67.1×0.25
16.775

Divide

9. $23 \overline{) 71.3}$ **3.1**

10. $5.6 \overline{) 11.76}$ **2.1**

11. What are the factors of 24? **1, 2, 3, 4, 6, 8, 12, and 24**

12. Which of the following are prime?

13 16 **41** 58

Calculate

13. $24 \times \frac{1}{4}$ **6**

14. $24 - \frac{1}{4}$ **96**

15. $\frac{1}{2} \times \frac{2}{3}$ **$\frac{1}{3}$**

16. $\frac{7}{10} \div \frac{1}{2}$ **$1\frac{2}{5}$**

17. $\frac{3}{5} + \frac{1}{4}$ **$\frac{5}{8}$**

18. $\frac{1}{4} + \frac{1}{3}$ **$\frac{11}{12}$**

19. $\frac{9}{10} - \frac{2}{5}$ **$\frac{1}{2}$**

20. $\frac{5}{6} - \frac{1}{4}$ **$\frac{7}{12}$**

21. 30% of 80 **24**

22. A car goes 80 km/h.
How far does it go in 5.5 h? **440 km**

Chapters 1-11: cumulative review 339

OBJECTIVE

To review and test selected concepts and skills previously covered

PACING

Level A All

Level B All

Level C All

USING THE BOOK

This page may be used for diagnostic and remedial as well as review purposes. Students should check their work, correct any errors, and review the pages that contain any problems of the type they missed. Some students can do this on their own while others may need help. If a large number of students have a particular problem incorrect, you may want to reteach that topic to the groups, then assign a duplicated worksheet to reinforce that topic or, refer to an appropriate skill card in the BFA Computational Skills Kit II or BFA Problem Solving Kit II.

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Skills Check Up—Chapters 1 to 5

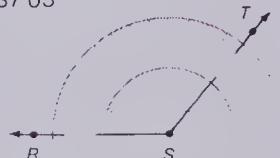
Give the correct answer for each: (a), (b), (c), or (d).

1. $\begin{array}{r} 27 \\ 376 \\ 7529 \\ 828 \\ + 4501 \\ \hline \end{array}$ (a) 13 571 (b) 13 271 (c) 14 271 (d) 13 261 (d)
2. $\begin{array}{r} \$8463.47 \\ - 790.58 \\ \hline \end{array}$ (a) \$7772.89 (b) \$8772.89 (c) \$7672.89 (d) \$7772.89 (c)
3. $46.0 - 31.63$ (a) 14.37 (b) 15.63 (c) 77.63 (d) 15.37 (a)

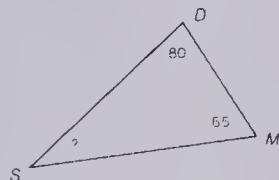
4. $\begin{array}{r} 75.76 \\ \times 4.8 \\ \hline \end{array}$ (a) 363.648 (b) 36.3648 (c) 363.548 (d) 373.648 (a)
5. $34 \overline{)90702}$ (a) 2668 R 23 (b) 2667 R 24 (c) 2678 R 14 (d) 2668 R 24 (b)

6. 637 047 rounded to the nearest hundredth is: (d)
(a) 600 (b) 700 (c) 637.1 (d) 637.05

7. What is the measure of $\angle RST$? (c)
(a) 50° (b) 80°
(c) 130° (d) 180°

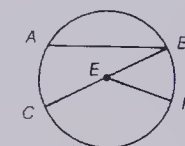


8. What is the measure of $\angle S$? (a)



- (a) 35° (b) 60° (c) 25° (d) 180°

9. Line segment EF is:
(a) a radius (b) a diameter
(c) a chord (d) parallel (a)



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10. This polygon is (c)
(a) a hexagon
(b) a pentagon
(c) an octagon
(d) a heptagon

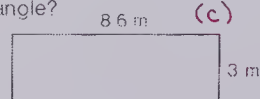


11. $5.6 \overline{)224}$ (b)
(a) 4 (b) 40
(c) 400 (d) 0.4

12. $\begin{array}{r} 635 \\ \times 0.001 \\ \hline \end{array}$ (b)
(a) 0.0635 (b) 0.635
(c) 6.35 (d) 63.5

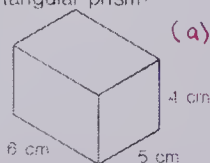
13. Which equation correctly solves this problem?
Susan spends \$1.20/d for bus fare. How much does she spend in five days? (a)
(a) $\$1.20 \times 5 = N$ (b) $\$1.20 + 5 = N$
(c) $5 - \$1.20 = N$ (d) $\$1.20 - 5 = N$

14. What is the perimeter of this rectangle? (c)



- (a) 25.8 m^2 (b) 11.6 m
(c) 23.2 m (d) 23.2 m^2

15. What is the volume of this rectangular prism?



- (a) 120 cm^3 (b) 15 cm^3
(c) 34 cm^3 (d) 26 cm^3

16. $5000 \text{ g} = \square \text{ kg}$ (c)
(a) 500 (b) 50
(c) 5 (d) 0.5

17. One litre of water has a mass of 1 kg. What is the mass of 1000 mL of water? (d)
(a) 1000 kg (b) 100 kg
(c) 10 kg (d) 1 kg

18. $8 \overline{)5}$ (c)

- (a) 0 (b) 0.62
(c) 0.625 (d) 1.6

19. It was -11°C . The temperature rose 12°C . What was the new temperature? (d)
(a) -23°C (b) 23°C
(c) -1°C (d) 1°C

20. How much time is there between 08:27 and 14:45? (b)
(a) 6 h (b) 6 h 18 min
(c) 6 h 45 min (d) 6 h 27 min

Basic skills check up: chapters 1 to 5 341

Skills Check Up—Chapters 6 to 11


Give the correct answer for each: (a), (b), (c), or (d).

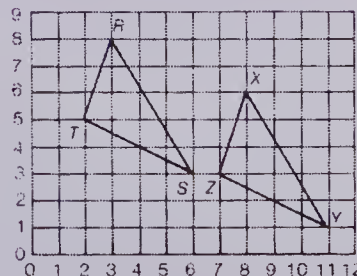
1. Which of these are prime numbers? **(a)**

2, 4, 6, 8

 (a) 2 (b) 4 (c) 6 (d) 8
2. The greatest composite number that is less than 20 is: **(b)**
 (a) 19 (b) 18 (c) 17 (d) 16
3. Which of these numbers is divisible by 5? **(b)**
 (a) 104 (b) 105 (c) 106 (d) 107
4. The greatest common factor of 10 and 30 is: **(d)**
 (a) 1 (b) 5 (c) 6 (d) 10
5. $4 \times 10^1 = \blacksquare$ **(c)**
 (a) 40 (b) 400 (c) 4000 (d) 640 000
6. $5^1 = \blacksquare$ **(b)**
 (a) 25 (b) 125 (c) 15 (d) 8
7. An equivalent fraction for $\frac{3}{5}$ is: **(d)**
 (a) $\frac{8}{5}$ (b) $\frac{6}{15}$ (c) $\frac{9}{25}$ (d) $\frac{18}{30}$
8. Which of the following fractions is greater than $\frac{3}{4}$? **(d)**
 (a) $\frac{6}{8}$ (b) $\frac{2}{3}$ (c) $\frac{15}{24}$ (d) $\frac{5}{6}$
9. $6\frac{1}{9}$ (a) $2\frac{7}{9}$ (b) $3\frac{1}{9}$
 $-3\frac{1}{4}$ (c) $3\frac{3}{8}$ (d) $9\frac{3}{8}$
(a)
10. $\frac{3}{8} - \frac{3}{4}$ **(c)**
 (a) $\frac{9}{16}$ (b) $\frac{9}{24}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$
11. $\frac{3}{7}$ of 28 **(b)**
 (a) 7 (b) 12 (c) 10 (d) 8
12. $\frac{4}{5} = \blacksquare$ **(c)**
 (a) 0.5 (b) 0.4 (c) 0.8 (d) 8.0
13. $\frac{7}{20} = \blacksquare$ **(d)** (a) 7% (b) 20% (c) 12% (d) 35%

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14. 9% as a decimal is: **(c)**
 (a) 9.0 (b) 0.9 (c) 0.09 (d) 90
15. 4% of \$150 **(b)**
 (a) \$600 (b) \$6 (c) \$60 (d) 600%
16. What is the average of 13, 40, 42, 5, 20, 18? **(a)**
 (a) 23 (b) 32 (c) 42 (d) 21
17. Use the scale 1:60. What is the actual height of this ladder? **(c)**

 (a) 20 cm (b) 18 cm (c) 180 cm (d) 57 cm
18. How much interest would be received on a \$500-deposit that earns 12% interest each year? **(c)**
 (a) \$12 (b) \$30 (c) \$60 (d) \$560
19. Linda bought a 10-speed bike for \$150. What was the final cost of the bike with a provincial sales tax rate of 7%? **(c)**
 (a) \$157 (b) \$160 (c) \$160.50 (d) \$139.50
20. Dave bought a bowl of soup for \$0.80, 2 sandwiches for \$0.65 each and a glass of milk for \$0.45. How much change should he get from a \$5-bill? **(a)**
 (a) \$2.55 (b) \$2.45 (c) \$4.10 (d) \$3.45
21. At an average speed of 14 km/h, how far could John cycle in 1.5 h? **(c)**
 (a) 7 km (b) 14 km (c) 21 km (d) 15.5 km
22. On the graph at the right, triangles RST and XYZ are related by a slide. Which vertex on RST matches vertex Z? **(d)**
 (a) Z (b) R (c) S (d) T
23. Which ordered pair names vertex R? **(a)**
 (a) (3, 8) (b) (8, 3) (c) (8, 6) (d) (3, 7)



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Extra Practice — Chapter One

Calculate.

1. (a) $\begin{array}{r} 8350 \\ 4259 \\ 7158 \\ + 2054 \\ \hline 21\ 821 \end{array}$ (b) $\begin{array}{r} 8563 \\ 285 \\ 4037 \\ + 74 \\ \hline 12\ 959 \end{array}$ (c) $\begin{array}{r} \$583.82 \\ 498.05 \\ 38\ 19 \\ 717.36 \\ + 300.98 \\ \hline \$\ 2138.40 \end{array}$ (d) $483 + 768 + 853 = 2104$

(e) $\$7653.82 + \$684.76 = \$8338.58$

2. (a) $\begin{array}{r} 469 \\ - 378 \\ \hline 91 \end{array}$ (b) $\begin{array}{r} 5716 \\ - 4839 \\ \hline 877 \end{array}$ (c) $\begin{array}{r} \$364.48 \\ - 87.39 \\ \hline \$\ 277.09 \end{array}$ (d) $6000 - 4876 = 1124$

(e) $\$8534.38 - \$5032.51 = \$3501.87$

3. (a) $\begin{array}{r} 48\ 57 \\ 9\ 35 \\ + 4\ 83 \\ \hline 62.75 \end{array}$ (b) $\begin{array}{r} 371.494 \\ 63.053 \\ + 156.04 \\ \hline 590.587 \end{array}$ (c) $\begin{array}{r} 5\ 852 \\ 3\ 950 \\ + 6\ 446 \\ \hline 16\ 248 \end{array}$ (d) $\begin{array}{r} 30\ 483\ 1 \\ 12\ 768\ 5 \\ 5\ 329\ 6 \\ + 86\ 476\ 8 \\ \hline 135.057 \end{array}$ (e) $\begin{array}{r} 9\ 487 \\ 3\ 017\ 4 \\ 8\ 15 \\ + 3\ 000\ 9 \\ \hline 23.6553 \end{array}$

4. (a) $\begin{array}{r} 8\ 4 \\ - 3\ 57 \\ \hline 4.83 \end{array}$ (b) $\begin{array}{r} 185\ 6 \\ - 84\ 93 \\ \hline 100.67 \end{array}$ (c) $\begin{array}{r} 63\ 49 \\ - 8\ 476 \\ \hline 55.014 \end{array}$ (d) $\begin{array}{r} 8563\ 7 \\ - 946\ 85 \\ \hline 7616.85 \end{array}$ (e) $\begin{array}{r} 476.03 \\ - 38\ 5 \\ \hline 437.53 \end{array}$

5. Write each number in words. (a) six million, four hundred eighty-three thousand, five hundred sixty-two (b) eight million, four hundred thousand, three hundred fifty-seven
(a) 6 483 562 (b) 8 400 357 (c) 81 305 768 (d) 753 000 500
(c) eighty-one million, three hundred five thousand, seven hundred sixty-eight (d) seven hundred fifty-three million, five hundred

6. Use = or \neq to make each number sentence true.
(a) $3+3+3 \neq 7$ (b) $438-78 \neq 360$ (c) $36+49 \neq 49+36$ (d) $83-69 \neq 24$

7. Copy and complete, using $>$ or $<$.
(a) $38 \bullet 38 >$ (b) $96 \bullet 12+9 >$ (c) $83 \bullet 83\ 74 <$ (d) $15-9 \bullet 7 <$

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Extra Practice — Chapter One

1. Write the value of each underlined digit.
(a) $\underline{3}201$ (2 hundreds) (b) $6\underline{0}7$ (0 tens) (c) $4\underline{3}29\ 1$ (4 thousands) (d) $7\underline{4}4\ 622$ (4 ten thousands) (e) $6.1\underline{1}3$ (3 thousandths)

Calculate.

2. (a) $\begin{array}{r} 3\ 112 \\ 0.218 \\ + 5.663 \\ \hline 8.993 \end{array}$ (b) $\begin{array}{r} 422.31 \\ 57.016 \\ + 313.9 \\ \hline 793.226 \end{array}$ (c) $\begin{array}{r} \$219.53 \\ 6.85 \\ + 2.99 \\ \hline \$\ 229.37 \end{array}$ (d) $7.8 + 0.811 = 8.611$

(e) $1.11 + 0.8 + 7.445 = 9.355$

3. (a) $\begin{array}{r} 45.941 \\ - 26.541 \\ \hline 19.400 \end{array}$ (b) $\begin{array}{r} \$68.50 \\ - 33.99 \\ \hline \$\ 34.51 \end{array}$ (c) $\begin{array}{r} 0.847 \\ - 0.608 \\ \hline 0.239 \end{array}$ (d) $0.9 - 0.052 = 0.848$

Round each number to the nearest value shown in brackets.

4. (a) 3540 (hundred) 3500 (b) 78 (ten) 80 (c) 8612 (thousand) 9000
(d) 442 881 (hundred thousand) 400 000 (e) 819 438 (ten thousand) 820 000

5. (a) 72 546 ($\frac{72.5}{10}$) (b) 9.117 ($\frac{9.12}{100}$) (c) 196 499 (whole number) 196
(d) 0.599 (whole number) 1 (e) 321.666 (hundredth) 321.67

6. Solve these equations.
(a) $\frac{8}{n} + 9 = 17$ (b) $\frac{\$46.25}{n} - \$12.75 = \$33.50$ (c) $56 + \frac{44}{n} = 100$
(d) $n - 28.5 = 63.8$ (e) $n + 4.113 = 6.8$
92.3 2.687

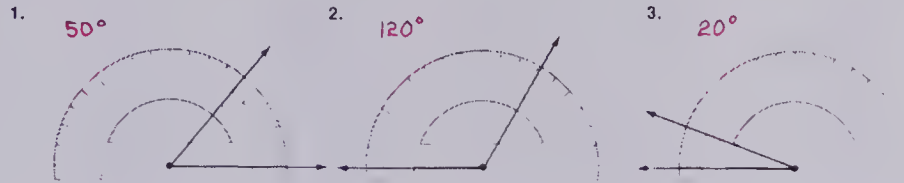
7. Arrange these numbers in order from smallest to largest.
(a) 91.9, 2.28, 615, 22.8, 15.15 2.28, 15.5, 22.8, 91.9, 615
(b) 45 637, 456.37, 45 637.1, 0.456 37, 45.637 1 0.456 37, 45.6371, 456.37, 45 637, 45 637.1

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Extra Practice—Chapter Two

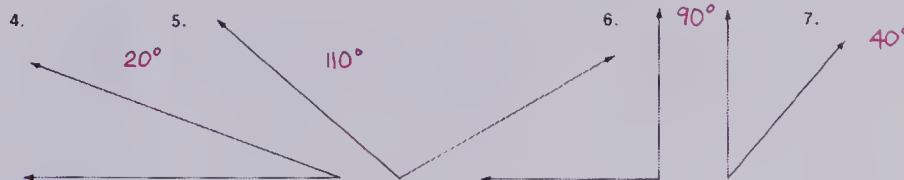
What is the measure of each angle?

35



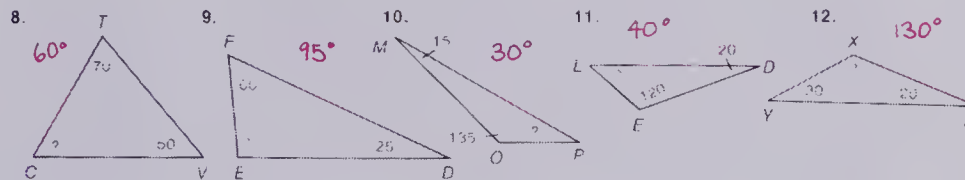
Use a protractor to measure each angle.

36



Calculate the measure of the third angle in each triangle.

43



Write an equation for each problem and solve.

$$132 - 6 = n; n = 22$$

$$12 + 36 = n; n = 48$$

53

13. Six forestry workers cut 132 logs in one day. On the average, how many logs did each cut?

14. There are 36 more firefighters than there are trucks. There are 12 trucks. How many firefighters are there?

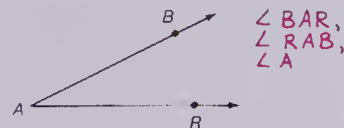
Extra Practice—Chapter Two

1. How long is the diameter?

44 mm

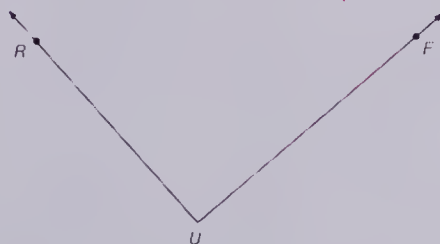


2. Name the angle three ways.



3. Use a protractor.

What is the measure of $\angle RUF$? 92°

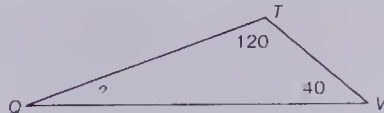


4. Match

- (a) 1. Straight angle
(b) 2. Obtuse angle
(c) 3. Right angle
(d) 4. Acute angle

5. Calculate the measure of $\angle Q$.

20°



6. Draw.

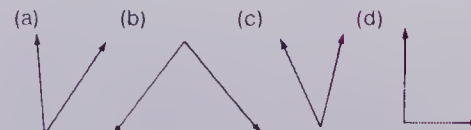
- (a) a pair of parallel lines
(b) a pair of intersecting lines
(c) a pair of perpendicular lines
(d) a circle with a radius 3 cm

(a) and (c)

7. Match.

- (a) Quadrilateral 1.
(b) Hexagon 2.
(c) Octagon 3.
(d) Pentagon 4.

8. Identify the congruent angles.



Extra Practice—Chapter Three

Calculate.

- | | | | | |
|---|---|---|---|---|
| 1. (a) $\begin{array}{r} 38 \\ \times 28 \\ \hline 1064 \end{array}$ | (b) $\begin{array}{r} 70 \\ \times 36 \\ \hline 2520 \end{array}$ | (c) $\begin{array}{r} 57 \\ \times 14 \\ \hline 798 \end{array}$ | (d) $\begin{array}{r} 39 \\ \times 54 \\ \hline 2106 \end{array}$ | (e) $\begin{array}{r} 68 \\ \times 71 \\ \hline 4828 \end{array}$ |
| 2. (a) $\begin{array}{r} 946 \\ \times 85 \\ \hline 80410 \end{array}$ | (b) $\begin{array}{r} 800 \\ \times 46 \\ \hline 36800 \end{array}$ | (c) $\begin{array}{r} 734 \\ \times 50 \\ \hline 36700 \end{array}$ | (d) $\begin{array}{r} \$4.39 \\ \times 86 \\ \hline \$377.54 \end{array}$ | (e) $\begin{array}{r} \$5.26 \\ \times 14 \\ \hline \$73.64 \end{array}$ |
| 3. (a) $\begin{array}{r} 543 \\ \times 482 \\ \hline 261726 \end{array}$ | (b) $\begin{array}{r} 985 \\ \times 217 \\ \hline 213745 \end{array}$ | (c) $\begin{array}{r} 646 \\ \times 702 \\ \hline 453492 \end{array}$ | (d) $\begin{array}{r} \$9.83 \\ \times 197 \\ \hline \$1936.51 \end{array}$ | (e) $\begin{array}{r} \$6.49 \\ \times 583 \\ \hline \$3783.67 \end{array}$ |
| 4. (a) $\begin{array}{r} 8432 \\ \times 100 \\ \hline 843200 \end{array}$ | (b) $\begin{array}{r} 9176 \\ \times 100 \\ \hline 917600 \end{array}$ | (c) $\begin{array}{r} 25 \\ \times 1000 \\ \hline 25000 \end{array}$ | (d) $\begin{array}{r} 768 \\ \times 1000 \\ \hline 768000 \end{array}$ | (e) $\begin{array}{r} 4105 \\ \times 1000 \\ \hline 4105000 \end{array}$ |
| 5. (a) $\begin{array}{r} 48.81 \\ \times 7 \\ \hline 341.67 \end{array}$ | (b) $\begin{array}{r} \$30.53 \\ \times 9 \\ \hline \$274.77 \end{array}$ | (c) $\begin{array}{r} 976.97 \\ \times 8 \\ \hline 7815.76 \end{array}$ | (d) $\begin{array}{r} 487.65 \\ \times 4 \\ \hline 1950.6 \end{array}$ | (e) $\begin{array}{r} 999.99 \\ \times 5 \\ \hline 4999.95 \end{array}$ |
| 6. (a) $\begin{array}{r} 347 \\ \times 0.8 \\ \hline 277.6 \end{array}$ | (b) $\begin{array}{r} 482 \\ \times 0.3 \\ \hline 144.6 \end{array}$ | (c) $\begin{array}{r} 7642 \\ \times 0.4 \\ \hline 3056.8 \end{array}$ | (d) $\begin{array}{r} 15476 \\ \times 0.7 \\ \hline 10833.2 \end{array}$ | (e) $\begin{array}{r} 36401 \\ \times 0.6 \\ \hline 21840.6 \end{array}$ |
| 7. (a) $\begin{array}{r} 58 \\ \times 1.52 \\ \hline 88.16 \end{array}$ | (b) $\begin{array}{r} 59 \\ \times 8.35 \\ \hline 492.65 \end{array}$ | (c) $\begin{array}{r} 865 \\ \times \$9.42 \\ \hline \$8148.30 \end{array}$ | (d) $\begin{array}{r} 1834 \\ \times 2.08 \\ \hline 3814.72 \end{array}$ | (e) $\begin{array}{r} 5819 \\ \times 6.19 \\ \hline 36019.61 \end{array}$ |
| 8. (a) $\begin{array}{r} 5.3 \\ \times 3.6 \\ \hline 19.08 \end{array}$ | (b) $\begin{array}{r} 8.5 \\ \times 9.4 \\ \hline 79.9 \end{array}$ | (c) $\begin{array}{r} 50.6 \\ \times 5.3 \\ \hline 268.18 \end{array}$ | (d) $\begin{array}{r} 468.7 \\ \times 3.9 \\ \hline 1827.93 \end{array}$ | (e) $\begin{array}{r} 309.4 \\ \times 6.7 \\ \hline 2072.98 \end{array}$ |
| 9. (a) $\begin{array}{r} 39.6 \\ \times 3.48 \\ \hline 137.808 \end{array}$ | (b) $\begin{array}{r} 83.4 \\ \times 6.59 \\ \hline 549.606 \end{array}$ | (c) $\begin{array}{r} 44.37 \\ \times 3.41 \\ \hline 151.3017 \end{array}$ | (d) $\begin{array}{r} 18.76 \\ \times 5.38 \\ \hline 100.9288 \end{array}$ | (e) $\begin{array}{r} 8.37 \\ \times 20.6 \\ \hline 172.422 \end{array}$ |
| 10. (a) $\begin{array}{r} 4859 \\ \times 706 \\ \hline 3430454 \end{array}$ | (b) $\begin{array}{r} 8005 \\ \times 4007 \\ \hline 32076035 \end{array}$ | (c) $\begin{array}{r} 9413 \\ \times 5037 \\ \hline 47413281 \end{array}$ | (d) $\begin{array}{r} 68342 \\ \times 7012 \\ \hline 479214104 \end{array}$ | (e) $\begin{array}{r} 98603 \\ \times 4318 \\ \hline 425767754 \end{array}$ |

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Extra Practice—Chapter Three

Calculate.

- | | | | | |
|--|---|---|--|---|
| 1. (a) $\begin{array}{r} 412 \\ \times 1 \\ \hline 412 \end{array}$ | (b) $\begin{array}{r} 613 \\ \times 10 \\ \hline 6130 \end{array}$ | (c) $\begin{array}{r} 589 \\ \times 1 \\ \hline 589 \end{array}$ | (d) $\begin{array}{r} 66 \\ \times 10 \\ \hline 660 \end{array}$ | (e) $\begin{array}{r} 876 \\ \times 1 \\ \hline 876 \end{array}$ |
| 2. (a) $\begin{array}{r} 29 \\ \times 100 \\ \hline 2900 \end{array}$ | (b) $\begin{array}{r} 149 \\ \times 100 \\ \hline 14900 \end{array}$ | (c) $\begin{array}{r} 99 \\ \times 1000 \\ \hline 99000 \end{array}$ | (d) $\begin{array}{r} 493 \\ \times 10 \\ \hline 4930 \end{array}$ | (e) $\begin{array}{r} 6000 \\ \times 1000 \\ \hline 6000000 \end{array}$ |
| 3. (a) $\begin{array}{r} 66 \\ \times 34 \\ \hline 2244 \end{array}$ | (b) $\begin{array}{r} 48 \\ \times 18 \\ \hline 864 \end{array}$ | (c) $\begin{array}{r} 59 \\ \times 31 \\ \hline 1829 \end{array}$ | (d) $\begin{array}{r} 28 \\ \times 37 \\ \hline 1036 \end{array}$ | (e) $\begin{array}{r} 75 \\ \times 25 \\ \hline 1875 \end{array}$ |
| 4. (a) $\begin{array}{r} 127 \\ \times 346 \\ \hline 43942 \end{array}$ | (b) $\begin{array}{r} 622 \\ \times 446 \\ \hline 277412 \end{array}$ | (c) $\begin{array}{r} 587 \\ \times 630 \\ \hline 369810 \end{array}$ | (d) $\begin{array}{r} 449 \\ \times 419 \\ \hline 188131 \end{array}$ | (e) $\begin{array}{r} 128 \\ \times 372 \\ \hline 47616 \end{array}$ |
| 5. (a) $\begin{array}{r} 391 \\ \times 105 \\ \hline 41055 \end{array}$ | (b) $\begin{array}{r} 442 \\ \times 303 \\ \hline 133926 \end{array}$ | (c) $\begin{array}{r} 626 \\ \times 603 \\ \hline 377478 \end{array}$ | (d) $\begin{array}{r} 499 \\ \times 509 \\ \hline 253991 \end{array}$ | (e) $\begin{array}{r} 906 \\ \times 500 \\ \hline 453000 \end{array}$ |
| 6. (a) $\begin{array}{r} \$33.50 \\ \times 4 \\ \hline \$134.00 \end{array}$ | (b) $\begin{array}{r} \$61.95 \\ \times 5 \\ \hline \$309.75 \end{array}$ | (c) $\begin{array}{r} \$66.88 \\ \times 3 \\ \hline \$200.64 \end{array}$ | (d) $\begin{array}{r} \$0.29 \\ \times 7 \\ \hline \$2.03 \end{array}$ | (e) $\begin{array}{r} \$55.25 \\ \times 9 \\ \hline \$497.25 \end{array}$ |
| 7. (a) $\begin{array}{r} 821 \\ \times 0.6 \\ \hline 492.6 \end{array}$ | (b) $\begin{array}{r} 488 \\ \times 0.4 \\ \hline 195.2 \end{array}$ | (c) $\begin{array}{r} 329 \\ \times 0.5 \\ \hline 164.5 \end{array}$ | (d) $\begin{array}{r} 3645 \\ \times 0.8 \\ \hline 2916 \end{array}$ | (e) $\begin{array}{r} 22027 \\ \times 0.7 \\ \hline 15418.9 \end{array}$ |
| 8. (a) $\begin{array}{r} 602 \\ \times 4132 \\ \hline 2487464 \end{array}$ | (b) $\begin{array}{r} 78042 \\ \times 708 \\ \hline 55253736 \end{array}$ | (c) $\begin{array}{r} 1.4132 \\ \times 0.03 \\ \hline 0.042396 \end{array}$ | (d) $\begin{array}{r} 9.17 \\ \times 2.3004 \\ \hline 21.094668 \end{array}$ | (e) $\begin{array}{r} 3.819 \\ \times 22.2 \\ \hline 84.7818 \end{array}$ |

Write an equation for each problem, and solve.

9. Jan bought 2 record albums. Each album was \$8.98. How much did she spend?
 $2 \times \$8.98 = n$; $n = \$17.96$

10. Derrick delivers 46 papers each day. How many does he deliver in six days?
 $46 \times 6 = n$; $n = 276$

Extra practice — chapter three 349

Extra Practice—Chapter Four

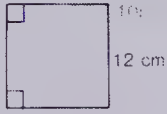
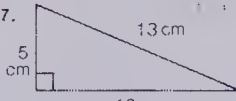

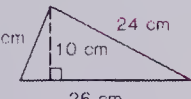
Calculate.

- (a) $10 \overline{)870}$ ⁸⁷ (b) $100 \overline{)2400}$ ²⁴ (c) $1000 \overline{)36\,000}$ ³⁶ (d) $1000 \overline{)521\,000}$ ⁵²¹
- (a) $8 \overline{)272}$ ³⁴ (b) $9 \overline{)1926}$ ²¹⁴ (c) $4 \overline{)2372}$ ⁵⁹³ (d) $5 \overline{)33\,705}$ ^{6\,741}
- (a) $56 \overline{)95\,437}$ ^{1704 R13} (b) $27 \overline{)76\,357}$ ^{2828 R1} (c) $85 \overline{)64\,768}$ ^{761 R83} (d) $93 \overline{)54\,376}$ ^{584 R64}
- (a) $7 \overline{)436.45}$ ^{62.35} (b) $3 \overline{)118.41}$ ^{39.47} (c) $6 \overline{)492.96}$ ^{82.16} (d) $9 \overline{)621.18}$ ^{69.02}

5. Copy and complete.

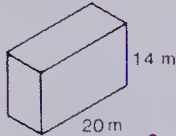
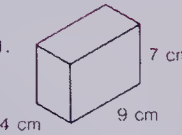
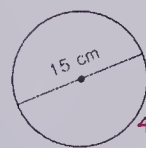

- (a) 5 cm = ■ mm ⁵⁰ (b) 8 km = ■ m ⁸⁰⁰⁰ (c) 3 km = ■ m ³⁰⁰⁰
 (d) 25 cm = ■ mm ²⁵⁰ (e) 60 cm = ■ m ^{0.6} (f) 1250 km = ■ m ^{1\,250\,000}
 (g) 750 mm = ■ cm ⁷⁵ (h) 900 m = ■ km ^{0.9} (i) 1000 mm = ■ m ¹
 (j) 2 L = ■ cm³ ²⁰⁰⁰ (k) 1 t = ■ kg ¹⁰⁰⁰ (l) 1 kL = ■ L ¹⁰⁰⁰

Calculate the perimeter and area of each.

-  ^{12 cm}
48 cm; 144 cm²
-  ^{5 cm, 12 cm, 13 cm}
30 cm; 30 cm²
-  ^{16 cm, 9 cm}
50 cm; 144 cm²
-  ^{26 cm, 10 cm, 24 cm}
62 cm; 130 cm²

Calculate the volume.

Calculate the circumference. Use $\pi = 3.14$.

-  ^{20 m, 8 m, 14 m}
2240 m³
-  ^{9 cm, 4 cm, 7 cm}
252 cm³
-  ^{15 cm}
47.1 cm
-  ^{8 cm}
50.24 cm

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Extra Practice—Chapter Four

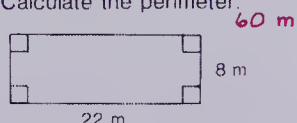
Calculate.

- (a) $90 \div 10$ ⁹ (b) $\$2600 \div 100$ ²⁶ (c) $4400 \div 100$ ⁴⁴ (d) $860\,000 \div 1000$ ⁸⁶⁰
- (a) $6 \overline{)1296}$ ²¹⁶ (b) $8 \overline{)4342}$ ^{542 R6} (c) $5 \overline{)2649}$ ^{529 R4} (d) $7 \overline{)83\,245}$ ^{11\,892 R1}
- (a) $34 \overline{)238}$ ⁷ (b) $52 \overline{)1404}$ ²⁷ (c) $93 \overline{)3908}$ ^{42 R2} (d) $39 \overline{)16\,088}$ ^{412 R20}
- (a) $81.6 \div 48$ ^{1.7} (b) $\$280.32 \div 6$ ^{46.72} (c) $56.52 \div 18$ ^{3.14} (d) $\$170.72 \div 44$ ^{3.88}

5. Copy and complete.

- (a) 2000 mm = ■ m ² (b) 500 cm = ■ m ⁵ (c) 2.5 km = ■ m ²⁵⁰⁰
 (d) 4 km = ■ m ⁴⁰⁰⁰ (e) 2000 kg = ■ t ² (f) 5000 L = ■ kL ⁵
 (g) 1 km + 2 km + 1 dam + 4 m = ■ m ¹²¹⁴ (h) 5 m + 3 dm + 6 cm + 4 mm = ■ m ^{5.364}

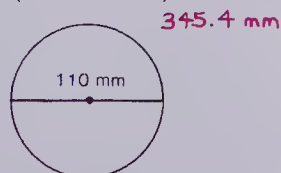
6. Calculate the perimeter.



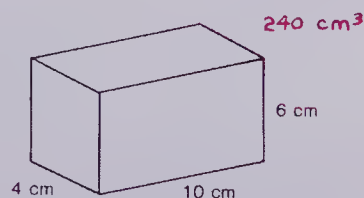
7. Calculate the area of the rectangle in Exercise 6.

176 m²

8. Calculate the circumference of the circle. (Use $\pi = 3.14$.)



9. Calculate the volume.



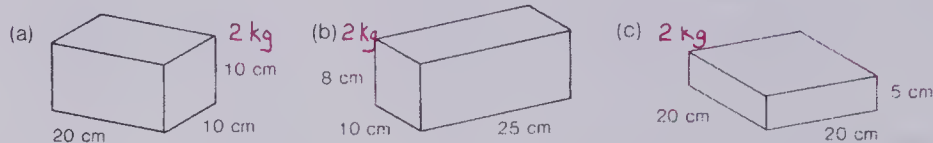
Extra practice — chapter four 351

Extra Practice—Chapter Five

Calculate

- (a) $43 - 0.01$ **43.00** (b) $509 - 0.1$ **509.0** (c) $617 - 0.01$ **617.00** (d) $3450 - 0.1$ **3450.0**
- (a) $0.6 \overline{) 438}$ **730** (b) $0.7 \overline{) 623}$ **890** (c) $5.6 \overline{) 336}$ **60** (d) $8.3 \overline{) 3569}$ **430**
- (a) $2.6 \overline{) 21.32}$ **8.2** (b) $4.7 \overline{) 25.38}$ **5.4** (c) $0.01 \overline{) 286.75}$ **28675** (d) $0.01 \overline{) 0.461}$ **46.1**
- (a) $0.001 \overline{) 13.56}$ **13560** (b) $4.2 \overline{) 1.47}$ **0.35** (c) $6.5 \overline{) 33.423}$ **5.142** (d) $64 \overline{) 16}$ **0.25**

5. 1 L of water has a mass of 1 kg
What is the mass of water that fills each carton?



6. An airplane left Vancouver at 06:00 for a direct flight to Toronto. It arrived in Toronto at 14:00. How long was the flying time? **5h**
7. A radio announcer in Winnipeg announces the time as "three thirty-two".
What would be the correct time for a listener in Prince Rupert? **one thirty-two**

Draw a picture for each problem. Solve.

8. Fee-Fie, the giant, is 273 cm tall.
Tom Thumb, the midget, is 87 cm tall.
How much taller is Fee-Fie than Tom Thumb? **186 cm**
9. A circular race track is 1.6 km long.
How far does a racing car go in 3 trips around the track? **4.8 km**

10. How much time from A to B?

A	03:15	02:30	09:40	23:10	23:45	21:11	14:14
B	07:45	05:10	14:10	00:10	01:15	02:23	07:07
	4h 30min	4h 30min	1h	1h 30min	5h 12min	16h 53min	

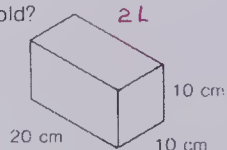
352 Extra practice — chapter five

Extra Practice—Chapter Five

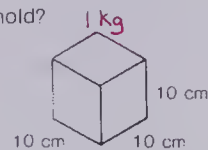
Divide

- (a) $0.2 \overline{) 76}$ **380** (b) $0.4 \overline{) 276}$ **690** (c) $0.9 \overline{) 252}$ **280** (d) $0.3 \overline{) 2982}$ **9940**
- (a) $1.5 \overline{) 510}$ **340** (b) $3.7 \overline{) 2257}$ **610** (c) $6.2 \overline{) 5766}$ **930** (d) $9.8 \overline{) 8624}$ **880**
- (a) $5.3 \overline{) 34.45}$ **6.5** (b) $4.6 \overline{) 30.82}$ **6.7** (c) $3.3 \overline{) 19.14}$ **5.8** (d) $7.5 \overline{) 60.75}$ **8.1**
- (a) $0.29 \overline{) 1.74}$ **6** (b) $0.51 \overline{) 3.06}$ **6** (c) $0.67 \overline{) 21.44}$ **32** (d) $0.94 \overline{) 25.38}$ **27**
- (a) $56 \overline{) 478.296}$ **8.541** (b) $48 \overline{) 695.328}$ **14.486** (c) $3.9 \overline{) 205.53}$ **52.7**

6. How many litres of water will this container hold?



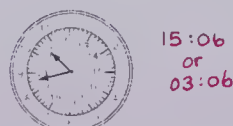
7. How many kilograms of water will this container hold?



8. Write the time 3 h 30 min after:



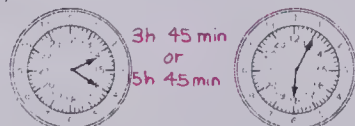
9. Write the time 4 h 23 min after:



10. How long

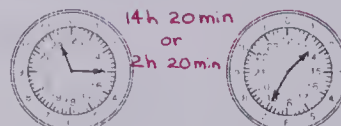
(a) from

to



(b) from

to



11. The temperature was 8°C ,
and then dropped 22°C .

What would be the new temperature? **-14°C**

12. The temperature was -12°C ,
and then rose 6°C .

What would be the new temperature? **-6°C**

Extra practice — chapter five 353

Extra Practice — Chapter Six

- Write 3 multiplication facts for:
 $1 \times 24 = 24$; $2 \times 12 = 24$; $1 \times 42 = 42$; $2 \times 21 = 42$; $1 \times 60 = 60$; $2 \times 30 = 60$; $1 \times 72 = 72$; $2 \times 36 = 72$;
 (a) 24 $3 \times 8 = 24$. (b) 42 $3 \times 14 = 42$. (c) 60 $3 \times 20 = 60$ (d) 72 $3 \times 24 = 72$.
 Also: $4 \times 6 = 24$. Also: $6 \times 7 = 42$ Also: $4 \times 15 = 60$; $5 \times 12 = 60$; Also: $4 \times 18 = 72$; $6 \times 12 = 72$;
 $6 \times 10 = 60$. $8 \times 9 = 72$
- List the set of factors for:
 (a) 18 $1, 2, 3, 6, 9, 18$ (b) 30 $1, 2, 3, 5, 6, 10, 15, 30$ (c) 40 $1, 2, 4, 5, 8, 10, 20, 40$ (d) 54 $1, 2, 3, 6, 9, 18, 27, 54$
- Write as a product of prime factors:
 (a) 20 $20 = 2 \times 2 \times 5$ (b) 22 $22 = 2 \times 11$ (c) 30 $30 = 2 \times 3 \times 5$ (d) 54 $54 = 2 \times 3 \times 3 \times 3$
 $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$ $90 = 2 \times 3 \times 3 \times 5$ (e) 108 $108 = 2 \times 2 \times 3 \times 3 \times 3$ (f) 121 $121 = 11 \times 11$
 (e) 64 (f) 90 (g) 108 (h) 121
- Write the numbers from this group that are prime numbers 2, 7, 17, 41
 2, 7, 8, 14, 17, 27, 35, 41, 48, 51, 57, 69
- Write the greatest common factor for each pair.
 (a) 10 and 30 10 (b) 14 and 35 7 (c) 12 and 42 6 (d) 20 and 30 10
- Write the least common multiple for each pair
 (a) 3 and 4 12 (b) 6 and 10 30 (c) 4 and 7 28 (d) 5 and 8 40
 (e) 3 and 12 12 (f) 6 and 15 30 (g) 8 and 12 24 (h) 12 and 20 60
- Evaluate.
 (a) 5×10^2 500 (b) $2^1 \times 3^2$ 72 (c) $5^2 \times 2^2$ 100
 (d) $(7 \times 10^3) + (2 \times 10)$ 7020 (e) $(4 \times 10^4) \times (3 \times 10^2) + (2 \times 1)$ 120 000 002
- Write each of the following in expanded notation.
 (a) 842 $= (8 \times 10^2) + (4 \times 10^1) + (2 \times 1)$ (b) 1982 $= (1 \times 10^3) + (9 \times 10^2) + (8 \times 10^1) + (2 \times 1)$ (c) 100 025 $= (1 \times 10^5) + (2 \times 10^2) + (5 \times 1)$
 (a) 842 (b) 1982 (c) 100 025

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Extra Practice — Chapter Six

- Write 3 multiplication facts for:
 $1 \times 16 = 16$; $2 \times 8 = 16$; $1 \times 48 = 48$; $2 \times 24 = 48$; $1 \times 84 = 84$; $2 \times 42 = 84$; $1 \times 100 = 100$; $2 \times 50 = 100$;
 (a) 16 $4 \times 4 = 16$. (b) 48 $3 \times 16 = 48$. (c) 84 $3 \times 28 = 84$. (d) 100 $4 \times 25 = 100$.
 Also: $4 \times 12 = 48$; $6 \times 8 = 48$. Also: $4 \times 21 = 84$; $6 \times 14 = 84$; $7 \times 12 = 84$. Also: $5 \times 20 = 100$; $10 \times 10 = 100$.
- Write the set of factors for:
 (a) 28 $1, 2, 4, 7, 14, 28$ (b) 32 $1, 2, 4, 8, 16, 32$ (c) 48 $1, 2, 3, 4, 6, 8, 12, 16, 24, 48$ (d) 64 $1, 2, 4, 8, 16, 32, 64$
- Write as a product of prime factors.
 (a) 21 $21 = 3 \times 7$ (b) 25 $25 = 5 \times 5$ (c) 38 $38 = 2 \times 19$ (d) 50 $50 = 2 \times 5 \times 5$
 (e) 54 $54 = 2 \times 3 \times 3 \times 3$ (f) 75 $75 = 3 \times 5 \times 5$ (g) 84 $84 = 2 \times 2 \times 3 \times 7$ (h) 96 $96 = 2 \times 2 \times 2 \times 2 \times 3$
- Write the numbers from the group below that are:
 (a) divisible by 2 $12, 18, 22, 36, 48, 54, 60$ (b) divisible by 3 $12, 15, 18, 36, 48, 54, 60$ (c) divisible by 4 $12, 36, 48, 60$ (d) divisible by 9 $18, 36, 54$
 12, 15, 18, 22, 36, 48, 54, 60
- Write the greatest common factor for each pair.
 (a) 8 and 20 4 (b) 15 and 21 3 (c) 28 and 42 14 (d) 10 and 25 5
- Write the least common multiple for each pair.
 (a) 5 and 9 45 (b) 10 and 12 60 (c) 8 and 12 24 (d) 20 and 30 60
- Evaluate. Write as a single numeral.
 (a) 10^3 1000 (b) $3^2 \times 10^2$ 900 (c) $5^2 \times 10^3$ 25 000 (d) 7×10^6 7 000 000
 (a) 139 $= (1 \times 10^2) + (3 \times 10^1) + (9 \times 1)$ (b) 3005 $= (3 \times 10^3) + (5 \times 1)$
 (c) 84 000 $= (8 \times 10^4) + (4 \times 10^3)$ (d) 90 003 $= (9 \times 10^4) + (3 \times 1)$
- Write in expanded notation.
 (a) 139 (b) 3005 (c) 84 000 (d) 90 003

Extra practice — chapter six 355

Extra Practice—Chapter Seven

1. Replace each \bullet , using $<$ or $>$ 188

(a) $\frac{1}{2} \bullet \frac{2}{5} >$ (b) $\frac{3}{4} \bullet \frac{7}{10} >$
 (c) $\frac{5}{8} \bullet \frac{3}{5} >$ (d) $\frac{1}{4} \bullet \frac{1}{3} <$

2. Complete to make equivalent fractions. 188

(a) $\frac{4}{5} = \frac{\blacksquare 24}{30}$ (b) $\frac{1}{6} = \frac{\blacksquare 5}{30}$
 (c) $\frac{5}{8} = \frac{\blacksquare 35}{56}$ (d) $\frac{9}{7} = \frac{\blacksquare 36}{28}$

Calculate. 187 191

3. (a) $\frac{7}{8} - \frac{1}{8} = \frac{6}{8} = \frac{3}{4}$ (b) $\frac{3}{4} + \frac{3}{4} = 1\frac{1}{2}$ (c) $\frac{5}{9} + \frac{4}{9} = 1$ (d) $\frac{5}{8} - \frac{1}{3} = \frac{7}{24}$ (e) $\frac{1}{6} + \frac{4}{5} = \frac{29}{30}$ (f) $\frac{9}{10} - \frac{3}{4} = \frac{3}{20}$

4. (a) $\frac{11}{12} + \frac{5}{8} = 1\frac{13}{24}$ (b) $\frac{7}{8} - \frac{1}{6} = \frac{17}{24}$ (c) $2\frac{1}{3} + 5\frac{1}{6} = 7\frac{3}{6} = 7\frac{1}{2}$ (d) $3\frac{4}{5} - 1\frac{1}{2} = 2\frac{3}{10}$ (e) $7\frac{3}{8} + 5\frac{3}{5} = 12\frac{39}{40}$ (f) $3\frac{2}{3} - 1\frac{8}{8} = 1\frac{16}{24} = 1\frac{2}{3}$

5. (a) $\frac{3}{4}$ of 28 21 (b) $\frac{2}{5}$ of 40 16 (c) $\frac{1}{2}$ of 7 $3\frac{1}{2}$ (d) $\frac{2}{5} \times \frac{5}{8} = \frac{1}{4}$

6. (a) $\frac{7}{12} \times \frac{4}{5} = \frac{7}{15}$ (b) $3 \times 2\frac{1}{2} = 7\frac{1}{2}$ (c) $5\frac{1}{4} \times 1\frac{1}{7} = 6$ (d) $1\frac{1}{5} \times 6\frac{2}{3} = 8$

7. (a) $7 \div \frac{1}{3} = 21$ (b) $\frac{3}{4} \div \frac{1}{4} = 3$ (c) $\frac{5}{8} \div \frac{1}{3} = 1\frac{7}{8}$ (d) $\frac{8}{9} \div 2 = \frac{4}{9}$

8. Write as a mixed numeral. 197

(a) $\frac{17}{4} = 4\frac{1}{4}$ (b) $\frac{25}{8} = 3\frac{1}{8}$
 (c) $\frac{83}{10} = 8\frac{3}{10}$ (d) $\frac{17}{16} = 1\frac{1}{16}$

9. Write as a decimal. 200 215

(a) $\frac{7}{10} = 0.7$ (b) $\frac{19}{20} = 0.95$
 (c) $\frac{8}{25} = 0.32$ (d) $\frac{5}{11} = 0.4545\ldots$

10. Solve for N. 207

(a) $N + \frac{2}{3} = 1\frac{1}{3}$ (b) $\frac{7}{8} - N = \frac{1}{4}$ (c) $N + \frac{1}{4} = \frac{7}{12}$
 (d) $\frac{1}{3} \times N = 4$ (e) $N \times \frac{1}{2} = \frac{1}{6}$

Extra Practice—Chapter Seven

1. Write the reciprocals for each. 209

(a) $\frac{2}{5} = \frac{5}{2}$ (b) $\frac{3}{4} = \frac{4}{3}$
 (c) $\frac{7}{4} = \frac{4}{7}$ (d) $3 = \frac{1}{3}$

2. Complete to make equivalent fractions. 188

(a) $\frac{1}{2} = \frac{\blacksquare 5}{10}$ (b) $\frac{3}{5} = \frac{\blacksquare 24}{40}$
 (c) $\frac{7}{9} = \frac{35}{\blacksquare 45}$ (d) $\frac{5}{4} = \frac{\blacksquare 25}{20}$

Calculate. 187 191

3. (a) $\frac{7}{10} - \frac{1}{10} = \frac{6}{10} = \frac{3}{5}$ (b) $\frac{9}{10} - \frac{7}{10} = \frac{2}{10} = \frac{1}{5}$ (c) $\frac{7}{8} + \frac{1}{8} = 1$ (d) $\frac{4}{5} - \frac{1}{3} = \frac{7}{15}$ (e) $\frac{5}{6} + \frac{2}{5} = \frac{37}{30} = 1\frac{7}{30}$ (f) $\frac{7}{10} + \frac{1}{4} = \frac{19}{20}$

4. (a) $\frac{5}{12} + \frac{3}{8} = \frac{19}{24}$ (b) $\frac{5}{4} - \frac{5}{6} = \frac{5}{12}$ (c) $2\frac{1}{2} + 3\frac{1}{6} = 5\frac{4}{6} = 5\frac{2}{3}$ (d) $5\frac{7}{8} - 2\frac{3}{4} = 3\frac{1}{8}$ (e) $1\frac{5}{6} + 4\frac{2}{5} = 6\frac{7}{30}$ (f) $7\frac{4}{5} - 3\frac{1}{3} = 4\frac{7}{15}$

5. (a) $\frac{2}{3}$ of 18 12 (b) $\frac{4}{5}$ of 30 24 (c) $\frac{1}{3} \times 5 = 1\frac{2}{3}$ (d) $\frac{3}{5} \times \frac{5}{6} = \frac{1}{2}$

6. (a) $\frac{7}{10} \times \frac{5}{8} = \frac{7}{16}$ (b) $3\frac{1}{2} \times 1\frac{1}{3} = 4\frac{2}{3}$ (c) $3\frac{1}{3} \times 1\frac{1}{5} = 4$ (d) $7\frac{2}{3} \times 2 = 15\frac{1}{3}$

7. (a) $3 - \frac{1}{2} = 2\frac{1}{2}$ (b) $\frac{4}{5} - \frac{1}{5} = \frac{3}{5}$ (c) $\frac{3}{4} - \frac{1}{2} = \frac{1}{4}$ (d) $\frac{6}{7} - 3 = -2\frac{5}{7}$

8. Solve for N. 207

(a) $\frac{3}{8} + N = \frac{7}{8}$ (b) $\frac{7}{10} - N = \frac{1}{2}$ (c) $9 \times N = 3$

9. Write as a decimal. 200 215

(a) $\frac{17}{100} = 0.17$ (b) $\frac{4}{5} = 0.8$
 (c) $\frac{7}{25} = 0.28$ (d) $\frac{43}{50} = 0.86$

10. Write as a mixed numeral. 197

(a) $\frac{50}{9} = 5\frac{5}{9}$ (b) $\frac{19}{5} = 3\frac{4}{5}$
 (c) $\frac{35}{8} = 4\frac{3}{8}$ (d) $\frac{43}{7} = 6\frac{1}{7}$

11. Write as an improper fraction. 197

(a) $7\frac{3}{5} = \frac{38}{5}$ (b) $3\frac{5}{8} = \frac{29}{8}$ (c) $2\frac{3}{4} = \frac{11}{4}$ (d) $5\frac{5}{6} = \frac{35}{6}$

Extra Practice—Chapter Eight

- Write two equivalent ratios for each.

(a) 7 to 10 $\frac{14}{20}, \frac{21}{30}, \text{etc.}$ (b) 3 to 8 $\frac{6}{16}, \frac{9}{24}, \text{etc.}$ (c) $\frac{9}{12} = \frac{3}{4}, \frac{6}{8}, \text{etc.}$ (d) 7 to 5 $\frac{14}{10}, \frac{21}{15}, \text{etc.}$
- Find the missing term in these equivalent ratios.

(a) $\frac{3}{8} = \frac{\square}{40}$ $\frac{15}{40}$ (b) $\frac{10}{3} = \frac{\square}{12}$ $\frac{40}{12}$ (c) $\frac{5}{12} = \frac{40}{\square}$ $\frac{96}{12}$ (d) $\frac{11}{4} = \frac{33}{\square}$ $\frac{33}{12}$
- Calculate cross products, and state which pairs of ratios are equivalent. (a), (c), and (d)

(a) $\frac{3}{5}, \frac{12}{20}$ $3 \times 20 = 60$ $5 \times 12 = 60$ (b) $\frac{12}{8}, \frac{4}{3}$ $12 \times 3 = 36$ $8 \times 4 = 32$ (c) $\frac{3}{10}, \frac{21}{70}$ $3 \times 70 = 210$ $10 \times 21 = 210$ (d) $\frac{12}{8}, \frac{6}{4}$ $12 \times 4 = 48$ $8 \times 6 = 48$
- Copy and complete these ratios charts.

(a)

Jumbo paper cups	12	2	4	6	1	10	50	100
Total cost	84				7			

(b)

Pens	3	9	12		28	30
Total cost	57			323	418	
- Express each as a percent.

(a) $\frac{3}{10}$ 30% (b) $\frac{17}{20}$ 85% (c) $\frac{14}{25}$ 56% (d) $\frac{3}{4}$ 75%
 (e) 0.37 37% (f) 0.09 9% (g) 0.3 30% (h) 0.9 90%
- Express as decimals.

(a) 32% 0.32 (b) 50% 0.5 (c) 5% 0.05 (d) 7% 0.07
- Express as reduced ratios.

(a) 60% $\frac{3}{5}$ (b) 25% $\frac{1}{4}$ (c) 52% $\frac{13}{25}$ (d) 85% $\frac{17}{20}$
- Calculate.

(a) 10% of 35 3.5 (b) 6% of 50 3 (c) 1% of 900 9
 (d) 12% of 60 7.2 (e) 8% of \$7.50 $\$0.60$ (f) 20% of \$18.50 $\$3.70$
- Find the actual dimensions of this room.

scale ratio is 1:80.
 4 cm 320 cm
 6 cm 480 cm
- Tina received the following marks on some math tests: 85%, 72%, 68%, 75%. Calculate her average mark. 75%

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Extra Practice—Chapter Eight

- Write three equivalent ratios for each.

(a) $\frac{8}{10}, \frac{12}{15}, \frac{16}{20}, \text{etc.}$ (b) $\frac{5}{9}, \frac{10}{18}, \frac{15}{27}, \frac{20}{36}, \text{etc.}$ (c) $\frac{7}{3}, \frac{14}{6}, \frac{21}{9}, \frac{28}{12}, \text{etc.}$ (d) $\frac{6}{5}, \frac{12}{10}, \frac{18}{15}, \frac{24}{20}, \text{etc.}$
- Copy and complete.

(a) $\frac{2}{7} = \frac{\square}{21}$ $\frac{6}{21}$ (b) $\frac{9}{4} = \frac{\square}{20}$ $\frac{45}{20}$
 (c) $\frac{5}{11} = \frac{20}{\square}$ $\frac{44}{11}$ (d) $\frac{10}{3} = \frac{50}{\square}$ $\frac{15}{3}$
- Calculate cross products and state which pairs of ratios are equivalent. (a), (c), and (d)

(a) $\frac{1}{2}, \frac{9}{18}$ $1 \times 18 = 18$ $2 \times 9 = 18$ (b) $\frac{2}{3}, \frac{12}{21}$ $2 \times 21 = 42$ $3 \times 12 = 36$ (c) $\frac{3}{8}, \frac{9}{24}$ $3 \times 24 = 72$ $8 \times 9 = 72$ (d) $\frac{12}{16}, \frac{15}{20}$ $12 \times 20 = 240$ $16 \times 15 = 240$
- Express each as a percent.

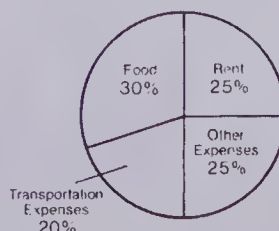
(a) $\frac{1}{4}$ 25% (b) $\frac{11}{20}$ 55% (c) $\frac{21}{25}$ 84% (d) $\frac{4}{5}$ 80%
 (e) 0.18 18% (f) 0.07 7% (g) 0.01 1% (h) 0.5 50%
- Express as decimals.

(a) 91% 0.91 (b) 40% 0.4 (c) 8% 0.08 (d) 2% 0.02
- Express as reduced ratios.

(a) 30% $\frac{3}{10}$ (b) 75% $\frac{3}{4}$ (c) 45% $\frac{9}{20}$ (d) 88% $\frac{22}{25}$
- Calculate.

(a) 30% of \$800 $\$240$ (b) 7% of \$60 $\$4.20$ (c) 15% of \$90 $\$13.50$
 (d) 20% of \$42.50 $\$8.50$ (e) 10% of \$75 $\$7.50$ (f) 6% of \$23.50 $\$1.41$
- During seven consecutive exercise days, Saline did the following situps: 12, 17, 23, 20, 21, 30, and 31. What was the average number of situps per day for that week? 22
- If the Gervais family monthly income after taxes is \$1400, calculate the amount spent each month on:

(a) rent $\$350$ (b) food $\$420$ (c) transportation $\$280$ (d) other expenses $\$350$



Extra practice — chapter eight 359

Extra Practice — Chapter Nine

248

1. Calculate the balance after each entry.

246

	Date	Item	Withdrawals	Deposits	Balance
	April 20	Bal. Fwd.			138.09
(a)	April 24		10.76		■ 127.33
(b)	April 27			20.00	■ 147.33
(c)	April 28		15.85		■ 131.48
(d)	April 30		7.98		■ 123.50

2. Calculate the interest per year.

- (a) \$50 at 13% **\$6.50**
 (b) \$100 at 14% **\$14**
 (c) \$6000 at 15% **\$900**
 (d) \$75 at 12% **\$9**
 (e) \$54.50 at 14% **\$7.63**

Copy and complete these charts.

251

- 3.

	Purchase Price	Regular Price	Discount Price	Discount Amount	Sale Price
(a)	bicycle	\$200	15%	■	■
(b)	winter coat	\$75	40%	■	■
(c)	shoes	\$27	25%	■	■
(d)	record	\$9.50	30%	■	■
(e)	sweater	\$37.50	50%	■	■

- \$30; \$170**
\$30; \$45
\$6.75; \$20.25
\$2.85; \$6.65
\$18.75; \$18.75

250

- 4.

	Purchase Price	Sales Tax Rate	Sales Tax Amount	Total Cost
(a)	\$50	7%	■ \$3.50	■ \$53.50
(b)	\$22.50	8%	■ \$1.80	■ \$24.30
(c)	\$250.50	6%	■ \$15.03	■ \$265.53
(d)	\$18 000	7%	■ \$12.60	■ \$19 260
(e)	\$235.25	8%	■ \$18.82	■ \$254.07

5. A jogger was running at 20 km/h. At this rate, how long (in minutes) would it take to run:

- (a) 1 km **3 min** (b) 5 km **15 min** (c) 10 km **30 min**
 (d) 50 km **150 min** (e) 13 km? **39 min**

261

6. A leaky tap drips 14 L/d.

How much water would be saved in a year (365 d) if this tap were repaired? **5110 L**

270

Extra Practice — Chapter Nine

248

1. Calculate the balance after each entry.

246

	Date	Item	Withdrawals	Deposits	Balance
	April 1	Bal. Fwd.	—		\$82.23
(a)	April 3	—	—	\$9.50	■ \$91.73
(b)	April 8	—	—	\$4.25	■ \$95.98
(c)	April 12	—	\$5.95	—	■ \$90.03
(d)	April 16	—	—	\$10.75	■ \$100.78

2. Calculate the interest.

- (a) \$500 at 10% **\$50**
 (b) \$800 at 12% **\$96**
 (c) \$2000 at 14% **\$280**
 (d) \$25 000 at 15% **\$3750**

Copy and complete these charts.

250

- 3.

	Purchase Price	Sales Tax Rate	Sales Tax Amount	Total Cost
(a)	\$30	7%	■	■
(b)	\$8.50	8%	■	■
(c)	\$135.50	6%	■	■
(d)	\$8000	7%	■	■
(e)	\$1500	8%	■	■

- \$2.10; \$32.10**
\$0.68; \$9.18
\$8.13; \$143.63
\$560; \$8560
\$120; \$1620

251

- 4.

	Regular Price	Discount Rate	Discount Amount	Sale Price
(a)	\$200	30%	■ \$60	■ \$140
(b)	\$25	20%	■ \$5	■ \$20
(c)	\$36	15%	■ \$5.40	■ \$30.60
(d)	\$75	35%	■ \$26.25	■ \$48.75
(e)	\$127.50	50%	■ \$63.75	■ \$63.75

5. Mr. Savatini calculates that he should save 35% on his home-heating bill by better insulating his home.

His normal yearly heating cost is \$860.

How much will he save by insulating? **\$301**

270

Extra Practice — Chapter Ten

- Solve. Mark each solution on a number line.
 - $N + 3 = 7$
 $N = 4$
 - $N - 2 = 4$
 $N = 6$
 - $6 + N = 8$
 $N = 2$
 - $N + 8 = 8$
 $N = 0$
- Write a related subtraction sentence for each. Solve.
 - $W + 14 = 34$
 $W = 20$
 - $R + 3.2 = 7.6$
 $R = 4.4$
- Write a related addition sentence for each. Solve.
 - $N - 12 = 14$
 $N = 26$
 - $M - 4.2 = 6.5$
 $M = 10.7$
- Solve. Graph the solution on a number line marked in tenths.
 - $M = 2.1 + 15$
 - $N - 3.1 = 16$
 $N = 19.1$
- Write a related division sentence for each. Solve.
 - $M \times 3 = 12$
 $M = 4$
 - $N \times 7 = 147$
 $N = 21$
- Write a related multiplication sentence for each. Solve.
 - $N - 3 = 13$
 $N = 39$
 - $N - 1.2 = 8.4$
 $N = 10.08$
- Use graph paper to graph these points. Connect the points as indicated.
 Join: $(-1, -3), (-1, -2), (-3, -2), (-3, -1), (-2, -1), (-3, 0), (-3, 1), (-4, 1), (-3, 2), (-3, 3), (-2, 4), (0, 4), (-1, 3), (-2, 1), (-1, -1), (-1, 3)$.
 Make a large dot at $(-2, 2)$.
 Join: $(0, 1), (0, 2), (-2, 1), (0, 0)$.
- 750 birds were counted altogether. How many evening grosbeaks were counted? **75**

Evening Grosbeaks
- Draw a broken line graph to show the accumulated centimetres of snowfall.

Time	04:00	08:00	12:00	16:00	20:00	00:00
Centimetres	0	10	45	75	90	96

Calculate.

- $3 + 7 = 10$
- $2 + 6 = 8$
- $5 + 2 = 7$
- $3 + 2 = 5$
- $3 - 5 = -2$
- $3 - 1 = 4$
- $3 - 5 = -8$
- $3 - 5 = -2$

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Extra Practice — Chapter Ten

Solve. Mark the solution on a number line.

- $3 + N = 7$
 $N = 4$
- $12 + N = 15$
 $N = 3$
- $N + 33 = 33$
 $N = 0$
- $N - 3 = 1$
 $N = 4$
- $N - 2 = 5$
 $N = 7$
- $N - 0 = 4$
 $N = 4$
- $W - 5 = 0$
 $W = 5$
- $T + 4 = 9$
 $T = 5$

Write a related subtraction sentence for each. Solve.

- $N + 8 = 17$
 $N = 17 - 8; N = 9$
- $W + 7 = 23$
 $W = 23 - 7; W = 16$
- $X + 12 = 48$
 $X = 48 - 12; X = 36$
- $Y + 26 = 42$
 $Y = 42 - 26; Y = 16$
- $M + 2.3 = 6.8$
 $M = 6.8 - 2.3; M = 4.5$
- $M + 8.9 = 27.8$
 $M = 27.8 - 8.9; M = 18.9$
- $W + 23.3 = 46.1$
 $W = 46.1 - 23.3; W = 22.8$
- $Z + 42.3 = 106.2$
 $Z = 106.2 - 42.3; Z = 63.9$

Write a related addition sentence for each. Solve.

- $A - 3 = 9$
 $A = 9 + 3; A = 12$
- $B - 8 = 26$
 $B = 26 + 8; B = 34$
- $C - 15 = 32$
 $C = 32 + 15; C = 47$
- $E - 38 = 56$
 $E = 56 + 38; E = 94$
- $H - 4.1 = 6.2$
 $H = 6.2 + 4.1; H = 10.3$
- $J - 6.7 = 12.3$
 $J = 12.3 + 6.7; J = 19.0$
- $K - 16.2 = 18.8$
 $K = 18.8 + 16.2; K = 35.0$
- $M - 56.9 = 56.9$
 $M = 56.9 + 56.9; M = 113.8$

Write a related division sentence for each. Solve.

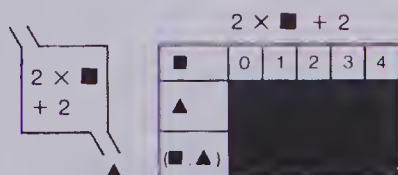
- $A \times 2 = 8$
 $A = 8 \div 2; A = 4$
- $D \times 3 = 27$
 $D = 27 \div 3; D = 9$
- $N \times 8 = 72$
 $N = 72 \div 8; N = 9$
- $N \times 12 = 144$
 $N = 144 \div 12; N = 12$
- $K \times 15 = 300$
 $K = 300 \div 15; K = 20$
- $N \times 1.2 = 1.44$
 $N = 1.44 \div 1.2; N = 1.2$
- $M \times 2.5 = 62.5$
 $M = 62.5 \div 2.5; M = 25$
- $N \times 3.1 = 93$
 $N = 93 \div 3.1; N = 30$

Write a related multiplication sentence for each. Solve.

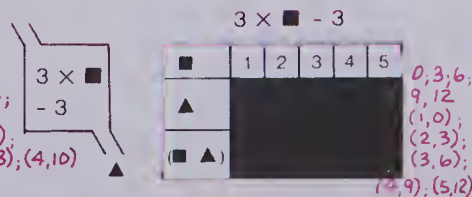
- $B - 2 = 12$
 $B = 12 \times 2; B = 24$
- $C \div 5 = 15$
 $C = 15 \times 5; C = 75$
- $N - 6 = 17$
 $N = 17 \times 6; N = 102$
- $M - 12 = 10$
 $M = 10 \times 12; M = 120$
- $N - 1.5 = 4.5$
 $N = 4.5 \times 1.5; N = 6.75$
- $M - 1.3 = 3.9$
 $M = 3.9 \times 1.3; M = 5.07$
- $K - 2.8 = 8.96$
 $K = 8.96 \times 2.8; K = 25.088$
- $M - 0.6 = 0.9$
 $M = 0.9 \times 0.6; M = 0.54$

Copy and complete each table.

41.



42.



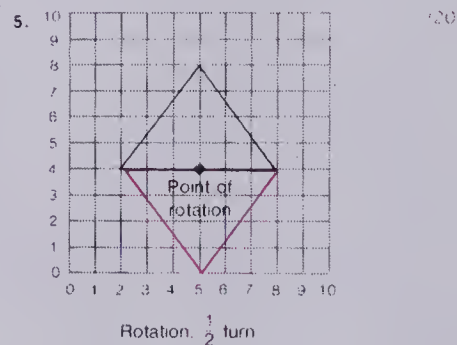
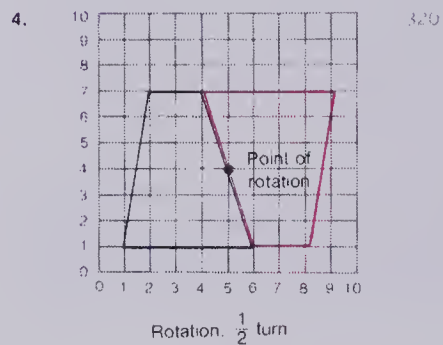
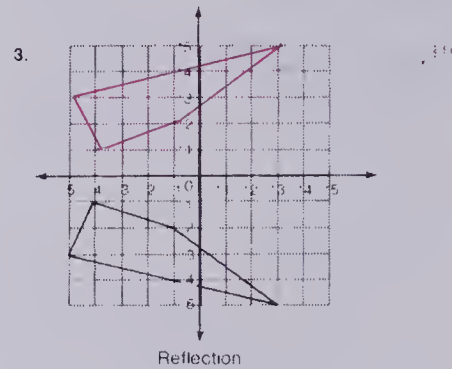
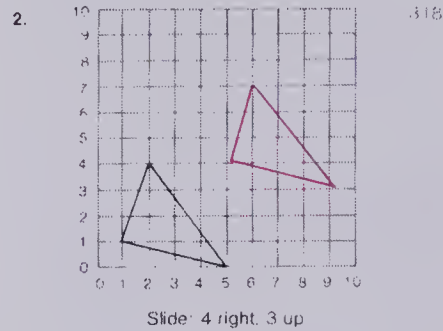
Extra practice — chapter ten 363

Extra Practice—Chapter Eleven

- Use this shape.
Make a pattern that has rotational symmetry.



Copy and draw the images as indicated.

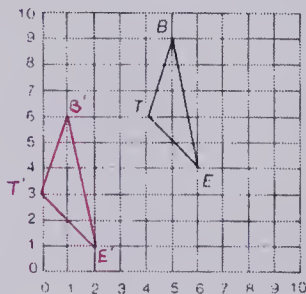


- Draw a segment of any length. 325
Construct the perpendicular bisector.
- Draw any angle. 327
Construct the bisector of the angle.

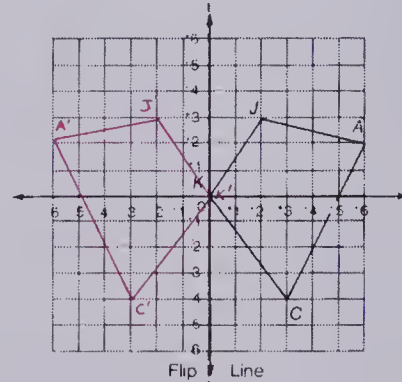
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Extra Practice—Chapter Eleven

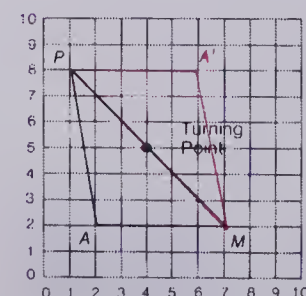
- Copy $\triangle BET$ on graph paper. 318
Draw the image for the slide
left 4, down 3.



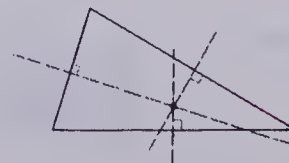
- Copy quadrilateral JACK. 316
Draw the image for a reflection
over the flip line.



- Copy $\triangle PAM$. Draw the image 320
for a rotation of $\frac{1}{2}$ turn about
the turning point.



- Draw any triangle. 326-327
Construct the perpendicular
bisectors of each side, so that
the bisectors intersect.
Repeat with other triangles.
Do the bisectors always
intersect at one point? Yes



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